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The (In)Efficiency of Emerging and Developed Markets: An Analysis from Fractal Theory

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ABSTRACT

The objective of this article is to study the behavior of the stock markets in emerging countries (BRICS) and developed countries (USA, England, Germany, and Japan), aiming to identify the evolution of their degree of efficiency between June 2007 and July 2021 based on the hypothesis of fractal markets. Using the Hurst exponent, the fractal dimension, and entropy approximation, it was built a market efficiency index. Among the main results, inconstancy of the efficiency indices over time was identified, which is consistent with previous studies within the field of econophysics. In addition, most of the inefficiency is due to the presence of deterministic elements in asset price variations, with a similarity in the efficiency level between the groups of emerging and developed countries, except for the case of China, which presented a singular behavior, which motivates new studies in this market. Finally, the results indicate the relevance of the cointegration effects of the analyzed markets, which is reflected in the inefficiencies of these markets over time.

INTRODUCTION

Since the 1960s, researchers in the field of finance have sought to develop theories that explain the behavior of financial assets and their agents, with the efficient market hypothesis (EMH), formalized by Fama (1970), as the main result. However, recent empirical studies have indicated that asset prices do not behave in a purely random way, highlighting issues related to informational asymmetry and divergence of investors' expectations (Kahneman & Tversky, 2013; Shiller, 2003). In this context, new theoretical currents have been developed to explain market behavior, providing a counterpoint to EMH.

From a neopositivist perspective, econophysics emerges as a branch of market efficiency analysis, which is based on the fractal characteristics of the market and chaos theory (Peters, 1994). In this perspective of the so-called fractal markets hypothesis (FMH), the market has random movements in the short run due to new information; however, patterns of behavior could be identified in the long run (Karp & Van Vuuren, 2019). In this way, asset price movements would be based on moments of local mean holding or reversal over time.

Models based on this hypothesis have a more flexible character compared to those based on the EMH, since they do not presuppose any a priori economic theory to prepare their analyses. In this way, the theories are elaborated a posteriori, always based on the actual data analyzed, which makes this approach have the potential to complement the economic literature (Jovanovic & Schinckus, 2013).

In this context, this paper is guided by the following question: How did the degree of market efficiency, based on fractal analysis, evolve in emerging and developed countries during the period analyzed? Based on this question, the objective of this paper was to investigate the behavior of the stock market of emerging countries (BRICS) and developed countries (USA, England, Germany, and Japan), aiming to identify the (co)evolution of their degree of efficiency from a longitudinal perspective based on the fractal markets hypothesis, considering the sample window from June 2007 to July 2021.

To this end, the methodology proposed by Kristoufek and Vosvrda (2014) was used, with the construction of an efficiency index with moving windows, in order to capture short- and long-term memory effects, as well as the entropy of the time series of market returns. Thus, it was sought to analyze the hypothesis that the markets of developed and emerging countries follow the behavior advocated by the EMH in different time windows, against the hypothesis that they have inefficiencies in their joint movements (comovements).

Among the research justifications, there is no consensus in the literature on the issue of market efficiency, and, according to Karp and Van Vuuren (2019), the identification of market behavior in different time windows is relevant for resource allocation, for investment strategies, and for understanding the dynamics of the comovements of emerging and developed countries. According to Mensi et al. (2016), both in market and economic terms, the BRICS requires specific studies, considering its role in the world economy and its peculiarities. Additionally, the identification of possible differences in the behavior of the degree of efficiency between emerging and developed markets is opportune.

Furthermore, the present study focuses on a longitudinal analysis of a data sample exceeding 10 years, both for the efficiency index generated and for the elements that compose it. Thus, the scope of the work is to understand how the degree of efficiency of the selected markets has behaved over the years, as well as to perform a comparative analysis of the results between the groups of emerging and developed countries.

From a practical point of view, this study also enables the identification of less efficient markets under the EMH perspective. In this context, it can assist in the selection of the market to invest in, given the greater chance of arbitrage. From a theoretical point of view, the study aims to validate the analyses of previous studies as well as deepen their discussions about the behavior of emerging and developed markets in light of EMH. Finally, it also aims to contribute to the literature in methodological terms by exploring the fractal strand of time series analysis, which is still little used in finance studies in the national literature.

THEORETICAL BACKGROUND

Efficient market hypothesis — EMH

The EMH theory is derived from Bachelier's (1900) speculation theory, which uses a random walk model to describe the behavior of asset prices in the market. In this way, prices reflect all available information, plus a random effect of several external factors. Expectations of the economic value and cash flow generation of companies are quickly incorporated into market prices, which makes it impossible to obtain abnormal profits consistently (Fama, 1970).

Three main conditions can be defined for the existence of an efficient market: the absence of transaction costs; informational symmetry; and consensus on the effects of this information on asset pricing. In this sense, the value of assets would reflect all past information and future expectations, which would be homogeneous, so that the estimates made by market agents would be the best approximation of the intrinsic value of securities in the market (Fama, 1970). In his seminal paper, Fama (1970) also presented three forms of market efficiency:

1. *Weak form*: past price information for an asset is incorporated into market prices, and there is no margin for predicting future prices, because asset pricing is based on a random walk model.
2. *Semi-strong form*: the market promotes the incorporation of public information into asset prices quickly and accurately.
3. *Strong form*: presupposes the inclusion of private and secret information in asset prices, where not even insiders could obtain abnormal returns, thus making arbitrage in the market impossible.

Later, Fama (1991) reviewed studies that analyzed EMH, which led to criticisms of his theory. Thus, the author stated that, given the reality of the market, there are indeed some momentary imperfections, but in the long run, and with reduced transaction costs and liquidity in the price discovery mechanism, EMH can consider information asymmetry and transaction costs, thus making its assumptions valid, at least for the weak version of efficiency formalized in Fama (1970).

Fractal theory and fractal markets hypothesis — FMH

According to Peters (1994), fractal theory refers to a relationship of local randomness in conjunction with global determinism, thus associating itself with the principles of chaos theory. Applying this theory to financial markets, and considering information as a random element, the way the market will interpret it would follow a deterministic pattern. This idea was the basis for the formalization of the fractal markets hypothesis (FMH) (Mandelbrot, 1999).

Under this hypothesis, the market, even with random movements in the very short term, maintains its general structure of evolution as it expands over the time horizon in a manner similar to a fractal. Thus, the theory proposes that the market assumes long-term patterns similar to those of the medium term, which, in turn, also assume patterns similar to those of the short term, presenting persistence or reversal movements to the averages that cause the variations in the prices.

One of the pioneering studies for the FMH conception was that of Elliott (1938), who analyzed capital market behavior using the theory of fractals. In the end, the author identified a cycle of five trend waves (points 1 to 5) and correction movements (points a, b, c), representing self-similarity patterns typical of a fractal (Kotyrba et al., 2013). Figure 1 exemplifies a basic Elliott wave pattern. In general, the FMH relates existing upswings and downswings in different periods to make an estimate of future price cycles, assuming their structure is maintained over the long term.

Figure 1. Example of a cyclic motion described by Elliot.
Source: Own elaboration.

The applicability of FMH in empirical work in finance has been used, above all, to verify levels of market efficiency. Studies that have applied this theory to the stock market, such as those by Nekrasova et al. (2018), Caporale et al. (2016), Kristoufek and Vosvrda (2014), and Kristoufek and Vosvrda (2013), have mainly used three metrics to measure market efficiency, and which will be presented below.

The first of these is the Hurst exponent (H), a parameter associated with the level of autocorrelation and long-term persistence. Its value varies between 0 and 1, with values below 0.5 indicating a negative long-term correlation; values above 0.5 indicate a positive long-term correlation; and, with $H = 0$, there is an absence of long-term memory, which is associated with a purely random process, expected in an efficient market.

The second analysis metric is the fractal dimension (*D*) able to measure the effect of local persistence. In an efficient market, its value would be 1.5, indicating the absence of short-term correlations. With values below 1.5, a local persistence movement is observed, while with $D > 1.5$, a local antipersistence effect is perceived, which is reflected as short-term mean reversion movements. This measure is also associated with capturing the herd effect in financial markets (Kristoufek & Vosvrda, 2013).

It is also noteworthy that this measure was originally calculated according to topographic, texture, and multidimensional irregularity analyses. Thus, adaptations for the calculations of this metric for financial time series were necessary (Gneiting et al., 2012).

By the process of self-similarity, that is, the identification of patterns independently of the scale analyzed, $H + D = 2$, that is, one could describe short-term memory as a function of long-term memory and vice versa. However, this property is not completely observed in financial time series, because although it is possible to identify patterns of movements in asset prices, there is no exact maintenance of these patterns, and there is the presence of external shocks that can change the trend of these time series (Kristoufek & Vosvrda, 2014).

