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Implementing and Evaluating Critical Thinking Skills Developing Lesson Plans for Prospective Teachers

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Abstract

In this study, a technology-supported teaching module was designed to support the advancement of critical thinking as determined by Facione (1990). The lesson plans of the designed teaching module were applied to future mathematics teachers in primary schools. In the study, worksheets constitute teaching model applications lesson plans and it is aimed to evaluate the statements of the pre-service teachers on the worksheets regarding solutions, suggestions and results within the framework of critical thinking skills. During the fall term of the 2019-2020 educational year, we carried out the research study with fourth-graders enrolled at the Faculty of Education, Primary Mathematics Education of a state university in the Marmara Region in Turkey for one semester. This study utilized a case study design, also known as qualitative research. Worksheets that constitute the designed lesson plans served as instruments for collecting data. Using descriptive analysis and content analysis, we analyzed the answers provided by prospective teachers on the worksheets. The previous study concluded that the designed lesson plans were compatible with the critical thinking sub-skills determined by Facione, and contributed to the critical thinking skills of future teachers.

Keywords: critical thinking, lesson plans, preservice mathematics teachers.

1. Introduction

Our age shows irresistible development with rapidly developing information and communication technologies. Keeping up with this development has become the mandatory policies of countries that want to stay strong. The need to update education systems has arisen due to reasons such as intense developments in all areas of life, progress in science and technology, increase in the amount of information, the effect of social media, globalization, and rapid flow of ideas. For this reason, it has been faced with the need to redefine the aims and objectives of education in order to create individuals who meet the requirements of the time in which they live. Organizations such as the United Nations Educational, Scientific and Cultural Organization (UNESCO) and the European Council on Education have drawn attention to the increased activity of students in the learning process, emphasis on personal development, creativity, autonomy, the development of thinking and especially the enhancement of critical reasoning (Maričić, 2015).

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- A lesson plan aimed at fostering the development of critical thinking skills assists pre-service mathematics teachers in enhancing their critical thinking abilities.
- Teaching transformation geometry acquisitions can be incorporated into lesson plans that emphasize critical thinking skills.
- In the designed lesson plan activities, GeoGebra software enhances students' motivation and boosts the learning environment.

When the definitions of critical thinking are examined, Paul and Elder (2013) explain it as a mental process formed by analyzing, evaluating and reorganizing the quality of an individual's perception about a particular topic, issue or problem in a thoughtful manner. Kurnaz (2013) expressed critical thinking as investigating the truth as a whole, with all its positive and negative aspects, visible and invisible to us, and making a judgment about it (p. 12). According to Halpern (2014), critical thinking is the process of using cognitive abilities or techniques that improve the possibility of success. Egmir (2016), on the other hand, expressed critical thinking as a systematic process of doubting, using critical questions in order to improve our thinking, carry out more effective ideas and reach better decisions. As can be understood from the definitions of critical thinking, adapting such an important skill to education systems is important in order to achieve a critical society structure consisting of individuals who can think critically (Egmir, 2018).

Critical thinking, a highly developed cognitive competency subject to the study, was defined in the Delphi panel organized by the American Philosophical Association, consisting of 46 scientists who are experts in their fields. As a result of the discussion, critical thinking was defined as explaining the evidential, conceptual, methodological, critical, contextual issues on which a purposeful, self-controlled judgment is based, resulting in interpretation, analysis, evaluation and inference (Facione, 1990). Facione (1990) grouped the criteria of critical thinking under six main headings: interpretation, analysis, evaluation, inference, explanation, and self-regulation. These skills are explained by Facione as follows:

> (1) *Interpretation*: Being aware of the problem, defining, interpreting, categorizing, making meaning and coding, expressing one's ideas in their own words, distinguishing the main ideas of a text from sub-ideas, determining the purpose, theme and point of view of the author
>
> (2) *Analysis*: Examining ideas, revealing and analyzing arguments/arguments, revealing the similarities and differences between two different approaches given as a solution to a problem, revealing the main idea of a text with the relationships between sentences or paragraphs, defining the claims that have not been clearly revealed and arranging a composition graphically by itself.
>
> (3) *Evaluation*: It is to judge the logical power of the claims based on the facts, to reveal the relevance of the existing claim to the situation at hand, its applicability to this situation or its effects in this situation, to define the perception, experience, situation, belief or judgment of the individual and to evaluate the reliability of the indicators.
>
> (4) *Making Inferences*: Creating assumptions and hypotheses, considering the information in question, making conclusions or meaning from data, judgments, principles, concepts, questions, definitions, questioning the evidence, predicting and interpreting alternatives, developing options for that problem when faced with a problem, to synthesize related ideas.
>
> (5) *Explanation*: To define methods and results, to judge processes, to present the reason for something in the form of convincing and consistent results, to refer to the criteria that constitute the justification of judgments, to create diagrams that organize the findings on the subject, to create graphs showing the connections between concepts and ideas, to present research results and the criteria used to reach

these results, to propose and defend good reasons for causal and conceptual explanations of events or perspectives, to point out the evidence that leads to acceptance or rejection of the author's situation on a topic.

