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TO DETERMINE STABILITY OF THE ROAD TRAIN WITH O1-CATEGORY TRAILER

Summary. In the work improved system of equation of plane-parallel movement of the road train with single-axle O1-category trailer. Defined lateral reactions on the vehicle and trailer wheels at body roll, wheels slip caused by body roll and also developed road train spatial mathematical model in a transverse plane. This model is used to study the road train course stability with O1-category trailer. It is shown that the spatial model of road train with no vehicle and trailer wheels inclination has the same divergent instability characteristic as the plane road train layout.
Keywords: road train, trailer, mathematical model, motion, wheel, stability

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1. INTRODUCTION

Small and medium business development in Ukraine has led to an increase in demand for trailers used in a coupling with light vehicles. These are the O1- and O2-categories trailers. Accordingly, the O1-category trailer is, so-called, "light" trailer. In addition, O2-category trailers can be used with M1 vehicles, which are often called "heavy". For these trailers, which are operated, as a rule, by private entrepreneurs and amateurs, very important are parameters concerning loading on a tractor and trailer, in particular cargo location in a trailer. The trailer must be loaded evenly over the floor area of the trailer or van, and unit loads shall be located and fixed above the axle or paired axles. The centre of mass arrangement above the trailer axle ensures a normal load on the hitch ball [11].

Load displacement and, respectively, the trailer centre of mass in running order forward of the trailer wheels axle causes an increase in the load on the vehicle traction coupling device. This leads to more than just pinning the vehicle's rear part to the road, moving back the vehicle's center of mass and lifting its front. This mass distribution impairs the front wheels road adhesion and the car becomes less controllable. In addition, due to a loosening of the road adhesion, braking on the front wheels do not generate sufficient braking force, particularly required when the trailer is in motion.

It is unacceptable to load the trailer so that its centre of mass is reversed behind the trailer wheels axles. If the load on the hitch ball is low, the trailer will swing vertically. Its vibrations will lift the vehicle's rear part, making the rear wheels less tractable, which can lead to skid on slippery or wet roads and during turns.

It is clear that improving the efficiency of road trains by increasing speed shall not be detrimental to traffic safety [4]. Therefore, study of road trains stability with O1-category trailers is an urgent task.

2. PROBLEM

Many work has been done to improve the operation of freight road trains. However, low-tonnage road trains have practically fallen out of sight of scientists, both in our country and abroad. It is worth noting that in some countries, the relevant experience has been accumulated in the design and trailers production for passenger cars. However, there is almost no specific data of parameters selection method used by foreign manufacturers in the design and trailers production. First of all, it concerns the parameters of sustainability and handling [5].

In practical terms, in the development of new vehicles, as well as the modernization of existing ones, it becomes important not only the cause of instability, but also the reaction of the car to it and driving actions of the driver, which are ambiguous and unstable. Therefore, it is assumed that vehicle stability and handling shall be ensured directly by its design parameters.

The qualitative stability assessment is based on A. Lyapunov's general stability theory. This only establishes the fact that the received random deviations from the given motion are increased or decreased. To measure stability in the mathematical theory of motion stability special methods have been developed. The work [5] shows that almost all parameters of the car and trailer links affect the road train handling and motion stability. This influence is related to the geometry and position of the vehicle center of mass, tire characteristics [1, 3, 9, 10], number of axles and their placement in the base [3], road train control system [1].

The success of road vehicle stability depends on the correct choice of the simulation scheme which best reflects the most important factors, influencing this operating property and from the accurate assessment of the pneumatic tire forces interacting with the road [5].

In the paper [8] it is shown that development of compact and easy-to-use mathematical models of articulated vehicles for motion planning, control and motion localization becomes more important in the era of intelligent transport systems, especially when there is a need for reliable motion forecasts for automated freight road trains and public vehicles of different kinematic structures.

In the paper [13] a universal mathematical model of the articulated bus (AB) is proposed, which includes both the rotation dynamics of the AB and its controlled axles. All possible AB configurations are presented, both on the bus and trailer axle. It is shown that the resulting model can be applied to the design of a motion stability controller. Although the focus of this article is on buses, the proposed approach actually covers any multi-axle articulated vehicles such as trucks, tractor trailers.