Finally, the entropy approximation proposed by Pincus (1991) was included, aiming to capture the effects of the magnitude of randomness present in the series of returns. According to Pincus and Kalman (2004), entropy is a measure that captures the complexity of a system. In a random system, the entropy of the system will be high, while in a deterministic system, it will be low. In this respect, a completely efficient financial market will have maximum entropy, while a completely inefficient market will have no entropy at all.

Recent literature

From the analysis of short-term and long-term memory, Kristoufek and Vosvrda (2013) created a ranking for the market efficiency of 41 countries. The Japanese stock market was found to have the highest efficiency. In addition, the indices of markets in Eurozone countries showed the highest concentration of efficient markets, while the indices of Latin American, Asian, and Oceania markets showed the worst indices. This conclusion is in line with the findings of studies such as those of Oprean and Tănăsescu (2013) and Cajueiro and Tabak (2004). Furthermore, it was found that short-term memory was the major cause of the inefficiency of the analyzed indices.

Ikeda's (2017) study considered 137 globally relevant market indices through *H* analysis in simulations via bootstrap. His findings highlight the oscillations of the metric over the years analyzed, with changes in the long-term memory of the time series. Furthermore, the series analyses enabled the verification of expected FMH behaviors for the market indices under consideration.

The study by Karp and Van Vuuren (2019), in addition to providing a literature review of the evolution of FMH comparing it to EMH, analyzes the *H* and *D* of the American, British, and South African markets. Among the most important points of the paper, it highlights that the magnitudes of variations in the *Ds* impacted the magnitudes of price variations in the markets. In addition, it was possible to notice that the tendency was to present greater changes at times of maintenance of the long-term trend.

Miloş et al. (2020) analyzed the returns of seven Central and Eastern European markets via detrended fluctuation analysis (DFA), finding evidence of inefficiencies in these markets. Nevertheless, the authors hypothesize that the level of development of countries can impact the behavior of the *H*. However, the authors hypothesize that the level of development of countries may impact the behavior of the DFA, especially with regard to its autocorrelation, which, when negative, implies cyclical movements regarding market efficiency.

More recently, Lahmiri and Bekiros (2020) studied the entropy and volatility of several markets, including financial and cryptocurrency markets, in periods before and after the peak of the 2020 pandemic. Among the results, one can see an increase in the randomness of these markets at times after the pandemic peak and that the volatilities of the stock market, captured via analysis of the S&P500, and those of cryptocurrencies show interdependence both before and after the crisis.

In the same direction, Dima et al. (2021) analyzed the Chicago Board Option Exchange Volatility Index (VIX) to investigate the behavioral effects during the COVID-19 crisis from an adaptation of Kristoufek and Vosvrda's (2013) index. The authors identified that market efficiency is variant over time and that both fundamentalist and technical analysis could be employed for arbitrage purposes, with the efficiency of these techniques depending on the level of inefficiencies present in the market, thus corroborating studies such as those of Mitra (2012).

A common point to highlight in previous studies that have analyzed the degree of market efficiency based on FMH is that they consider either an entire period, or a subdivision of the sample into two points with reference to a specific period. From this perspective, the justification of this work was reinforced, whose focus is not only to analyze the efficiency of markets, but also to verify how this degree of efficiency behaved throughout the analyzed period.

METHODOLOGY

Data

In defining the emerging markets to be analyzed, the stock market indices of Brazil, Russia, India, China, and South Africa (BRICS) were chosen, due to both the growth potential of these markets and the growing interest of investors in this group (Mensi et al., 2016).

However, from an economic point of view, it is worth noting the heterogeneity in the composition of the BRICS. China and India have relatively closed economies with a greater presence of state control than the others. Brazil and Russia have economies based on the export of commodities and with significant state presence in specific sectors. Finally, South Africa is the country that has developed the least within the group, although its potential is recognized (Cheng et al., 2007). Thus, although these countries are treated as a group, it is important to keep in mind their economic differences and local specificities, which may cause their stock markets to behave differently over the sample years.

The choice of developed countries, USA, Germany, UK, and Japan, was due to the global wealth they concentrate (29.93% in the case of the USA, 6.96% in the case of Japan, 4.07% in the case of Germany, and 3.98% in the case of the UK), according to the Global Wealth Report 2021, in addition to their capital markets having the 10 largest capitalizations according to the World Data Bank. It is also noteworthy that such countries have already been included in similar analyses in previous studies, such as Mitra (2012) and Al Nasser and Hajilee (2016), which facilitates the comparative analysis of the results.

In operational terms, daily data from the main market indices of each of the countries was used: Brazil (IBRX), Russia (MOEX), India (NSE), China (SSEC), South Africa (JTOPI), United States (DJI), Germany (DAX), England (FTSE), and Japan (N225), containing information from June 1, 2007 to July 31, 2021, totaling about 3,500 observations per market index. The daily closing price data was collected via Yahoo Finance.

From the quotations, the logarithmic returns of the indices were calculated. It is also noteworthy that no data treatment was used. This is justified because, according to Schinckus (2010) and Jovanovic and Schinckus (2013), data treatment via winsorization or outlier exclusion, for example, alters the true distribution of data, which is the main object of analysis using fractal models.

This period was chosen because it includes three stressful moments for the market: the subprime crisis in 2008, the fall in commodity prices between 2013 and 2016, and the COVID-19 pandemic. Thus, it was possible to verify the potential of increased arbitrage opportunities in these periods, which is expected according to both EMH and FMH.

Efficiency index

The proposed index is based on the work of Kristoufek and Vosvrda (2014), an adaptation of the index proposed in Kristoufek and Vosvrda (2013), in which the authors built it based on fractal effects, that is, geometric shapes that can be reduced into smaller elements with the same shape. The index developed by the authors is based on the sum of differences between empirical and expected values in an efficient market for a given number of metrics.

The original model uses three metrics to compose the index: first-order serial correlation (ρ_1), the *H*, and the *D*. However, Kristoufek and Vosvrda (2014) identified ρ_1 as redundant for capturing short-run effects, since these dynamics are already captured in *D*, allowing entropy to be used instead of this metric.

In view of the diversity of methods for calculating *H* and *D*, the authors indicate the methodologies that are suitable for short-time series, useful for the sample windows investigated in the present study. For *H*, the authors proposed the use of two alternative approaches, namely local Whittle estimator (LW) and Geweke and Porter-Hudak method (GPH).

The LW consists of a semi-parametric estimator calculated via maximum likelihood (Robinson, 1995). The first step in the calculation is to identify the time series' spectrum, which can be approximated by the periodogram $I(\lambda_j) = \frac{1}{T}\sum_{t=1}^{T} \exp(-2\pi_{it}\lambda_j)_{xt}$ with T being the sample size, j being different subsamples, so that $j = 1,2, \ldots m$ with $m < T/2$ and $\lambda_j = 2\pi_{it}/T$. Thus, H_{LW} is obtained from the minimization of $R(H) = \log\left(\frac{1}{m}\sum_{j=1}^{m}\lambda_j^{2H-1}\right) - \frac{2H-1}{m}\sum_{j=1}^{m} log\lambda_j$, with $0 \leq H < 1$. Additionally, H_{LW} has consistency properties and is asymptotically normal.

The modeling via GPH is based on Gaussian noise fractionation, assuming a spectral function of the type $\log f(\lambda) \propto (H-0{,}5)\log\left[4seno^2\left(\frac{\lambda}{2}\right)\right]$. The exponent H_{GPH} will also be normally asymptotically distributed (Geweke & Porter-Hudak, 1983).

Whereas for *D*, Kristoufek and Vosvrda (2014) point to the methodologies *G* by Genton (1998) and *HW* by Hall and Wood (1993) as suitable for short-time series.

The methodology for calculating the metric G is based on Genton's robust variogram estimator (Genton, 1998), which can be defined as $V_2(l/n) = \frac{1}{2(n-1)}\sum_{i=1}^{n}\left(\frac{X_i}{n} - \frac{X_{(i-l)l}}{n}\right)^2$ where n indicates the size of the subseries of X, and l the size of the windows used in the clustering. Based on these calculations, it is possible to construct D_G, expressed by $D_G = \frac{\sum_{l=1}^{L}(s_l - \bar{s})\log(V_2(l/n))}{2\sum_{l=1}^{L}(s_l-\bar{s})^2}$ with $s_l = \log(l/n)$ and s being the average of s_l, and, according to Davies and Hall (1999), making $L = 2$ reduces the bias of the metric.

