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THE NEW PROCESSING METHOD OF QUASI-PERIODIC PULSE SIGNALS USING WAVELET ANALYSIS AND HERMIT TRANSFORM

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A B S T R A C T

The article considers the basic principles of constructing a method for processing quasi-periodic pulse signals. The method is based on the combined use of wavelet analysis and the Hermite transform, in particular, the Gauss-Hermite function. Wavelet transform is considered as a cross-correlation function. The Gauss-Hermite functions are used as a basis in the wavelet analysis. Approbation of the method is carried out as on a test signal, in the form of a rectangular pulse with additive noise, which at some point in time has a local inhomogeneity, also on the real signal received from the bearings of the gas turbine engine.

1. INTRODUCTION

The analysis of form and parameters of quasi-periodic pulse signals against the background of noise, which is a slowly changing process, is an integral part of the information processing procedure in order to diagnose the state of an object. For example, signals appear during the functioning of dynamic systems: various machines and mechanisms, living organisms and other objects. In most cases, these objects operate in a cyclical manner. In mechanical systems, this is due to the presence of rotating parts, in a living organism - the leading centers of nervous excitation. Very often, such systems are subject to random disturbances. In particular, in a biological object due to various pathologies, disturbances in the rhythm of its functioning occur. In mechanical systems due to interaction with the environment, under the influence of a time factor or other reasons, disturbances in the cyclicity of work occur. If these violations go beyond the permissible limits, then there is a reason to draw a conclusion about the malfunction of machines and mechanisms or about a person's disease.

The most common processing method to state the value of quasiperiodic impulse systems is spectral analysis (Barkova et al., 2019). Due to well-studied mathematical apparatus, it allows you to detect various defects in the system. Spectral analysis confidently copes with the task of diagnosing dynamic systems if the defect is of a periodic nature. This is expressed by the characteristic frequency components in the spectrum of the signal under investigation. But if the defect is not periodic or quasi-periodic, has a random character, then the efficiency of the spectral analysis is significantly reduced.

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This problem can be partially solved using the windowed Fourier transform. In this case, the frequency resolution depends on the duration of the window, and the dynamics of the system functioning is assessed by using the average frequency. It is difficult to estimate the quasiperiodicity of the system from the average frequency, thereby reducing the accuracy of its diagnostics. In addition, using this processing method, we are faced with the uncertainty of the choice of the window width. With a small width, we lose exactly in frequency. With a large width, the estimate of quasiperiodicity is lost.

Thus, to solve the above problem, it is necessary to measure the distance between quasiperiodic pulses. Then we are faced with the need to adapt to the shape of the impulses so that the assessment of the dynamics of their repetition is more accurate. This is complicated by the fact that the shape of the pulses is constantly changing, i.e. the system under study is nonstationary.

The method that is based on the combined use of wavelet analysis (Jiang & Mahadevan, 2011) and Hermite transform was developed to eliminate the above disadvantages.

2. BASIC PRINCIPLES OF THE METHOD

The traditional interpretation of the wavelet transform is based on the consideration of basis functions as time-limited oscillation segments. Variation their scale is equivalent to changing the frequency composition of these functions (Vorobyov & Gribunin, 1999).

$$W_s(a,\tau) = \int_{-\infty}^{\infty} S_{in}(t)\psi_{a,\tau}(t)dt \qquad (1)$$

As can be seen from (1), the wavelet coefficients in the general case are determined by the integral transformation of the signal. Continuous wavelet transform $W_s(a,\tau)$ is the scalar product of the process under study $S_{in}(t)$ and basis functions $\psi_{a,\tau}(t)$.

The basic functions are real, defined on a certain interval and fluctuate around the abscissa axis:

$$\psi_{a,\tau}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-\tau}{a}\right) \qquad (2)$$

τ – shift parameter, shows the location in time, a – scale parameter.