Interconnections between the axles and links of multi-link vehicles may cause specific oscillatory trailers' behaviour during vehicle manoeuvres. The paper [7] shows that such fluctuations are the vehicle kinematic properties direct consequence. If some trailers of the so-called n-trailer system are connected with each other by means of hitch-mechanism, the kinematic system model is more complicated than in the standard case [2]. At the same time, more complex equations can be interpreted as a work result of the virtual steering wheels located on hitch-mechanism, with a rotational angle which is non-linear feedback from the initial configuration state. This is also sufficient to suggest that a general control problem may be embedded in a corresponding n-trailers multistage system.

Theoretically, such s model should best reflect the real vehicle. Research of the car as a system of multiple bodies connected by holonomic and non-holonomic bonds, leads to the complex mechanical system study with many degrees of freedom and described by the system of high order differential equations. However, as noted in the paper [6], the complexity of the mathematical model does not always give a positive effect because in determining mass dimensions (masses, moments of inertia) and characteristics of the connections errors are unavoidable, the combination of which leads to inaccuracies in outcomes determination. Therefore, when studying vehicle stability, flat or spatial models are increasingly used, taking into account the non-linear motion of its axle wheels. In this context, the aim is to study the influence of constructive and operational factors of road train with O1-category trailer on motion stability.

3. RESEARCH

When studying road trains, stability is generally considered the plane-parallel motion of it links. It is believed that the normal reaction of the supporting surface to the wheels on the starboard and port sides are the same. Under this condition, the motion stability is considered for plane road train model. However, if the trailer centre of mass is high, it is possible to significantly change supporting surface reaction on it wheels. Therefore, it is necessary to consider the road train motion in both horizontal and longitudinal vertical and transverse planes. In controlled road train theory and modeling are rather justified assumptions [1], that the road train is moving on a level horizontal surface; the unsprung mass is considered not to be heeling; control influence on the motion parameters of the road train is carried out via steered wheels of traction vehicle. Consequently, steering changes are not taken into account; gaps of the hitch-mechanism are not taken into account; the longitudinal speed of the road train is constant; the distance between the road train links does not change due to the small folding angles. Road train components are totally rigid bodies: the load on the road train is located so that the traction vehicle centre of mass, trailer and also its hitch-mechanism are located in the vertical symmetry plane of the link; the main trajectory is the trajectory of the towing vehicle centre of mass.

Basic kinematic and dynamic properties of the road train with trailer, as a single mechanical system of bodies, depend on the physical phenomena that arise during movement of all its elements and their interactions with each other. In turn, these phenomena are determined by the geometry and structure of the road train.

In the paper [6] the system of equations for plane-parallel motion of the road train with a single-axle trailer (Figure 1), recorded in the form:

- for the longitudinal velocity of towing vehicle centre of mass

$$\begin{aligned}&(m+m_1)(\dot{V}-U\omega)+c\omega^2 m_1-\left[m_1 d_1(\dot{\omega}-\ddot{\varphi})\sin\varphi_1-(\omega-\dot{\varphi})^2\cos\varphi_1\right]=\\&-\left(X_1\cos\theta_1+Y_1\sin\theta+X_1^{'}\cos\theta^{'}+Y_1^{'}\sin\theta^{'}\right)-\left(X_2+X_2^{'}\right)-\left(X_3+X_3^{'}\right)\cos\varphi-\left(Y_3+Y_3^{'}\right)\sin\varphi;\end{aligned} \quad (1)$$

- for the transverse velocity of towing vehicle centre of mass

$$\begin{aligned}&(m+m_1)(\dot{U}-V\omega)-c\omega^2 m_1-\left[m_1 d_1(\dot{\omega}-\ddot{\varphi}_1)\cos\varphi_1+(\omega-\dot{\varphi}_1)^2\sin\varphi_1\right]=\\&-\left(X_1\sin\theta-Y_1\cos\theta+X_1^{'}\sin\theta^{'}-Y_1^{'}\cos\theta^{'}\right)+\left(Y_2+Y_2^{'}\right)-\sum_{i=1}^{2}\left(X_3+X_3^{'}\right)\sin\varphi+\left(Y_3+Y_3^{'}\right)\cos\varphi;\end{aligned} \quad (2)$$