On the other hand, the metric HW is based on the box-counting process and the use of stepwise scaling of absolute deviations, originally proposed by Hall and Wood (1993). The absolute deviations of a time series X are computed as $A(l/n) = \frac{1}{n}\sum_{i=1}^{[n/l]}|X_{il/n} - X_{(i-1)l/n}|$, which represents the absolute deviations of the time series X of size n and grouped into windows of size l. Thus, the estimator is measured via $D_{HW} = 2 - \frac{\sum_{l=1}^{L}(s_l - \bar{s})\log\left(A\left(\frac{l}{n}\right)\right)}{2\sum_{l=1}^{L}(s_l-\bar{s})^2}$ with $s_l = log\ (l/n)$ and s being the mean of s_l, and, as in the previous case, with $L = 2$, there is a reduction in the bias of the metric.

In the present study, as in Kristoufek and Vosvrda (2014), it was chosen to use the entropy approximation of Pincus (1991). Once defined that $1 \le i \le T-m+1$, one can develop a measure of autocorrelation based on the lags of a time series of the type $C_i^m(r) = \frac{\sum_{i=1}^{T-m+1} 1_{d(i,j)} \le r}{T-m+1}$ such that $1_{d(i,j)}$ is a binary measure that equals 1 if $d(i,j) = \max_{k=1,2,\ldots m}(|X_{i+k-1} - X_{j+k-1}|) \le r$ and 0 otherwise, thus being a measure that measures the distance between different lags of a time series X as a function of a parameter $r \in R$.

Thus, one can define $C^m(r) = \frac{1}{T-m+1}\sum_{i=1}^{T-m+1} C_i^m(r)$ as the mean of $C_i^m(r)$. This average is used as an entropy approximation through the correlation dimension according to the equation $EA = \lim_{r\to 0}\lim_{T\to\infty}\frac{\log[C^m(r)]}{\log(r)}$.

The general formula for efficiency index (EI) is expressed in Equation 1, in which M_i is the i-th of n estimated metrics for the index, M_i^* is the expected value for M_i in an efficient market and R_i is the range of variation of M_i. Thus, according to the authors EI assumes values from 0 to $\sqrt{n}/_2$, indicating a completely efficient or inefficient market, respectively.

$$EI = \sqrt{\sum_{i=1}^{n}\frac{(M_i - M_i^*)^2}{R_i}} \quad (1)$$

First, H, D, and EA were calculated by the methodologies described previously and then, according to Equation 1, EIs were calculated considering 756 working days, approximately three years, with a daily moving window, totaling about 2.743 EIs per market.

Based on the historical series of the indices and the elements that compose it, analyses were performed to verify patterns of behavior of the degree of efficiency and anomalies in the markets during the selected period. The data collected was organized in an MS-Excel spreadsheet, and then used to construct the index in R software using the packages *pracma*, *fractaldim*, and *LongMemoryTS*.

Cointegration analysis

In order to complement the analyses, the cointegration between the markets was analyzed, aiming to capture the contagion effect between the groups analyzed. For this, the R packages *urca* and *vars* were used.

First, the augmented Dickey-Fuller (ADF) unit root test was performed. For cases in which the variables at the level were not statistically identified as stationary, the test was rerun considering the first difference. In parallel, the long-run cointegration tests between the variables were performed using the Johansen test (1988), which, based on a vector error correlation model (VECM), checks whether two or more variables that have the same order of integration present a stationary linear combination. Equation 2 describes the multivariate modeling used, which is a p-order error correction vector modeling (VECM(p)).

$$\Delta Y = \phi_0 D_t + \Pi Y_{t-p} + \Gamma_1 \Delta Y_{t-1} + \cdots \Gamma_{p-1}\Delta Y_{t-p+1} + \nu_t \quad (2)$$

In this equation, $\Gamma_i = \phi_1 + \cdots \phi_i - I_n$, ϕ_k is the matrix of coefficients of the VAR model, $\Pi = \phi_1 + \cdots \phi_p - I_n$, and D_t comprises the set of deterministic elements, such as trend and intercept. The Johansen test tests both the ranks and the eigenvalues of the matrix Π. For selecting the number of lags used, Akaike's criterion (AIC) was used. In the VECM(p) models, only the intercept was considered as an integral of D_t. Figure 2 summarizes the flowchart of the procedures performed in the study.

PRESENTATION OF RESULTS

Hurst exponents

In order to capture the effects of long-term memory, the Hs via local Whittle estimator and Geweke and Porter-Hudak methodology were used. Figure 3 illustrates the variations of the average of the calculated exponents. From the results summarized in Figure 3, one can verify oscillations of the metric over time, which corroborate previous results in the literature, regarding the FMH on the degree of efficiency of a market not being static.

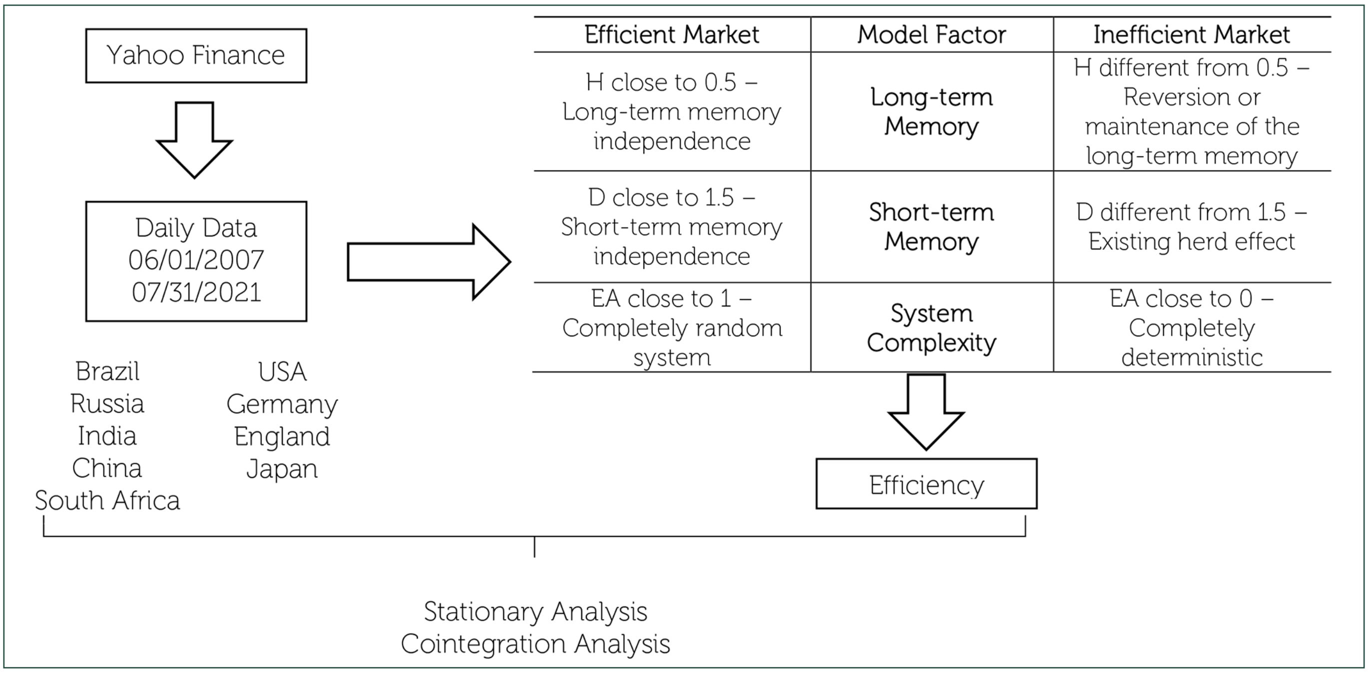

Figure 2. Flowchart of the study procedures.
Source: Own elaboration.