The pulses of a quasi-periodic signal can have an arbitrary shape. Therefore, for more efficient diagnostics, it is necessary to use orthogonal basis functions. The most popular orthogonal wavelets are the Haar and Daubechies (Daubechies, I., & Heil, 1992). They have a number of limitations and disadvantages. Haar wavelets poorly describe smooth functions, and Daubechies functions have an asymmetric shape, which narrows the area of their practical use. Thus, with the help of classical wavelets, it is not possible to take into account all the features of the signal shape. In addition, when processing real signals, it is necessary to construct basis functions based on the discrete recording of the signal, and this is absent in the classical interpretation of the wavelet transform. As a basic function, it is proposed to use the Gauss-Hermite functions (FGH), defined in the Hermite transform. The energy of the FGH is concentrated on a limited interval in both the time and frequency domains. Therefore, FGH work with a finer time-frequency localization. This representation of the process under study is less robust to noise, but it retains a much larger number of characteristic features of the signal. The inherent localization of the FGH in the time domain makes them very suitable for representing the bearing support signals in the form of a generalized Fourier series based on these functions.

The FGH have the following form (Martens, J-B., 1990):

$$\Psi_n(t) = H_n(t)\exp\left(-0.5t^2\right)/\sqrt{n!2^n\sqrt{\pi}} \qquad (3)$$

where $H_n(t)$ – Hermite polynomials, n – FGH order.

Transformation (1) can be considered as a cross-correlation function of the signal and the basis function sliding over it. The higher the upper limit of the correlation integral, the greater the degree of similarity between the signal and the function. Taking into account the above said, substituting expression (3) into expression (1) taking into account expression (2), we have the following correlation integral (Balakin & Shtykov, 2019):

$$R_{out}(a,\tau) = \int_{-\infty}^{\infty} S_{in}(t)\tilde{S}_{ptrn}(a,t-\tau)dt \qquad (4)$$

where $\tilde{S}_{ptrn}(a,t)$ – basic function.

In turn, the basic function has the following form:

$$\tilde{S}_{ptrn}(a,t) = \frac{1}{\sqrt{a}}\sum_{n=0}^{\infty} W(n,n_c)A_n(a)\Psi_n(t/a) \qquad (5)$$

where $A_n(a)$ – the spectrum of the signal in the FGH basis, $W(n,n_c)$ – the smoothing window for weakening the Gibbs effect in the FGH space, n – the number of the FGH.

Let us consider a test example of the calculation: a rectangular pulse with additive noise, which at some point in time has a local inhomogeneity in the form of a third order FGH with a fixed scale parameter (figure 1, a).

Since the inhomogeneity is characterized by one feature, the correlation integral (4) has the following form:

$$R_{out}(a,\tau) = \int_{-\infty}^{\infty} S_{in}(t)\Psi_3(a,t-\tau)dt \qquad (6)$$

By varying the scale parameter, the duration of the local inhomogeneity and its location can be found. The results of such a procedure are reflected in the form of level lines (figure 1, b). Having found the local maximum of the surface in figure 1, b, one can find the location (t = 0) of the local inhomogeneity and its duration 0.2. The result of calculating the correlation integral (6) with the scale parameter $a = 0.2$ is shown in figure 1, c.