- for towing vehicle angular velocity

$$\begin{aligned}&I\omega+\left[\dot{\omega}c-(U+V\omega)\right]cm_1+cm_1 d_1\left[(\dot{\omega}-\ddot{\varphi})\cos\varphi+(\omega-\dot{\varphi})^2\sin\varphi\right]=\\&H\left(X_1\cos\theta+Y_1\sin\theta-X_1^{'}\cos\theta^{'}-Y_1^{'}\sin\theta^{'}\right)+\varepsilon\left(X_1+X_1^{'}\right)+\\&+a\left[\left(Y_1\cos\theta-X_1\sin\theta\right)+\left(Y_1^{'}\cos\theta^{'}-X_1^{'}\sin\theta^{'}\right)\right]+\left[\left(X_2-X_2^{'}\right)H_1-\left(Y_2+Y_2^{'}\right)b\right]-\\&-c\sum_{i=1}^{2}\left[\left(X_3+X_3^{'}\right)\sin\varphi_1+\left(Y_3+Y_3^{'}\right)\cos\varphi\right];\end{aligned} \quad (3)$$

- for trailer angular velocity

$$\left[I_1+m_1 d_1^2\right]\times(\dot{\omega}-\ddot{\varphi}_1)+m_1 d_1\left[\left(\dot{V}-U\omega+c\omega^2\right)\sin\varphi+\left(V\omega-\dot{U}-c\omega^2\right)\cos\varphi\right]=I_1\left(Y_3+Y_3^{'}\right)+M \quad (4)$$

In the system of equations (1-4), the following designations are adopted:

v, *u*– point "C" longitudinal and lateral velocity projection (meaning, p. C velocity projections on the axis of dynamic coordinate system, directly linked to the towing vehicle);

φ_1 – the folding angle of cinematically independent road train links, rad;

$M_K = f(\varphi_1, \dot{\varphi}_1)$ – trailer turn resistance moment, N×m;

$X_{i,j}, Y_{i,j}, Z_{i,j}$ – longitudinal, lateral and vertical reactions of supporting surface to road train wheels;

a, *b*, *c*, d_1, c_1, l_1– road train layout options, m.

The plane-parallel motion system of equations (1-4) should be supplemented by road train equations in a transverse plane.

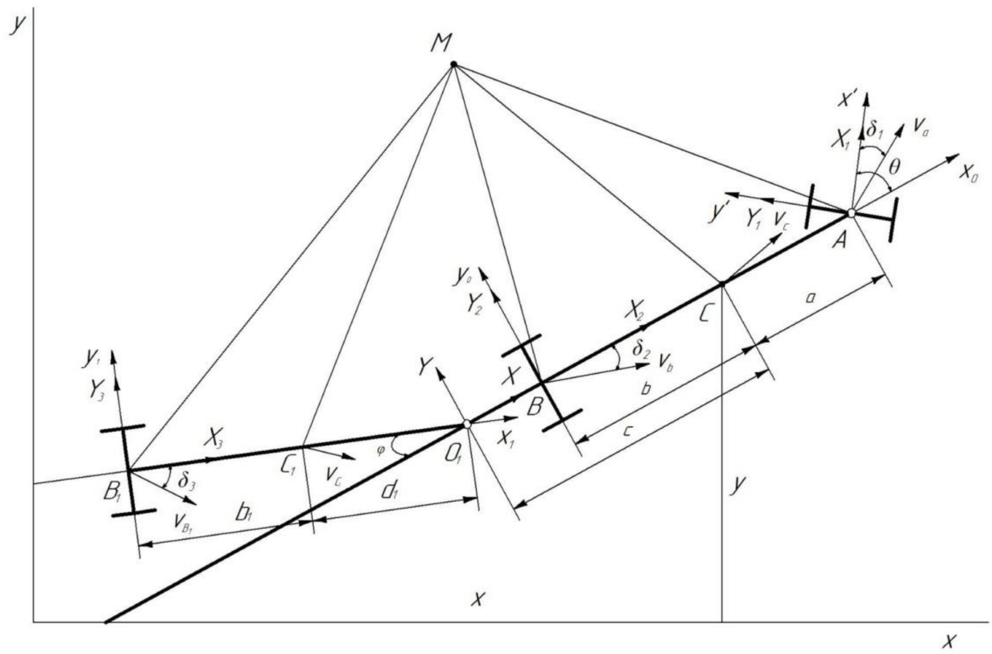


Fig. 1. Road train turn scheme

Paper [12] presents constructing method of the vehicle mathematical model in a transverse plane. This methodology can also be used to construct such model of road train with an O1-category trailer.