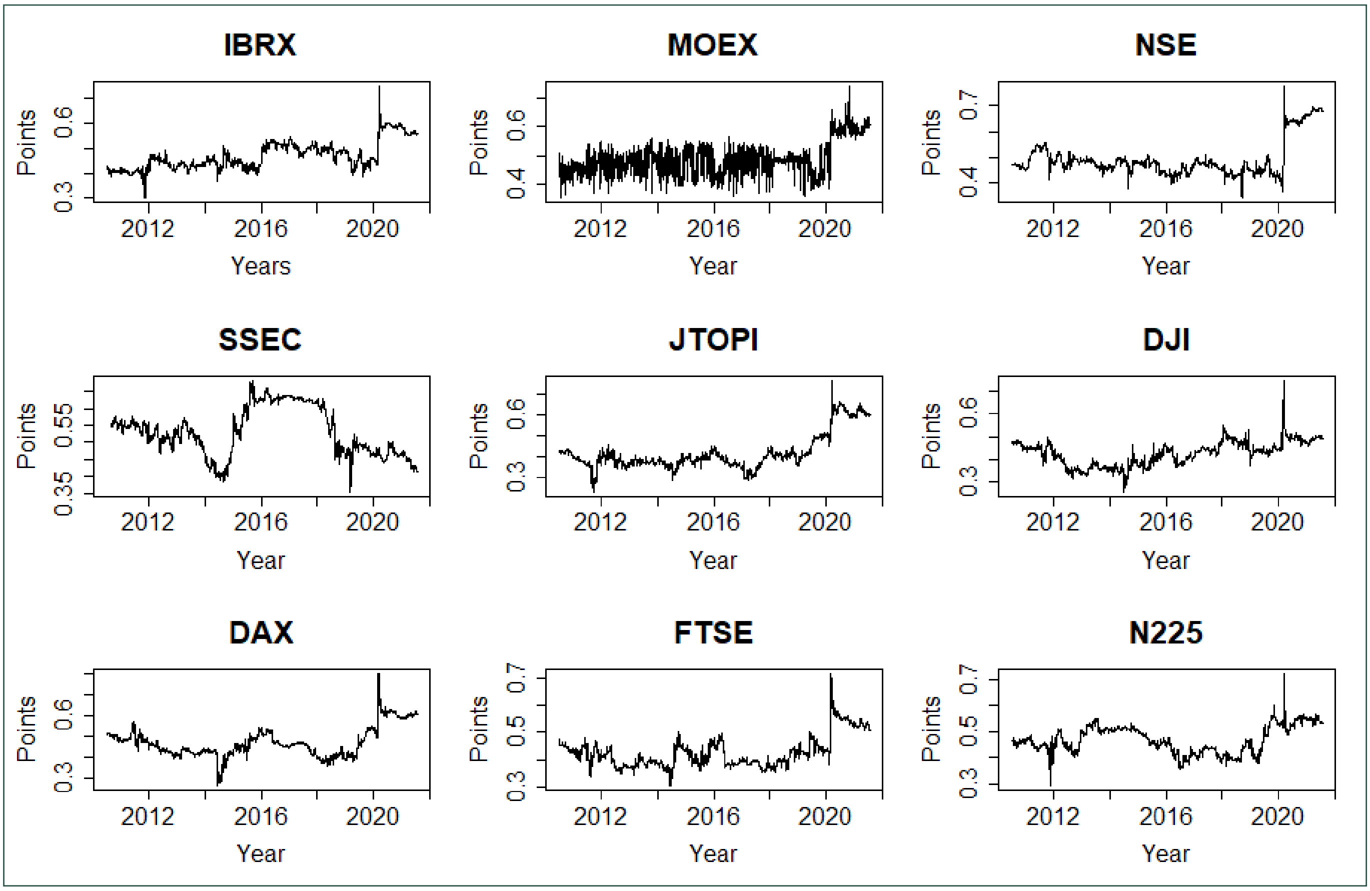

Figure 3. Average Hurst exponents of market indices.
Source: Own elaboration.

The *Hs* values oscillated on average between 0.7430 and 0.3010, with mean values around 0.4647. Thus, for the analyzed indices, there are occasional moments of maintenance and mean reversion; however, the mean values are close to 0.5, indicating no expressive effects related to long-term memory. This finding was also observed in previous studies, such as Kristoufek and Vosvrda (2014) and Karp and Van Vuuren (2019).

In the case of the MOEX, it was noticed a more unstable behavior of *H*. However, its values still remain close to 0.45, signaling a maintenance of reversion movements of the short-term average more constant than in other markets. Besides this, it is also highlighted that, except for the case of the Chinese index, the other market indices present a similarity regarding the behavior of *H*.

Furthermore, the ADF tests revealed that the *Hs* series are nonstationary, considering a significance level of 5%. This conclusion is graphically visible in moments of abrupt market oscillations, as occurred due to the pandemic between 2020 and 2021; there was an average increase of 25% in the value of the *H*. The exception, again, is the Chinese case, whose increase was more discrete compared to other markets.

As far as volatility is concerned, one can verify the existence of the stylized fact of oscillations of its variation pattern, which allows categorizing the series as being heteroscedastic. The indices with higher volatility were NSE, SSEC, JTOPI, and DAX, which allows concluding that these markets are the ones that present the greatest oscillations in terms of long-term memory, making them attractive to investors in search of arbitrage operations, especially in times of greater turbulence.

Fractal dimensions

Complementing the previous analyses, *Ds* were calculated from the Genton and Hall-Wood metrics with the objective of analyzing the effects of short-term memory. This measure is associated with the perception of investors that current prices will follow the bullish or bearish trend existing in the market in the short term. Figure 4 illustrates the variations of the average of the two metrics in the analyzed period.

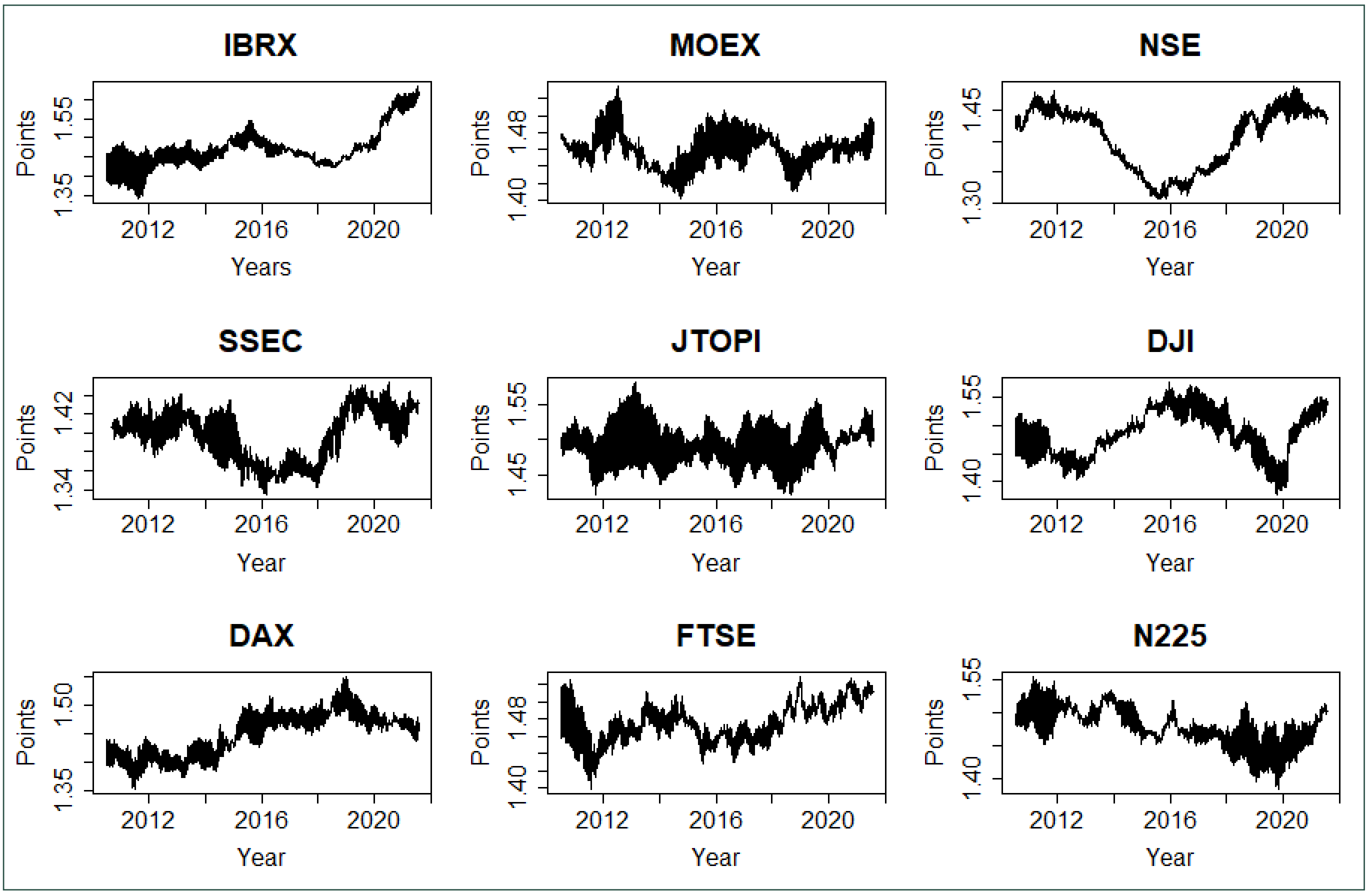

Figure 4. Average of fractal dimensions of market indices.
Source: Own elaboration.

In general, it can be seen that the results of the *Ds* results fluctuate between 1.3306 and 1.6327, with a median around 1.2406. Besides this, it is possible to observe once again an approximation between the mean and the median, thus concluding the existence of a symmetry of the distributions of *Ds*, although they are not normally distributed.

Moreover, according to the unit root test, it can be rejected that the time series analyzed are stationary, a fact that can also be observed graphically, so that a tendency to reduce this bias is observed during moments of crisis, given its approximation to the value of 1.5.