(6) *Self-regulation*: Observing an individual's own cognitive activities, the components used in these activities and the results achieved, questioning their own judgments, especially using the skills at the analysis stage, self-examination and self-correction, sensitively examining issues such as thoughts, personal bias and interest in a contradictory situation, to remind yourself that the author of a text or paragraph must keep his or her ideas separate.

When the objectives of primary and secondary education mathematics curricula are examined, it is emphasized that it is necessary to raise critical thinking individuals (MEB, 2018). In order to achieve this goal, focusing critical thinking in mathematics lessons, including practices, activities, and homework that aim to develop critical thinking skills in the content of lesson planning will provide significant benefits in the development of students' critical thinking skills. In order to achieve this development in students, it will play an important role to determine the opinions of teachers on the mentioned issues and to take appropriate steps within the scope of the obtained ideas.

Critical thinking is a cognitive process that can be taught with effective methods (Kokdemir, 2012). At this point, teachers have great responsibilities. The functionality of education systems that aim to develop critical thinking skills will be thanks to teachers who will apply it in lessons. Norris (1985) stated that students should gain critical thinking skills by teachers (cited in Sezer, 2008). Ennis (1991) stated that the most important element in teaching critical thinking skills is "teacher". Demirci (2000) stated that students' ability to learn to think critically depends on the training of teachers or faculty members on this subject (cited in Semerci, 2003). In this context, it was stated in the Teacher Training Strategy (2017-2023) published by the Ministry of National Education that the functionality of education systems is closely related to the qualifications of teachers and plays a key role (MEB, 2017).

The study sample includes teachers who will become the future of education, so it is crucial for prospective teachers to be able to think critically after graduation. This research is important as it provides a model for the stated goals.

In line with today's educational needs, another issue that should be integrated into curricula is the necessity of integrating technology into classroom lessons. Dynamic geometry software (DGS) provides an opportunity to create and explore environments by increasing the imagination power in mathematics. When these paths are opened, the student will be able to analyze, make assumptions and generalize. Thus, the student's problem-solving skills will improve (Baki, 2001). According to Guven and Karatas (2003), with DGS, students can easily enter the research environment and have the opportunity to explore, make assumptions, test, reject, formulate, and explain. Sanders (1998) stated that while creating a strong teaching and learning environment with dynamic mathematics software, it also provides a basis for analysis and deduction, evidence and creative thinking (cited in Corekcioglu, 2019).

Among the mathematical subjects where DGS can be used, transformation geometry gains draw attention. According to the geometry standards in NCTM (2000), it is stated that students should think about translation, reflection and rotation transformations, which are three important elements of transformation geometry (cited in Borazan, 2019). DGS environments offer convenient environments for transformation geometry teaching. Transformation geometry supports students' creativity in math class. Thanks to the transformation geometry, students understand how they can transform geometric shapes in two-dimensional space, discover the properties of some rules and shapes (Ince, 2012).

When the positive effects of dynamic geometry software specified in the literature on its users and the definitions and explanations determined by Facione (1990) for critical thinking skills are examined, there is a belief that DGS environments can be beneficial for critical thinking development. To this end, the focus is on assessing the technology-supported teaching model lesson plans designed to promote the acquisition of critical thinking abilities determined by Facione (1990) in terms of solutions, suggestions and results obtained by pre-service teachers. In this context, it is intended to give an example of instructional procedures that can contribute to the development of critical thinking skills in the literature. In order to fulfill this objective, an answer to the coming issue has been sought:

> *What does it involve in evaluating the process of technology-supported teaching model applications designed to promote the enhancement of critical thinking abilities by assessing pre-service teachers' expressions on their worksheets within the framework of critical thinking skills?*

2. Method

2. 1 *Research pattern*

Qualitative research is a study in which qualitative data collection methods (observation, interview, document analysis) are used, and a qualitative process is followed to reveal events and perceptions in their natural environment in a realistic and holistic manner (Simsek & Yildirim, 2008). The study was modeled according to the holistic single case pattern of the Case Study method, one of the qualitative research methods. In a holistic single case design, there is only one unit of analysis (an individual, an institution, a program, a school, etc.) (Aytacli, 2012). It is expressed as a method that can include all data collection tools such as case study, interview, observation, questionnaire, and document. In case studies, it is aimed to enlighten the theories with the questions "How?", "Why? and what?" (Cepni, 2012). Within the scope of this study, a holistic single-case design was used in order to provide the basis and guide new teaching model applications to be designed to develop critical thinking skills based on the expressions of the elementary mathematics teacher candidates in the designed lesson plans.