We take that the road train is moving on a horizontal surface with constant speed, no vertical motion and rotation of the vehicle and trailer body around its transverse axis (galloping). That is, for each link of road train there are three degrees of freedom, in particular lateral motion along the transverse axis, rotating motion around the vertical axis (yawing) and rotating motion around the longitudinal axis (roll).

The design model for each link of a road train consists of an unsprung and sprung mass. The roll axis runs parallel to the supporting surface; the centre of mass of road train each link lies on a vertical axis, with this same axle intersects the axis of the roll, which coincides with the axis of O_x (Figure 2).

In this case, unsprung and sprung masses are positioned relative to the center of mass *m* of the car so that the sum of their moments relative to the center of mass is equal to zero and the system of equations for road train model transverse movement shall be written in the form:

$$ma\left(\dot{u}_a + v_a\omega_{Za}\right) = m_a h_{C_{Za}}\dot{\omega}_{Xa} + \left(Y_1 + Y_2\right); \tag{5}$$

$$L_{ZZa}\dot{\omega}_{Za} - L_{ZXa}\dot{\omega}_{Xa} = aY_1 - bY_2 - J_{Ka}\frac{v_a}{r_{Ka}}\omega_{Xa}; \tag{6}$$

$$I_{XXa}\dot{\omega}_{Xa} - I_{XZa}\dot{\omega}_{Za} = -\Delta N_{2a}\frac{B_a}{2} + m_a g h_{CZa} \times \sin\varphi_a + m_a(\dot{u}_a + v_a\omega_a)h_{CZa} \times \cos\varphi_a + \\ + \Delta N_{1a}\frac{B_a}{2} + I_{Ka}\frac{v_a}{r_{Ka}}\omega_{Za}; \tag{7}$$

$$L_{ZZn}\dot{\omega}_{Zn} - L_{ZXn}\dot{\omega}_{Xn} = dY - J_{Kn}\frac{v_n}{r_{Kn}}\omega_{Xn} \tag{8}$$

In the system of equations (5-8), the following designations are adopted:

$\dot{u}_{a,n}$ – lateral acceleration (along O_y axis), m/s^2;
$v_{a,p}$ – vehicle and trailer longitudinal velocity (along the O_x axis), m/s;
$w_{Za,n}$ – vehicle and trailer angular velocity around the vertical axis - O_z, 1/s;
$\dot{w}_{Xa,n}$ – angular acceleration around the longitudinal axis - O_x, 1/s^2;
$\Sigma Y_{a,n}$ – the sum of the lateral forces (along the O_y axis), N;
$h_{CZa,n}$ – vehicle and trailer transverse inertia arm, m;
$m_{a,n}$ – vehicle, trailer mass, kg;
$m_{a,n}(\dot{u}_{a,n} + v_{a,n}w_{a,n})h_{cZ_{a,n}}$ – vehicle and trailer moment of inertia, kg×m^2;
$M_{x,n}$ – vehicle and trailer external forces moment, N×m;
ΔN_{12a}, ΔN_{22a} – variation of vertical load on the second side of front and rear wheels, kg;
φ – vehicle sprung masses roll angle, rad;
B_a – vehicle track, m;
$K_{Z_{22a,n}}$ – vehicle and trailer suspension strength coefficient;
$\frac{B}{2}\varphi$ – vehicle and trailer resilient element of the suspension deflection, m×rad.