It is worth noting that, on average, results between 1.4 and 1.6 make up approximately 86.86% of the time series, indicating that the series analyzed have low short-term herd effect bias. The least efficient markets, according to this metric, would be the Indian (NSE) and the Chinese (SSEC), with 40.90% and 48.03%, respectively, of the observations with deviations greater than 0.1 from the expected value of an efficient market, as advocated by the FMH.

Finally, it is highlighted that in general the deviations are more concentrated in values below 1.5, consistent with previous studies such as those by Kristoufek and Vosvrda (2013) and Karp and Van Vuuren (2019). This finding indicates evidence of short-term memory persistence movements, signaling a predominance of local mean maintenance of the indices.

In terms of volatility, the standard deviations ranged from 0.0474 to 0.0051, with a median value of 0.0198. The least volatile indices were MOEX, FTSE, and SSEC, with deviations of 0.0220, 0.0235, and 0.0281. The most volatile were IBRX, NSE, and JTPI, with deviations of 0.0524, 0.0499, and 0.0328, indicating that these markets have a greater tendency to oscillate in terms of the degree of local average maintenance, and are therefore riskier for investors.

Entropy approximation

Finally, the last element calculated to construct *EI* was *EA*, a metric that measures the complexity of the financial markets system. Figure 5 shows the graphical results of the calculated metrics. As can be seen in the graphs, the values of the metrics are on average between the ranges of 0.05686 and 0.2175, with an overall mean and median of about 0.4527 and 0.4599. Thus, it is seen that there are deterministic effects in the series of index returns, which violates the random walk hypothesis proposed by EMH. This deviation was detected in studies such as those of Lahmiri and Bekiros (2020) and Kristoufek and Vosvrda (2014).

As in the case of *Ds*, the series exhibits nonstationary behavior, with moments of growth and decrease over time, correlating with the result of Delgado-Bonal (2019). In addition, at times of abrupt changes in the index prices, a reduction in the randomness of the series is perceived, which is then corrected. Therefore, it is concluded that there are relevant deterministic trends that can be used for arbitrage, although there are times when these are less significant.

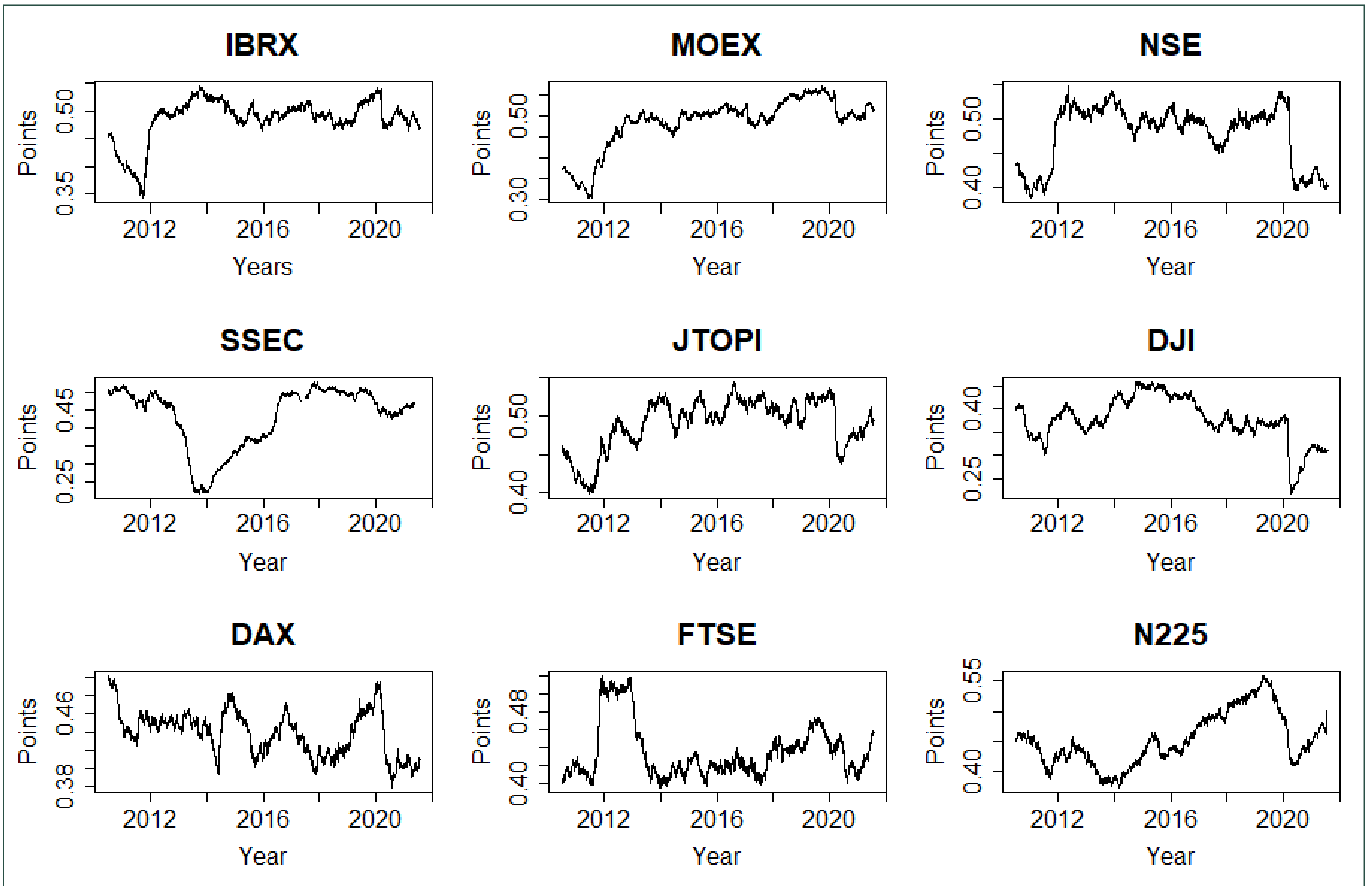

Figure 5. Entropy approximations of market indices.
Source: Own elaboration.

Furthermore, it can be seen that the total sample is below the 0.8 level, with approximately 93% around 0.5, except for SSEC (China) and DJI (USA), which present *EA* concentrations at a higher level.

Thus, it is noteworthy that this is the indicator that is farthest from the expected values of an efficient market, which corroborates the results of Kristoufek and Vosvrda (2014). In general, 84.32% of the results indicate values around 0.5, which reinforces the analysis that there is a balance between deterministic and random factors in the analyzed markets.

The volatility analysis also allows verifying two important points. The first is that the average of the *EAs* standard deviations was 0.0155, with the Indian, Chinese, and American markets showing deviations higher than the average, respectively 0.0165, 0.0216, and 0.0172. As for the markets with lower deviations, the IBRX index (Brazil) stands out, with an average deviation of 0.0139. The N225 (Japan) has an average deviation of 0.0131 and the FTSE (United Kingdom) has an average deviation of 0.0127.

Thus, it can be verified that the markets studied present a balance between random and deterministic effects in their movements, which is not compatible with what is expected in efficient markets, in which random walk movements would be expected. In times of market stress, there is a tendency to reduce the randomness of returns, which implies a reduction in market efficiency, which is later reestablished. This finding is consistent with previous studies, such as Lahmiri and Bekiros (2020) and Kapecka (2013).

Efficiency indices

Figure 6 shows the results of the *EIs*, calculated for the sample. The graphic representation allows verifying the *EIs*. Thus, the oscillations in the behavior of efficiency are evident, and at no time a perfect efficient market was verified, which corroborates previous analyses of the validity of the FMH on this phenomenon, as presented by Meng et al. (2020) and Dima et al. (2021).

To verify the significance of the difference of the *EIs* obtained for each market, tests were performed to evaluate the significance of differences in variances and medians of the indices via the F-test and Wilcoxon's test. As a conclusion, it was found that, at a significance level of 10%, both the medians and the variances of each distribution are distinct from each other, so that it concludes differences in the distributions of the *EIs* calculated for each market.