2.2 *Research group*

Participants in this study are teacher candidates enrolled in a state university's primary education mathematics program in the Marmara Region. By using the appropriate sampling strategy, we chose our study sample. Appropriate sampling is the method of determining the sample from easily applicable units due to limitations in terms of time, money and labor (Buyukozturk et al., 2010).

The sample for this study is composed of 56 teacher candidates of 4^{th} grade in the 2019-2020 academic year fall semester primary school mathematics teaching program. The teacher candidates participating in the designed teaching model implementation process consist of 90% (50) female students and 10% (6) male students.

2.3 *Data collection tools*

Lesson plans were developed to guide prospective teachers in the teaching model implementation process and to collect research data from pre-service teachers. Lesson plans and worksheets were prepared by the researcher, and their opinions were taken and the worksheets were finalized by submitting them to the examination of two faculty members from the mathematics education department and one faculty member from the computer and instructional technology department. The worksheets were created from gap-filling, free-response questions and GeoGebra activities to be filled in by the future educators as part of the course preparation according to the prepared plans and to follow the lesson.

The worksheets aimed to teach the transformation geometry acquisitions in the high school curriculum, published by the Ministry of National Education in 2018, within the framework of the critical thinking skills determined by Facione (1990). The aim of this instruction is to boost prospective mathematics teachers' critical thinking skills.

Technology support has been used in teaching the achievements. Technology support in the research was provided by GeoGebra program, one of the dynamic geometry software. In the worksheets, GeoGebra applications took place as activities. Instructions are provided clearly and effectively.

2.4 *Data analysis*

In this research, the results were analyzed by means of “content analysis” and “descriptive analysis method”. In content analysis, we attempt to explain collected data through concepts and relationships. Descriptive analysis is used to explain situations in which the conceptual structure is clearly determined (Cepni, 2012). With the aid of content analysis, responses provided by the study participants to the tasks on the worksheets were analyzed and the answers coded. At the same time, descriptive analysis was used to review the participants’ responses to the questions and applications in the worksheets in terms of critical thinking sub-skills. Findings obtained by descriptive analysis and content analysis were interpreted comparatively.

2.5 *Introduction of the application*

At the beginning of the application, introductory lessons were given to develop the technology competencies (GeoGebra) of prospective teachers. This continued for 5 lesson hours for two weeks in total. GeoGebra features, which will be used frequently during the application in the lessons, were introduced and activities were carried out to understand and reinforce these features. The purpose of GeoGebra introductory courses is to prevent data loss / error / deficiency that may occur due to lack of knowledge using GeoGebra in the data (for critical thinking) to be obtained from pre-service teachers. In addition, “What is thinking? What is critical thinking?”, “What is the importance of critical thinking in education systems?”, “What is the role of teachers in critical thinking education?”, “What is the role of critical thinking in teaching mathematics?” An introductory presentation was made under the headings of critical thinking (1 course hour). Thus, the preliminary preparations for the implementation of the designed teaching model have been completed.

Lesson plans were prepared to direct the implementation process activities of the designed teaching model. These lesson plans were prepared according to the content-based teaching approach, one of the critical thinking teaching approaches, to develop the capability of critical thinking in prospective teachers. The acquisitions of the lesson plans were formed by the secondary school mathematics course transformation geometry outcomes. The applications of these acquisitions in teaching are designed to develop critical thinking skills determined by Facione (1990). The transformation geometry component is included in the course outcomes for 11^{th} and 12^{th} grades in secondary school mathematics. These acquisitions are presented in Table 1.