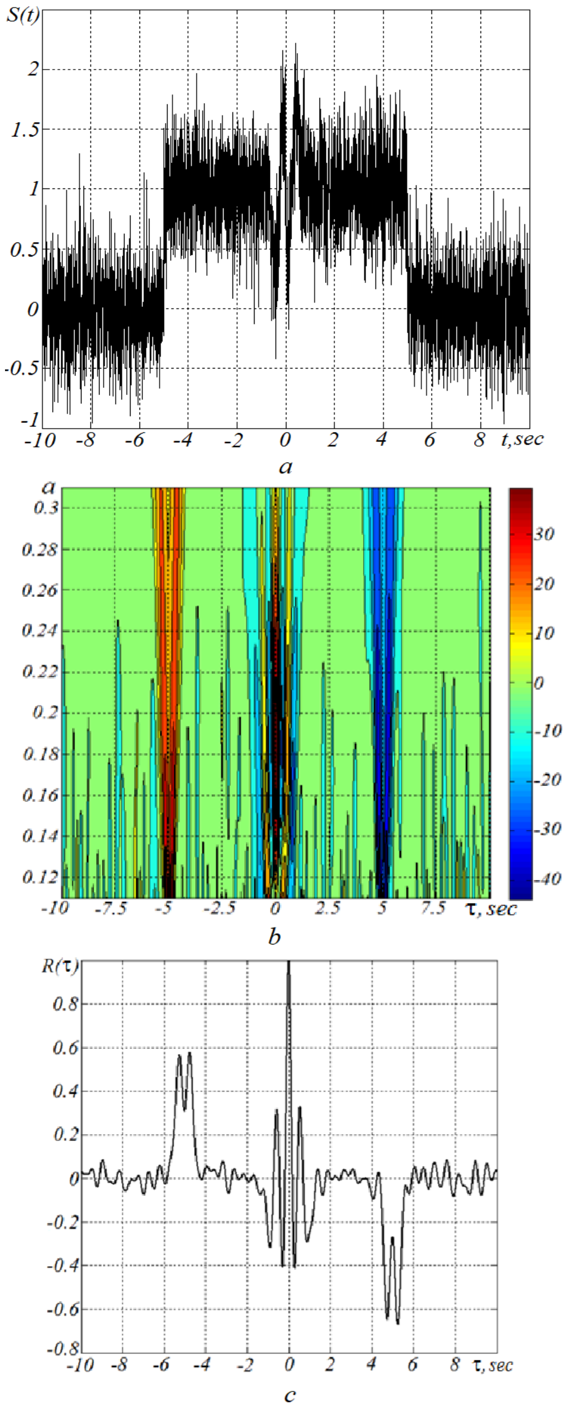

Figure 1. Processing results: a) - Rectangular pulse with a singularity in the form of a third order FGH, b) - the result of calculating the cross correlation function.

We can compare the obtained result with the classical wavelet transform. Figure 2 shows the wavelet spectrograms of a rectangular pulse, constructed with various basic functions. The construction of the basic functions was carried out with the standard tools of the MATLAB package, so the values of the scale parameter are different. It is advisable in this case to compare not quantitatively, but qualitatively: to identify the fine structures of the process under study.

Local inhomogeneity is more confidently distinguished by processing using the MHAT function, since it is adapted for analyzing complex signals due to its narrow energy spectrum and two moments (zero and first) equal to zero.

The key difference between the two processing approaches is that at the output of the algorithm under consideration we have a cross correlation function, which allows us to estimate the degree of similarity of the reference signal and the reference, and at wavelet processing, the spectral distribution.

The main idea of the processing method is as follows: from the investigated quasi-periodic pulse signal, we select a fragment, on the basis of which we construct the basis function (5). A fragment can represent a local inhomogeneity, the dynamics of which we want to trace. Next, we calculate the correlation integral (4), where the result is a cross correlation function, which is a complex surface with many local extrema, due to the scale variation and time shift. Supposing that a working system operates cyclically, we determine the distance between each extremum. If this distance is in the limit of the confidence interval, then the system is in good order, otherwise it is not. It is convenient to reflect such a dynamic picture using scatterograms. The speed of calculating the correlation integral can be increased by going to the frequency domain.

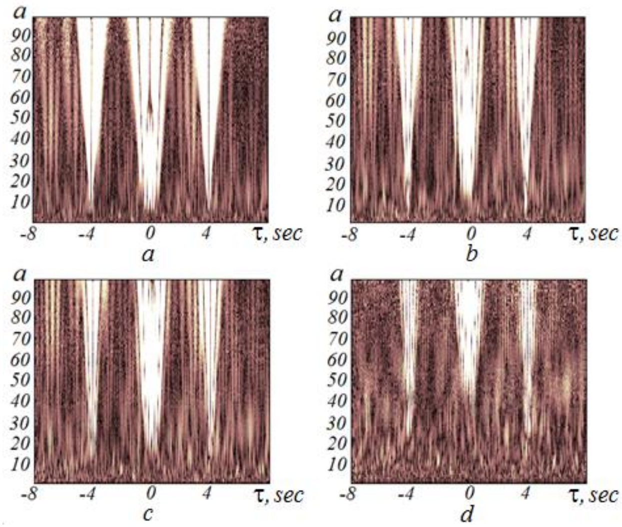

Figure 2. The results of processing using: a – MHAT function, b – Gauss 3 orders, c – Gauss 4 orders, d – Morlet wavelet