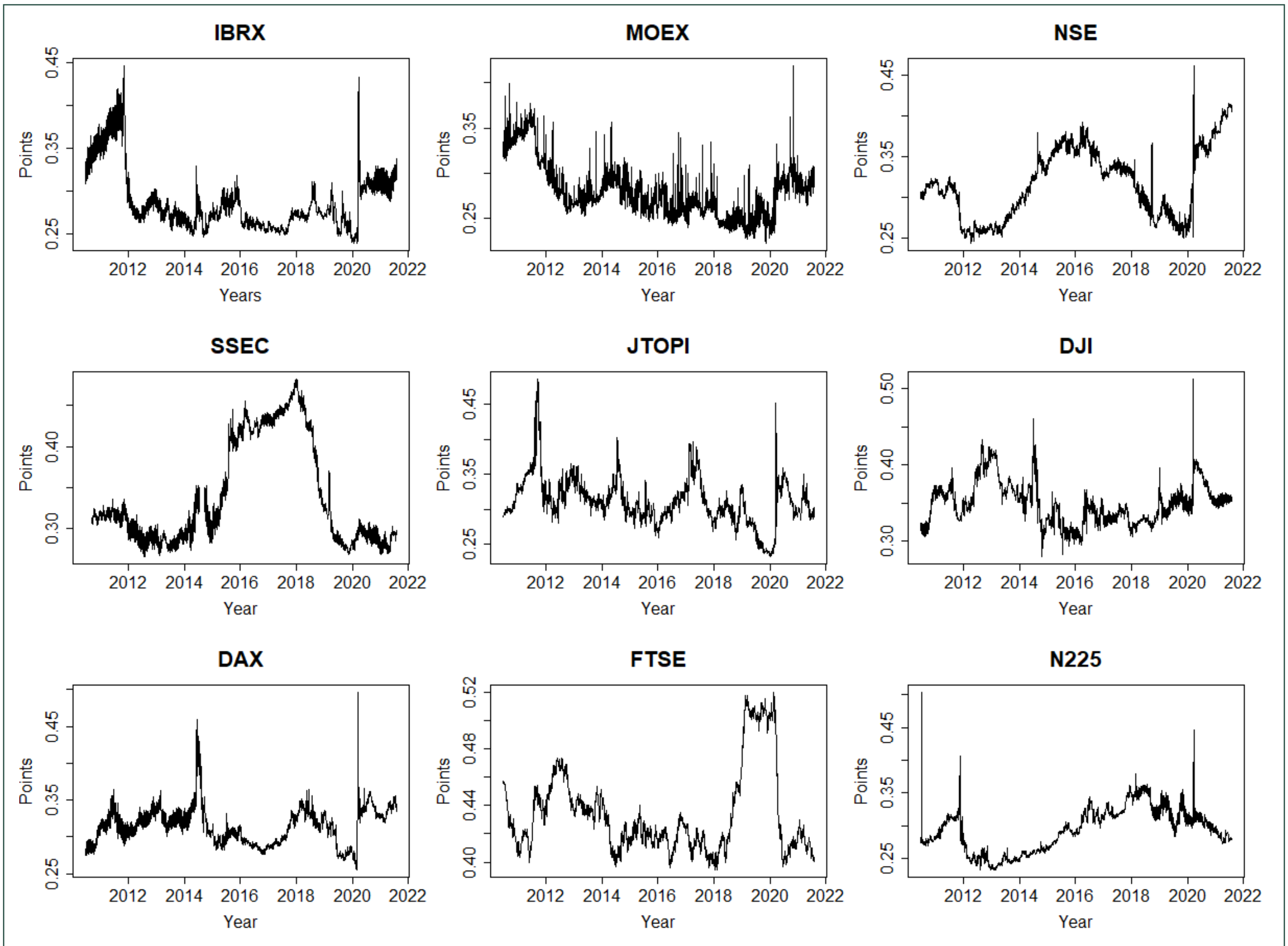

Figure 6. Market efficiency indices.
Source: Own elaboration.

Based on this information, as well as on Figure 7, one can further highlight that the highest concentration of the *EIs* is between 0.2 and 0.4 for about 98.40% of the calculated *EIs*. Considering also that the index oscillates between 0 and 1.1180, one can see that there is a predominance of inefficiencies in market movements in a magnitude between 17.86% and 35.78%.

However, it is necessary to highlight the Chinese case again. For the *EI* SSEC, its values are concentrated in about 39.92% of the cases in interval from 0.2 to 0.3, 32.53% between 0.3 and 0.4, and 27.55% between 0.4 and 0.5. As for the other indices, IBRX (Brazil), MOEX (Russia), and N225 (Japan) present the highest concentration of *EIs* < *0,3*, being the most efficient.

These results reinforce the evidence that none of the markets studied behaved in a fully efficient way during the studied period. Furthermore, the sensitivity of to abrupt changes in index prices indicates a loss of efficiency, so that in times of turbulence, such as the COVID-19 crisis, an increase in the *EI* is visible. The oscillatory behavior and the values of the index prices reflect a loss of informational efficiency. The oscillatory behavior and values of the *EIs* were also found in previous studies, such as those of Kristoufek and Vosvrda (2014) and Kapecka (2013).

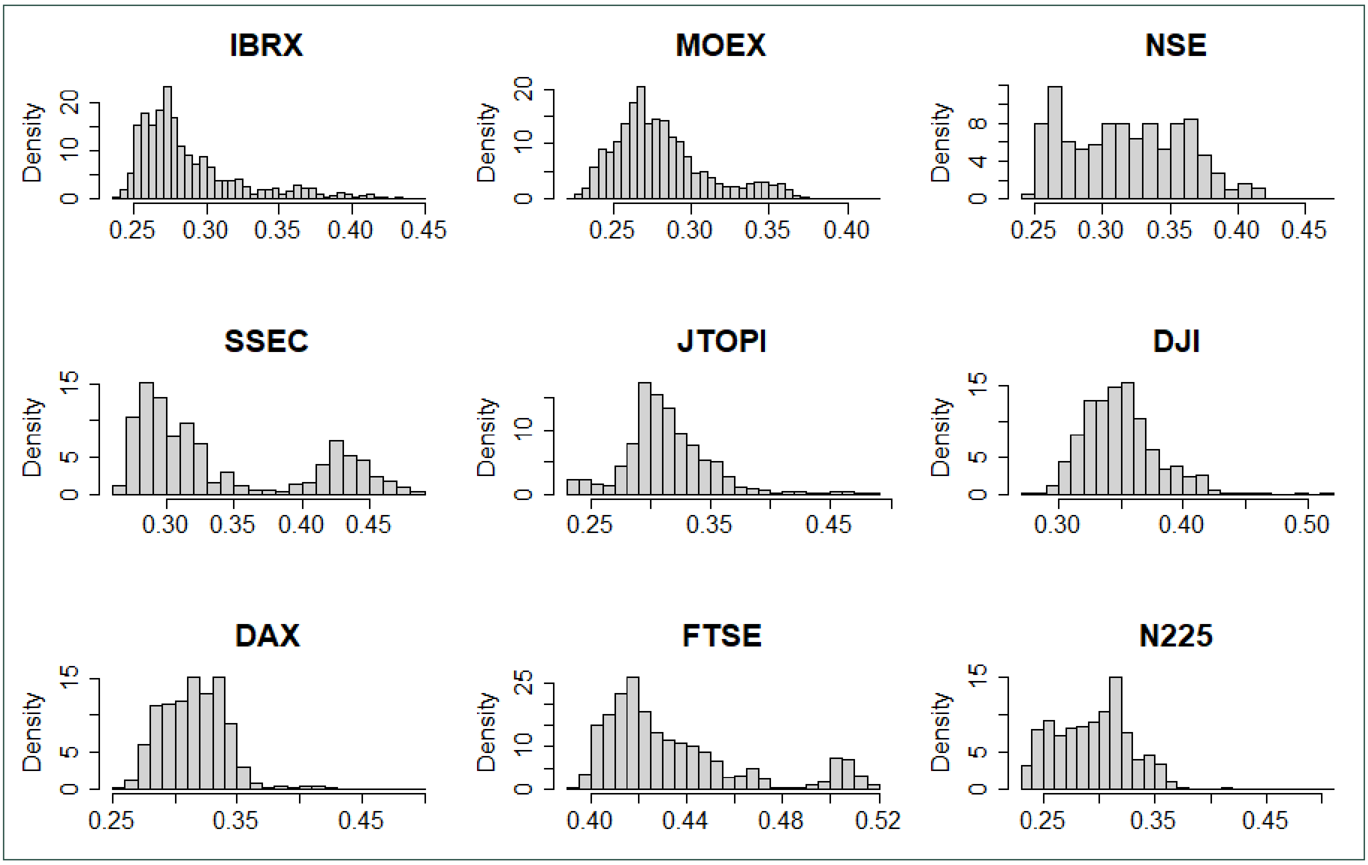

Figure 7. Distribution of efficiency ratios.
Source: Own elaboration.

In addition, although not very expressively, it is possible to identify that, in general, the BRICS countries have a higher concentration of *EI* < *0,2*. Thus, one can consider that emerging countries have smaller deviations from what is expected from an efficient market when compared to developed markets. This conclusion is also in line with previous studies, such as those of Kristoufek and Vosvrda (2013) and Kristoufek and Vosvrda (2014).