Table 1. Secondary school mathematics course transformation geometry topics and acquisitions

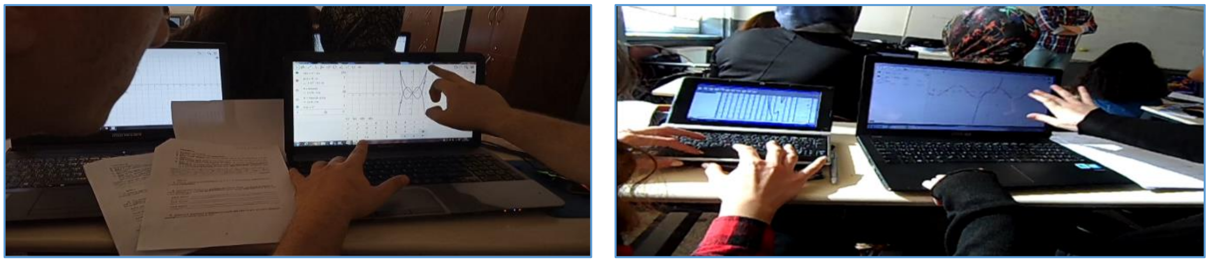
Grade	Number	Topics	Acquisitions
11	11.3.3.	Transformations of Functions	1. Draws new function graphs from the graph of a function with the help of transformations. a) The symmetry properties of the graph of odd and even functions are emphasized. b) $y=f(x)+b$, $y=f(x-a)$, $y=kf(x)$, $y=f(kx)$, $y=-f(x)$, $y=f(-x)$ graphs of conversions are given using information and communication technologies.
12	12.4.1.	Basic Transformations in the Analytical Plane	1. Finds the coordinates of the image under translation, rotation and symmetry transformations of a given point in the analytic plane. a) The concepts of translation, symmetry and rotation are reminded. b) Of the point; symmetries with respect to point, axes, line y = x, and a line and symmetries of the line with respect to the point are emphasized. Symmetries of the line with respect to the line are not included. c) Translation, symmetry and rotation are discussed with the help of information and communication technologies. 1. Solves problems related to basic transformations and their components. a) Modeling studies are included. b) Examples are made from nature and architectural works.

Working papers for each achievement given in Table 1 were prepared. Each worksheet is designed for only one acquisition. Table 2 shows which acquisitions the worksheets that make up the lesson plans are prepared for.

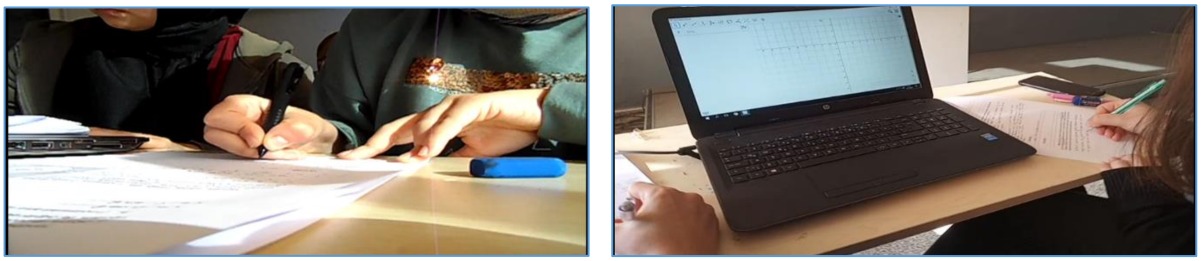
Table 2. Working papers and targeted acquisitions

Lesson plan	Acquisition Number	Acquisition
First worksheet	11.3.3.1.a	The symmetry properties of the graph of odd and even functions are emphasized.
Second worksheet	11.3.3.1.b	$y=f(x)+b$, $y=f(x-a)$, $y=kf(x)$, $y=f(kx)$, $y=-f(x)$, $y=f(-x)$ graphs of conversions are given using information and communication technologies.
Third worksheet	12.4.1.1	It is designed to teach the objectives of the acquisition towards the 'translational transformation'.
Fourth worksheet	12.4.1.1	It is designed to teach the objectives of the acquisition towards the 'translational transformation'.
Fifth worksheet	12.4.1.1	It is designed to teach the objectives of the acquisition towards the 'translational transformation'.

GeoGebra, one of the dynamic geometry software, was used in the applications. Lesson plans, which are at the center of teaching model applications, are the most important source and guide for achieving the goals of the research. It was developed to guide prospective teachers during the application process and to collect research data from pre-service teachers. Visuals of the teaching model application environment are given in Picture 1 and Picture 2.



Picture 1. Pre-service teachers' realization of GeoGebra applications



Picture 2. Teacher candidates' filling out worksheets

The application of the worksheets of the lesson plans was continued regularly every week from the 3rd week to the end of the 7th week (5 weeks in total). The worksheets were printed out and distributed to each prospective teacher every week before the lesson began. With the completion of the worksheet, the worksheets were collected from the pre-service teachers for later analysis.