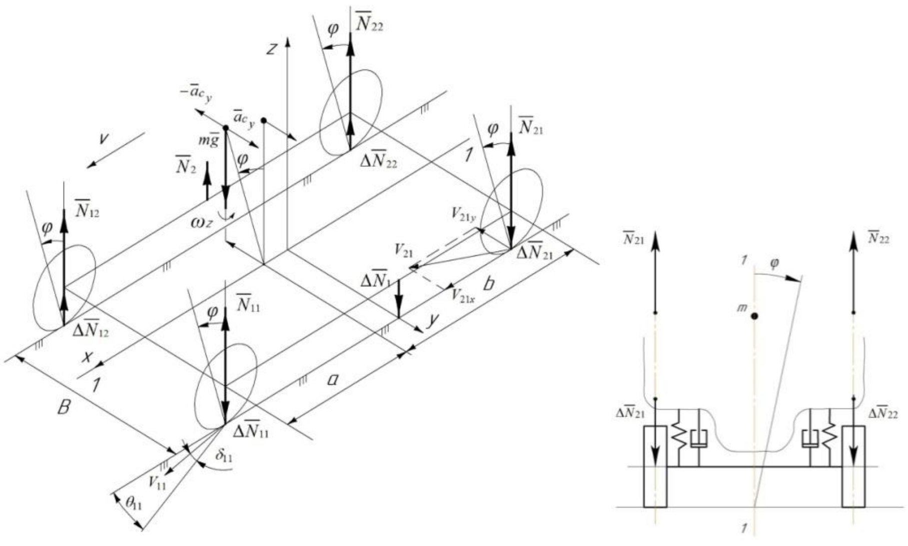
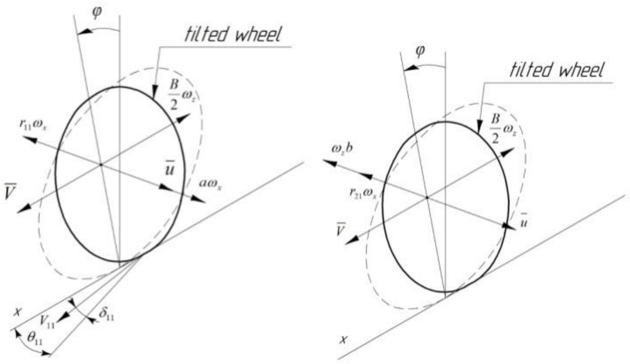


Fig. 2. Vehicle design model
Source: [9]

Vehicle and trailer body roll and the transverse inclination of their wheels will lead to a change in the withdrawal angles and thereby the forces of resistance to withdrawal.

Define withdrawal drag forces for wheels which are inclined due to vehicle and trailer roll. In Figure 3a the vehicle front axle wheel is presented, and in Figure 3b – the wheel of the vehicle rear axle and trailer axle.



(a) (b)

Fig. 3. Vehicle and trailer wheel rolling arrangement: front (a) and rear axle (b)
Source: [9]

Using (Figure 1) and (Figure 4), we have:

$$V_{21_{Xa,n}} = V_{a,n} - \frac{B_{a,n}}{2}\omega_{Za,n}; \tag{9}$$

$$V_{21_{Ya,n}} = \omega_{Za,n} b_{a,n} + r_{21a,n}\omega_{Xa,n} - u_{a,n} \tag{10}$$

Define the vehicle and trailer rear axle withdrawal angle:

$$\delta_{21a,n} = \frac{V_{21_{Ya,n}}}{V_{21_{Xa,n}}} = \frac{\omega_{Za,n} b_{a,n} + r_{21a,n}\omega_{Xa,n} - u_{a,n}}{V_{a,n} - \frac{B_{a,n}}{2}\omega_{Za,n}}; \tag{11}$$

$$\delta_{22a,n} = \frac{V_{22_{Ya,n}}}{V_{22_{Xa,n}}} = \frac{\omega_{Za,n} b_{a,n} + r_{22a,n}\omega_{Xa,n} - u_{a,n}}{V_{a,n} + \frac{B_{a,n}}{2}\omega_{Za,n}} \tag{12}$$

The same determination shall apply to the vehicle front wheel withdrawal angles:

$$\delta_{11a} = \theta - \frac{V_{11_{Ya}}}{V_{11_{Xa}}} = \theta - \frac{\omega_{Za} b_a - r_{11a}\omega_{Xa} + u_a}{V_a - \frac{B_a}{2}\omega_{Za}}; \tag{13}$$

$$\delta_{12a} = \theta - \frac{V_{12_{Ya}}}{V_{12_{Xa}}} = \theta - \frac{\omega_{Za} b_a - r_{12a}\omega_{Xa} + u_a}{V_a + \frac{B_a}{2}\omega_{Za}} \tag{14}$$

Wheel interaction with the supporting surface is expressed by the road track reaction as the withdrawal angle and vehicle and trailer body roll function, namely [3]:

$$Y_{ij} = \frac{k_{ij}\delta_{ij}}{\sqrt{1 + k_{ij}\left(\varphi^2 G_{ij}^2\right)^{-1}\delta_{ij}^2}} + \gamma_{ij}\varphi_{ij} \tag{15}$$

Where:
δ_{ij}, Yij – withdrawal angles and lateral reactions on the vehicle/trailer wheels, rad;
φ – adhesion coefficient between tire and supporting surface in transverse direction (we consider it a constant value for given road conditions);
k_{ij} – lateral withdrawal resistance coefficient;
γ_{ij} – proportionality coefficient depends on tire design, air pressure in it, normal load and supporting surface properties, on which the wheel is rolled [12];
φ_{ij} – vehicle and trailer roll angle, rad.