In this regard, two points should be highlighted: first, previous studies have had little focus on longitudinal analyses; second, a higher degree of inefficiency in developed countries may be related to the fact that these are the drivers of information with a global impact on other markets, in a possible advance relationship, which will be investigated through cointegration analysis.

DISCUSSION

Market efficiency

Having as reference the results of the metrics presented, it can be observed that, regarding the effects captured by the *Hs* of the analyzed markets, in general they present an oscillatory and nonstationary behavior with an average close to 0.5, a reference value for efficient markets. As for the *Ds*, it is possible to identify a trend toward values of *D* < *1.5*, that is, a divergence from what is expected in an efficient market. However, in the long run this effect is eliminated, as verified in the *Hs* analysis.

Thus, corroborated by previous studies, such as those of Kristoufek and Vosvrda (2013), Kristoufek and Vosvrda (2014), and Caporale et al. (2016), it can be seen that although there are peaks of maintenance and reversal of short-term and long-term averages, the markets show an average behavior close to that expected by EMH. This result is also in line with the considerations made by Fama (1991) that sporadic trends can occur in an efficient market, but these are not sustained in the long run. Thus, there is evidence that does not permit rejecting the weak form of the EMH of Fama (1970).

As for the *EAs*, it can be seen that this is the indicator that presents the greatest distortion to what is expected of an efficient market. In this sense, it would be the main responsible for the deviation of efficiency, in line with the study of Kristoufek and Vosvrda (2014). To illustrate this fact, based on the medians of the differences between the actual values and reference values of the *Hs*, *Ds*, and *EAs*, Figure 8 shows the composition of the median of .

From the graph, one can observe that about 70% of the market inefficiency is caused by the presence of deterministic elements in the price variations. Thus, it is possible not only to question Fama's (1970) EMH in its weak form, but also to reinforce the existence of patterns in the movements of stock market index prices, thus corroborating the application of the FMH.

This conclusion validates analyses whose primary goal is to model asset prices and returns for arbitrage purposes. Furthermore, there is no discrepancy in the composition of the index for BRICS and developed countries, indicating that the analyzed effects are found to approximately the same degree in both analyzed groups.

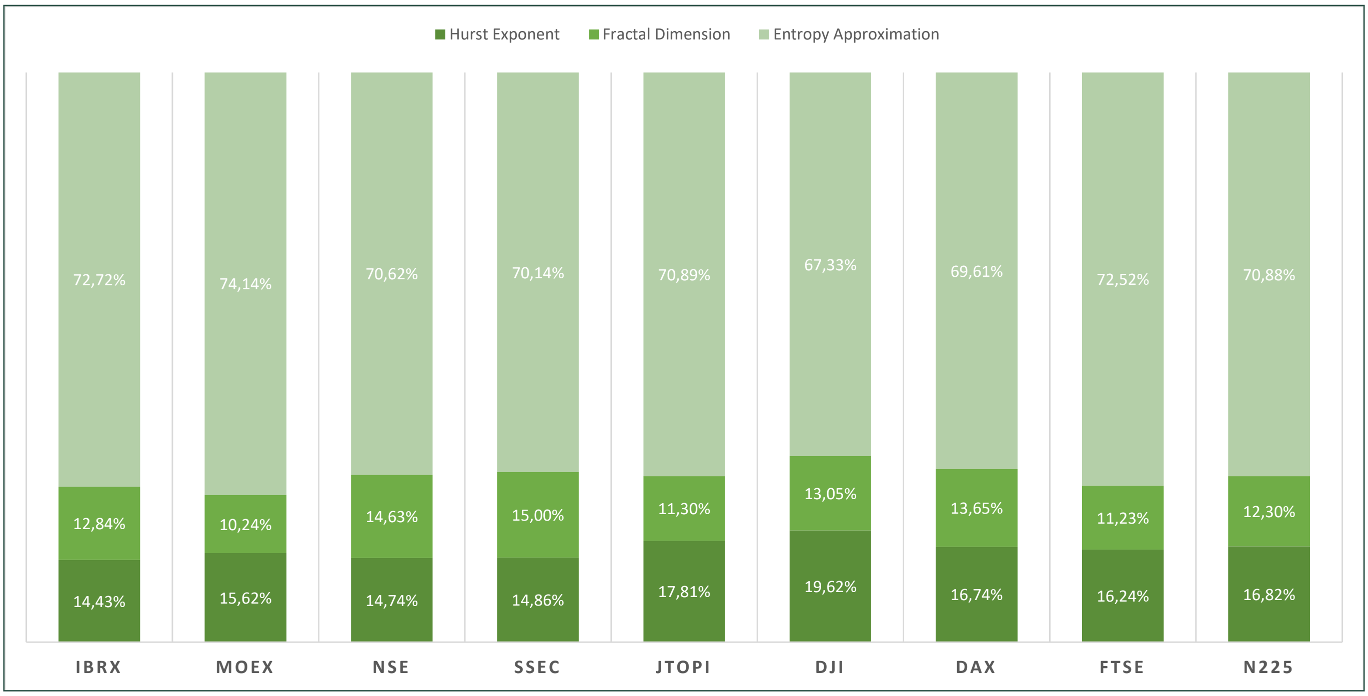

Figure 8. Composition of the median-based efficiency indices.
Source: Own elaboration.

However, it is also important to note that the Chinese index was the one that presented the greatest discrepancy in the behavior of the metrics used to assess market efficiency, with the highest concentration of deviations from the expected values of an efficient market when compared to the others. Thus, the validity of studies that focus specifically on China, such as Meng et al. (2020), is verified, given the particularities of the shocks that this market tends to suffer in relation to other markets.

Finally, on the elements that make up the *EI*, the correlation matrices of the *Hs*, *Ds*, and *EAs* of the nine markets analyzed were calculated, as well as their significance levels. Based on the findings, a positive relationship between *H* and *D* can be established, indicating a negative reaction between long-term and short-term memory maintenance. Entropy showed, in general, a negative correlation with the other two, which is consistent with the FMH, since the existence of trend maintenance indicates a reduction in the randomness of time series. These findings support the graphical approach to asset valuation, which makes use of, for example, short- and medium-term moving average reversion metrics for strategy making.

Furthermore, the adoption of the FMH broadens the range of distributional possibilities for families of financial asset prices. Thus, arbitrage would be possible by identifying the distribution, and its respective parameters, that best resemble the distribution of asset returns at a given time. Usually, distributions are used according to Lévy's stable alpha processes, which can incorporate elements such as leptokurtic syndromes and volatility clusters (Rickles, 2011). Furthermore, according to Lévy (1924), with $H = 0.5$ and $D = 1.5$, Lévy's process converges to a Brownian process, so one can characterize EMH as a particular case of FMH in which short- and long-term memory maintenance motions are close to zero.

Investigating the relationship of results to cross-market contagion effects

To better understand the time series characteristics of the analyzed markets, tests were performed to identify the cointegration of these markets. The results are available in Table 1. Firstly, it is noteworthy that, for the prices of the market indices analyzed, none of the series was identified as stationary at level. However, the first difference, equivalent to the return of the markets, was signaled as stationary by the tests.

Considering the BRICS groups, it can be seen that there is cointegration among almost all members of the group, especially the Brazilian case, which, in turn, presents cointegration with all other markets. This conclusion corroborates the analyses of Wang and You (2022), both in terms of the strong integration of the stock markets of the BRICS countries and the attractive potential of the Brazilian market for diversification and risk management purposes.

Table 1. Summary of the tests for unit root, market cointegration, and market efficiency.

Panel A — Augmented Dickey-Fuller unit root test							
Quote				EI			
Marketplace	ADF Level	ADF First Difference	Conclusion	Marketplace	ADF Level	ADF First Difference	Conclusion
IBRX	-1.95	-13.47***	I(1)	IBRX	-2.318	-16.33***	I(1)
MOEX	-1.99	-15.03***	I(1)	MOEX	-2.701	-18.99***	I(1)
NSE	-3.5	-13.78***	I(1)	NSE	-1.154	-18.98***	I(1)
SSEC	-2.77	-13.56***	I(1)	SSEC	-0.764	-14.76***	I(1)
JTOPI	-3.27	-15.35***	I(1)	JTOPI	-3.71**		I(0)
DJI	-2.72	-14.75***	I(1)	DJI	-3.44**		I(0)
DAX	-3.95	-14.76***	I(1)	DAX	-3.78**		I(0)
FTSE	-3.37	-15.11***	I(1)	FTSE	-3.58**		I(0)
N225	-3.34	-14.51***	I(1)	N225	-2.04	-17.75***	I(1)