For this, it is necessary to replace the operation of calculating the correlation integral by filtering in the spectral region. This procedure is carried out using the generalized Rayleigh formula. As a result, the correlation integral has the following form:

$$R_{out}(a,\tau)=\frac{1}{2\pi\sqrt{a}}\int_{-\infty}^{\infty}\dot{S}_{in}(\omega)\dot{K}_{n}(a\omega)\exp(j\omega\tau)d\omega \quad (7)$$

where $\dot{S}_{in}(\omega)$ – complex Fourier spectrum, $\dot{K}_{n}(a\omega)$ – complex filter gain, which is

$$\dot{K}_{n}(a\omega)=j^{n}\sqrt{\frac{\sqrt{\pi}}{n!2^{n-1}}}\exp\left(-0,5a^{2}\omega^{2}\right)H_{n}(a\omega) \quad (8)$$

Let us consider in more detail the operation of the method using the example of bearings of rotor supports.

3. SIGNAL PROCESSING OF BEARINDS OF A GAS TURBINE ENGINE

A gas turbine engine is a complex technical system with many different vibration sources (Balakin et al., 2021). One of these sources is the rotor bearing arrangements, the failure of which interrupts the further operation of the gas turbine engine. In good condition, the bearing arrangements function cyclically. However, it is at an early stage of development that the cyclicity is violated, which manifests itself in the form of quasi-periodicity of pulse repetition. Let us estimate the quasiperiodicity using the method presented.

Figure 3 shows fragments of three bearings. The first one is in good condition, the other two are defective. In accordance with the processing method, first one need to isolate the local heterogeneity from the process under study.

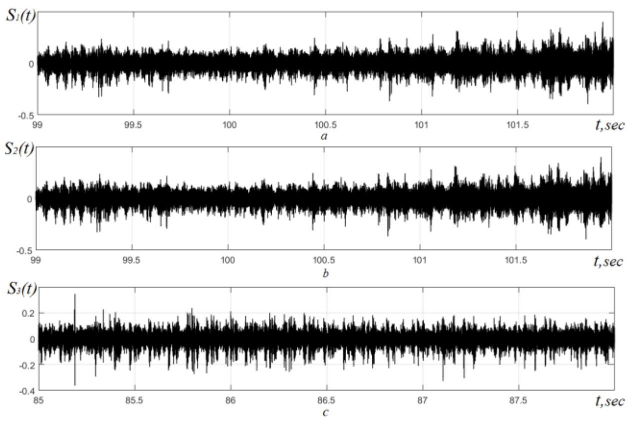

Figure 3. Bearing vibration recordings: a – no damage, b and c – there is damage

After analyzing the records of a bearing without a defect, a fragment was selected, which is shown in figure. 4, since it is most often found in the record.

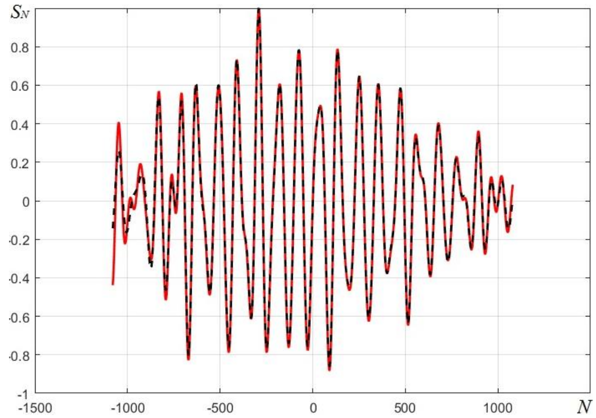

Figure 4. Basic function (black dash-dotted line), selected fragment (red solid), N - number of samples

Based on (5), we construct a basis function from a fragment of a record of a serviceable bearing. Given the approximation error, one can find the optimal number of FGH in (5). Figure 5 shows a 5% approximation error for various parameters of the scale and the number of FGH.