The resultant roll angles and loading (unloading) are used as a basis for calculating vehicle and trailer wheels withdrawal drag coefficient for further calculation of road train motion stability.

To determine road train motion stability, consider the equation in the variants. The system of equations (1-4) with balanced longitudinal forces are solvable provided that: $v = 0$, $u = 0$, $\omega = 0$, $\theta = 0$, $\dot{\theta} = 0$, $\varphi_1 = 0$, $\dot{\varphi}_1 = 0$ which corresponds to the road train straight line motion. We will take this motion as unperturbed and explore its stability according to A. Lyapunov. The system of linearized equations in general leads to almost imperceptible sustainability conditions (even in case of static instability). In case of X_1=0, X_2=0, X_3=0, system analysis is greatly simplified and road train critical velocity may be determined, and written as:

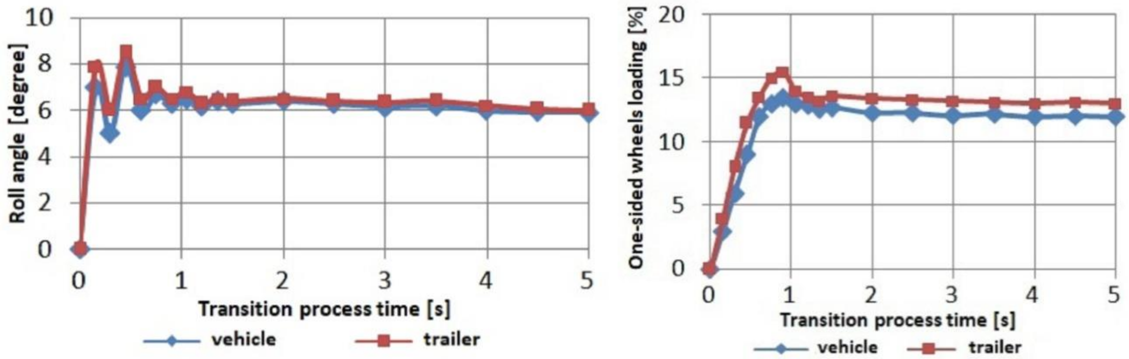
$$V_{кр}^2 = \frac{k_t k_1 k_2 L_1 l_1^2}{k_t k_1\{mL_1(k_1 a_1 - k_2 b) - m_1 L_1 k_2 l_1 + m_2 b_1[k_1(a+c) + k_2(c-b)]\}} \tag{16}$$

The analysis of cumbersome road train static stability conditions showed that aerodynamic drag forces do not affect the critical velocity (aerodynamic drag coefficient and therefore aerodynamic drag force is not included in the expression for critical velocity), and values of motion resistance coefficients on the road train first, second and third axles have almost no effect on critical velocity value. To this end, integrating the system of equations, which describes the plane-parallel movement and road train link movement in vertical plane by roll angles, was performed separately using Maple software.

Figure 4 shows the results of the vehicle and trailer body roll calculation, as well as after-loading of the vehicle and trailer wheels when driving in a circle, R=25 m; V=15 m/s.

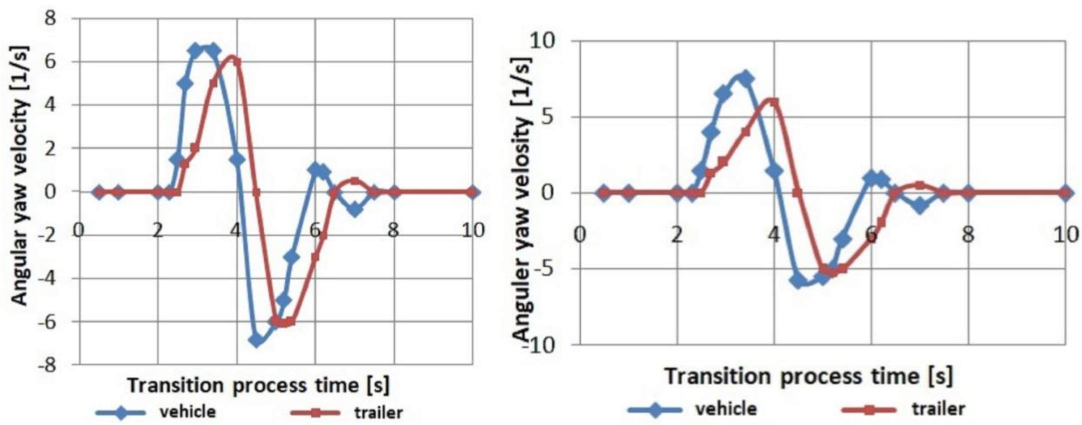
As shown by the reported dependencies, when the road train is moving in a circle, body roll and vehicles on-board load exceeds the same figures for trailer link. This is due to the lower trailer centre of mass (hg=0.65 m) compared to a towing vehicle (hg=0.83 m).