Panel B — Johansen's cointegration test: Quote				
Marketplace		Cointegration Vectors	Eigenvalue	Trace
IBRX	MOEX	r1	4.54***	4.54***
		r=0	752.9	752.9
IBRX	NSE	r≤1	7.2***	7.2***
		r=0	752.97	752.97
IBRX	SSEC	r≤1	9.33***	9.33***
		r=0	747.3	747.3
IBRX	JTOPI	r≤1	5.8***	5.8***
		r=0	718.53	718.53
IBRX	DJI	r≤1	9.64***	9.64***
		r=0	741.92	741.92
MOEX	NSE	r≤1	0.29***	0.29***
		r=0	30.95	30.95
SSEC	JTOPI	r≤1	2.42***	2.42***
		r=0	10.63	10.63
SSEC	DAX	r≤1	0.09***	0.09***
		r=0	31.28	31.28
SSEC	FTSE	r≤1	0.09***	0.09***
		r=0	34.13	34.13
DJI	FTSE	r≤1	0.13***	0.13***
		r=0	54.01	54.01
DJI	N225	r≤1	1***	1***
		r=0	29.56	29.56
DAX	FTSE	r≤1	0.02***	0.02***
		r=0	26.68	26.68

Panel B — Johansen's cointegration test: EI				
Marketplace		Cointegration Vectors	Eigenvalue	Trace
IBRX	MOEX	r≤1	10.35***	10.35***
		r=0	64.26	53.92
IBRX	NSE	r≤1	4.18***	4.18***
		r=0	15.92	11.74
IBRX	SSEC	r≤1	1.53***	1.53***
		r=0	14.49	12.96
MOEX	NSE	r≤1	4.6***	4.6***
		r=0	27.59	22.99
SSEC	DAX	r≤1	1.55***	1.55***
		r=0	35.13	33.58
DJI	FTSE	r≤1	14.42***	13.5***
		r=0	38.4	22.05
DAX	FTSE	r≤1	14.38***	13.48***
		r=0	51.07	31.12

Note. Source: Own elaboration. * p < .10, ** p < .05, *** p < .001.

As for the group of developed countries, a similar result is observed, with the American market being the only one cointegrated with the Japanese market, and the English market being the only one cointegrated with the German market, corroborating studies such as those of Agoraki et al. (2019) and Rizvi and Arshad (2017). Regarding the cointegration between the groups, the relationships of the Brazilian market with the American market and the Chinese market with the European markets are significant. This conclusion is consistent with the studies of Siddiqui et al. (2022), Rizwanullah et al. (2020), and Al Nasser and Hajilee (2016).

Thus, the presence of cointegration between the analyzed markets partly limits international portfolio diversification strategies, especially when considering markets in emerging and developed countries. However, the development of portfolios that mix assets from emerging and developed countries may be a viable option between markets from both groups for diversification purposes, corroborating the conclusions of Bhutto et al. (2020).

Finally, a cointegration check of the *EIs* was also performed, with the objective of identifying whether there is contagion among market inefficiencies. Firstly, it is interesting to verify that there is a contagion effect not only in the markets, but also in the inefficiencies between them regarding the maintenance of short- and long-term memory and deterministic effects.

However, it is necessary to point out that for the group of developed countries (except Japan), and for the South African stock market, the *EIs* series are stationary. This fact points to a greater stability in the level of efficiency of markets in these countries. In addition, according to the work of Doorasamy and Sarpong (2018), the South African capital market would have a behavior similar to that of the American one, besides being less volatile than that of other emerging countries. Thus, for these countries, it can be seen that the degree of efficiency, although oscillating, presents a stationarity around a value close to 0.3. Furthermore, given the difference in the integrality of these series, the hypothesis of cointegration between the *EIs* of these markets with the others, which in turn are not stationary, is rejected.

Thus, some points about the evolution of the efficiency of the analyzed markets are highlighted. Firstly, it was found that none of the analyzed markets, whether emerging or developed, presented full efficiency based on the Kristoufek and Vosvrda (2014) index. Moreover, the existence of deterministic effects in the series of returns not only was the element that most corroborated the market's inefficiency, but it also opens margins for arbitrage strategies based on the analysis of price trends.

Secondly, and considering the possibility of arbitrage, the presence of inefficiencies ends up generating windows of opportunities for abnormal gains, which, as they are detected and taken advantage of by various agents in the stock market, are no longer relevant, which implies a reduction of movements in the market focused on that strategy, which, in turn, ends up increasing market efficiency. Such conclusion is corroborated by the analyses of Dima et al. (2021), Balladares et al. (2021), and Sánchez-Granero et al. (2020), signaling that the existence of a higher degree of inefficiency enables the use of graphical analyses to obtain abnormal returns.

It is also noteworthy that this study corroborates the analysis of Schinckus (2011) on the validation of the applicability of modeling from the field of econophysics to complement the financial theory. Thus, the non-observance of EMH in its weak form in concomitance with the adequacy of the FMH principles highlights the need for studies and practical work to take into account the reality of time series properties in their elaborations, especially abandoning the equilibrium assumption.

Examples of methodologies that take such fractal properties into account are: the multifractal model of asset return, in which the asset price is modeled as a combination of Brownian motion and a random time warp according to the cumulative distribution of the multifractal measure of the asset mean; and Markov-switching multifractal, in which a hyperbolic decay for price autocorrelation is generated and models abrupt changes in volatility according to fractal patterns (Calvet & Fisher, 2008).

Finally, developed markets tend to have a smaller variation in the level of inefficiencies. This fact can be a reflection both of the greater development of the financial market in these countries and of a lower level of investor overreaction, greater economic stability, and lower impact of economic policy uncertainties (EPU), hypotheses that can be studied in future research.

FINAL CONSIDERATIONS

This paper aimed to analyze the evolution of market efficiency in emerging markets and compare the results with those of developed countries. For this objective, econophysics metrics were used to identify short- and long-term memory and complexity of the time series of market index returns.

First, it was identified the existence of divergences between the values of the three analyzed metrics with what is expected from efficient markets, especially regarding the randomness of prices. Furthermore, it was identified that the results regarding the behavior of market efficiency according to fractal analysis could be divided into two groups.

The first is composed of Brazil, China, India, Russia, and Japan. The first four are components of the BRICS, while the last is an Asian powerhouse. What these countries have in common is their dependence on the global macroeconomic context, particularly on the North American scenario. Moreover, this group has significant variations in market efficiency levels over the years, signaling unstable windows of higher arbitrage degree from the capital market point of view.

The second group, composed of the United States, Germany, England, and South Africa, corresponds to countries with levels of market inefficiencies that do not change statistically throughout the sample, thus reflecting greater stability in capital markets from the perspective of market efficiency. This verification may be a reflection of the existence of a lower degree of UPE and overreaction of investors in these countries, which, except for the South African case, are developed countries. However, it is reinforced that at no time in the sample were there periods in which there was full market efficiency.

Furthermore, the analysis of the long- and short-term memory structure contradicts Fama's (1991) argument that inefficiencies in the very short term may occur, but that in the long term the market converges to efficiency. It was found in this research that the presence of deterministic trend elements in the returns of the indices, which is not restricted to the group of emerging or developed countries, ends up going against the assumptions of randomness of the price movements of financial assets of EMH.

Thus, this study verifies and reinforces the need for modeling and empirical work not to be limited only to the adoption of EMH and its assumptions in their analyses. Moreover, the presence of inefficiencies detected from fractal properties also indicates the existence of arbitrage opportunities, which sheds light on studies that explore strategies based on graphical analysis to detect short-term trends in the market, as well as reinforces the applicability of FMH for financial analysis, such as the one undertaken in this study.

From a practical point of view, it is noteworthy that the study corroborates, through market cointegration analyses, the existence of a contagion structure between markets, especially when considering the component markets of the group of developed and emerging countries. However, except for the case of the Brazilian market with the American and the Chinese with the European ones, the practice of international diversification between emerging and developed markets as a means of risk dilution may be a strategy adopted by investors.

Furthermore, the Chinese market was also identified as the most frequent in the occurrence of inefficiencies. Thus, there are possibilities for research on arbitrage strategies in this market.

With regard to the limitations of this research, it is highlighted, firstly, that daily data was used, which limits the potential capture of inefficiency effects in an intraday manner. In addition, market indices were used as proxies for the stock markets of emerging and developed countries, so that only one time series was used to signal the efficiency of each of the markets.

For future studies, it is suggested to conduct replications of this research using intraday data to identify its fractal properties. Furthermore, it might be interesting to verify the robustness of the results for weekly and monthly data, aiming to help investors with longer investment horizons in their decision-making.

Another possibility is to analyze sets of assets from several countries instead of using market indices to avoid distortions regarding the real level of market inefficiency. Finally, formal tests and analysis between the results of fractal metrics and behavioral finance metrics are suggested, aiming to formalize the relationships between both. This would be interesting in the perspective of obtaining advances in financial theories and that behavioral finance can be seen as an a posteriori theoretical complementation to the analyses of fractals of financial markets.

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