The approximation error in the general case can be calculated using the following formula:

$$Er(a,n)=\left(\int_{-\infty}^{\infty}\left(\tilde{S}_{ptrn}(a,t)-S_{ptrn}(t)\right)^{2}dt\Big/\int_{-\infty}^{\infty}S_{ptrn}(t)^{2}dt\right)100\% \quad (9)$$

Then in accordance with (4), we construct the correlation function. Fragments of the cross-correlation function for bearing signals are shown in figure 5. By finding the distance between the extrema, for example, using the steepest descent method, one can display the dynamics of pulse repetition or quasiperiodicity.

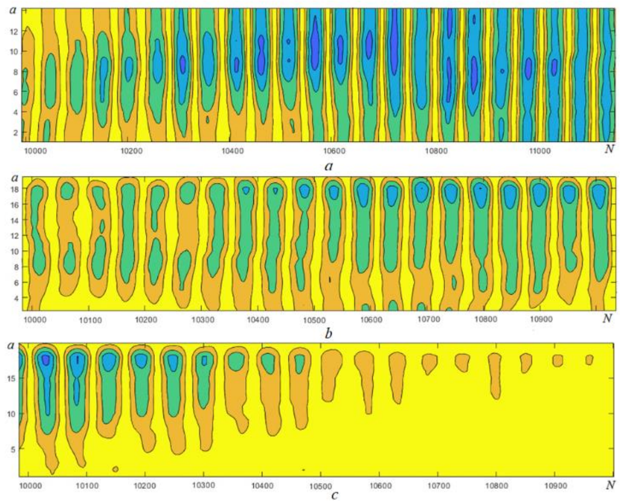

Figure 5. The result of calculating the cross-correlation function: a – no damage, b and c – there is damage

It is convenient to display quasi-periodicity using rhythmograms and scatterograms. More details about the procedure for constructing a rhythmogram and a scatterogram can be found in (Balakin et al., 2022).

4. CONCLUSIONS

The presented processing results both on the test signal and on the real signal allow us to state that the method based on the combined use of the wavelet transform and the Hermite transform can be applied to detect and estimate the quasiperiodicity of various local inhomogeneities. In turn, inhomogeneities can characterize both the correct operation of the system under study and the system defects. The choice depends on the thing, the dynamics of which process the researcher wants to trace. The key feature of the method is that it is possible to display the dynamics of pulse repetition of arbitrary shape in order to diagnose quasiperiodic pulse systems. The obtained processing results confirm the state of the system under study, in particular the bearing arrangements. The presented method in turn can be applied as an additional diagnostic tool to the proven classical spectral analysis.

References:

Balakin, D., & Shtykov, V. (2019). Using rhythmograms to diagnose mechanical systems *Journal of Physics: Conference Series*, *1399*, 1-6. doi: 10.1088/1742-6596/1399/4/044027

Balakin, D., Shtykov, V., Zubko, A., Shalimova, E., & Zayed, A. (2022). Principles of Diagnosing: The Technical Condition of the Bearing of the Gas Turbine Engine Support Using Rhythmogram and Scatterogram *Advances in Turbomachinery*, doi: 10.5772/intechopen.108400

Balakin, D., Zubko, A., Zubko, A., & Shtykov, V. (2021). Vibration diagnostics of gas turbine engines bearing assemblies technical condition with rhythmograms and scatterograms *Aerospace MAI Journal*, *28*(4), 151-162. doi: 10.34759/vst-2021-4-151-162

Barkova, N., Barkov, A., & Grishchenko, D. (2019). Vibration diagnostics of equipment units with gas turbine engines *Vibroengineering PROCEDIA*, *25*, 89-94. doi: 10.21595/vp.2019.20723

Daubechies, I., & Heil, C. (1992). Ten Lectures on Wavelets *Computers in Physics and IEEE Computational Science & Engineering*, *6*, 697. doi.org/10.1063/1.4823127

Jiang, X., & Mahadevan, S. (2011). Wavelet spectrum analysis approach to model validation of dynamic systems. *Mechanical Systems and Signal Processing*, *25*(2), 575-590.. doi:10.1016/j.ymssp.2010.05.012

Martens, J-B. (1990). The Hermite transform-theory *IEEE Transactions on Acoustics, Speech and Signal Processing*, *38*, 1595-1606. doi: 10.1109/29.60086

Vorobyov, V., & Gribunin, V. (1999). Theory and practice of wavelet transform *St. Petersburg: VUS*, 204 p.

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