Integration of the equation system describing the road train in the vertical plane, along with the equations describing the plane-parallel motion allows studying the ψ_i, γ_i variables, as well as lateral acceleration and angular yaw velocity when performing typical manoeuvres such as "steering wheel jerk" and "shuffle, S_l = 24 m", (Figure 5-7).



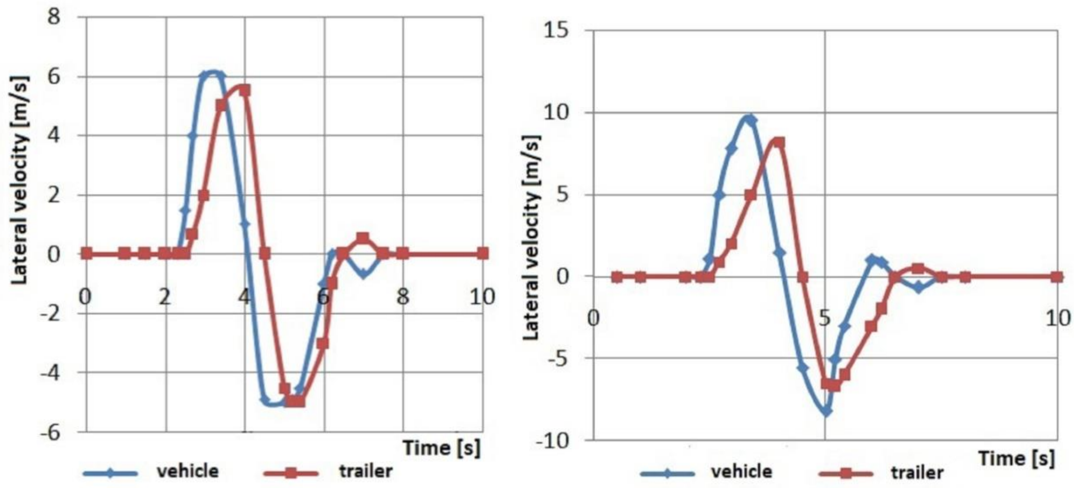
(a) (b)

Fig. 4. Vehicle roll angle change (a), vehicle and the trailer exterior wheels loading (b), when driving in a circle, R=25 m; v=15 m/s



(a) (b)

Fig. 5. Road train links angular velocity during the «shuffle» manoeuvre, v=15 m/s: (a) roll excluding; (b) roll including



(a) (b)

Fig. 6. Road train links lateral velocity during the «shuffle» manoeuvre, v=15 m/s: (a) roll excluding; (b) roll including

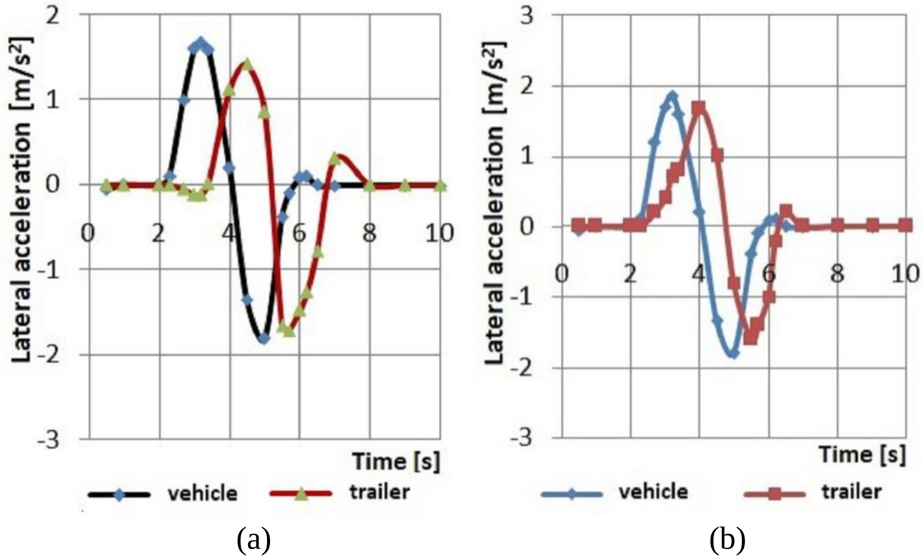


Fig. 7. Road train links lateral acceleration during the «shuffle» manoeuvre, v=15 m/s: (a) roll excluding; (b) roll including

As can be seen from the results of the calculation – highest roll and axle load, angular yaw velocity, lateral velocity and lateral acceleration of road train links are inherent to the vehicle, which is a limiting factor when performing different manoeuvres.

Road train links sustainability during the «shuffle» manoeuvre indicates the attenuation of road train links angular velocity and yaw velocity oscillations (Figure 5-6). The motion stability can be better measured by the magnitude of lateral accelerations, operating at the centre of mass of its individual links (Figure 7). Motion stability may be considered satisfactory if transverse accelerations at the centre of link masses do not exceed 0.45 g. The road train in question corresponds to this condition.

Figure 8 provides acceleration amplification coefficient as limiting factor at 15 m/s and "steering wheel jerk" manoeuvre.

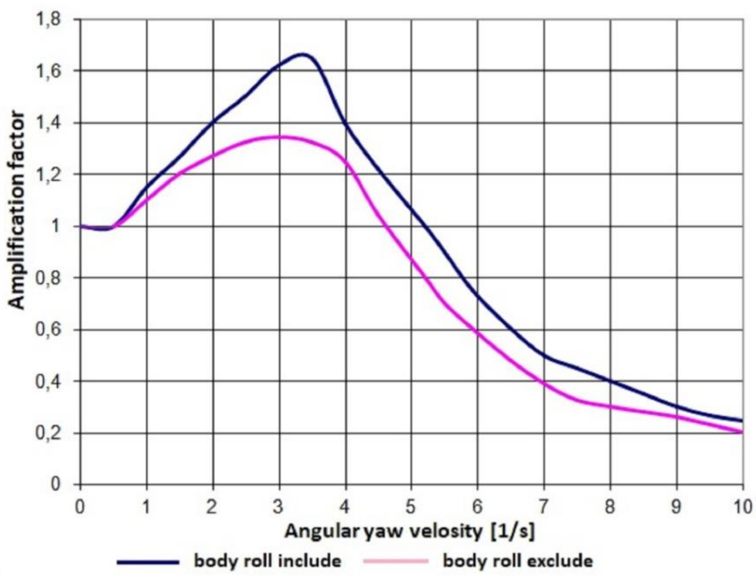


Fig. 8. Vehicle body lateral acceleration amplification factor to the angular yaw velocity

The analysis of the graphs also shows that vehicle body roll has a significant impact on road train motion stability when performing different manoeuvres. For example, vehicle body lateral acceleration amplification factor when body roll is taken into account and "steering wheel jerk" manoeuvre performing, increases by 19.92% in comparison with its absence and this should be considered when selecting a towing vehicle, in particular the chassis and suspension.

4. CONCLUSION

The system of equations for plane-parallel movement of road train with single-axle O1-category trailer has been improved, lateral reactions on the vehicle and trailer wheels have been determined when the body is rolled, withdrawal angles of their wheels caused by the body roll have been determined, and also developed a spatial mathematical model of road train in a transverse plane. This model has been used to study yaw stability control of the road train with O1-category trailer, which establishes that, for the road train in question by nominal loading parameters, air pressure in all tires, symmetrical trailer loading – motion stability is ensured. Also, the critical speed of straight-line movement – 33.97 m/s, oscillatory instability occurrence speed – 31.5 m/s, that is significantly higher than the normalized maximum speed value for road trains with O1-category trailers (25 m/s), and maximum lateral accelerations for manoeuvres such as "steering wheel jerk" and "shuffle" did not exceed the maximum permissible 0.45g.

Analysis of the spatial stability model in general requires further study, for example, it is possible flatter loss of stability, that will occur before divergent stability. The complexity of the analysis will be related to the definition of necessary suspension and tires characteristics, centrifugal moments of inertia affecting both the vehicle and trailer.

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