3. Findings

The worksheets prepared in order to teach the transformation geometry acquisitions of secondary education within the framework of critical thinking skills were distributed to the pre-service teachers during the education and were collected from the pre-service teachers upon the completion of the lesson. The worksheet was prepared for a total of five weeks. Each worksheet is designed for a different outcome. We assessed the answers provided by the participants to the questions and activities on the worksheets distributed to pre-service teachers by means of a content analysis. Teacher candidates' responses were analyzed using a descriptive analysis based on sub-skills of critical thinking. Analyzing the sample questions and applications, as well as the answers provided by the preservice teachers, was in-depth.

3.1 *Sample question and analysis from the first worksheet*

Starting with the question in Figure 1 below, to the lesson in which this worksheet was applied, it was aimed to determine the readiness of the teacher candidates for single and double functions, to remind the forgotten information and to correct the wrong information. With the part of the worksheet shown in Figure 1, it was aimed to increase the interpretation and explanation skills of the teacher candidates, one of the critical thinking sub-skills determined by Facione (1990).

Fonksiyon çeşitlerinden çift ve tek fonksiyon kavramlarını matematiksel biçimde tanımlayınız. Birer örnek veriniz.

Figure 1. Sample question from the first worksheet

Figure 1 mentions the following question:

Figure 1: *Define the concepts of even and odd functions from the types of functions mathematically. Give an example.*

Sufficient time was given to this question for prospective teachers to answer. When the answers of the question were examined, 4 pre-service teachers gave correct answers, 4 pre-service teachers gave wrong answers and 30 pre-service teachers gave incomplete answers. After the pre-service teachers gave their own answers, the formal mathematical definition was shared with the pre-service teachers.

“ $f: R \to R$, $y = f(x)$ function given.

Double function: $\forall x \in R$ için $f(-x) = f(x)$ f is a double function.

Single function: $\forall x \in R$ için $f(-x) = -f(x)$ f is the only function.”

After this stage, pre-service teachers were asked to make self-evaluations for their own answers and, according to the formal definition, they were asked to write down their deficiencies/mistakes, if any, in the missing/mistakes section of the worksheet given in Figure2.

Varsa;

EKSİKLERİM:

....
....

HATALARIM:

....
....

Figure 2. Sample question from the first worksheet (continued)

Figure 2 mentions the following:

If there is;

MY SHORTCOMINGS:

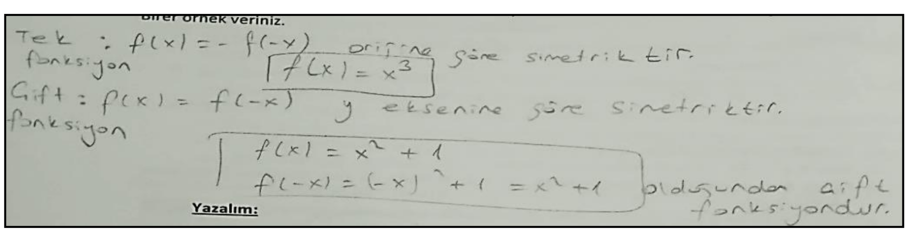
....

MY MISTAKES:

....

The worksheet in Figure 2 is designed for self-regulation, one of the critical thinking sub-skills determined by Facione (1990). Only one of the pre-service teachers who gave the wrong answer wrote his mistake in the section given in Figure 2. On the other hand, 26 of the 30 pre-service teachers who gave incomplete answers filled in Figure 2 and expressed their shortcomings. Table 3 includes the first answers of the teacher candidates and the examples they wrote in the part of my mistakes/shortcomings.

Table 3. Answer examples of teacher candidates from the first worksheet

Example1) Teacher candidate’s first answer	

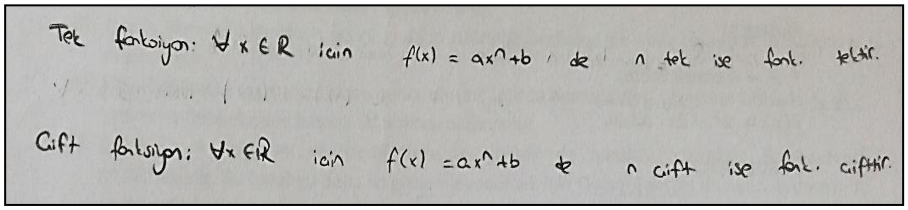
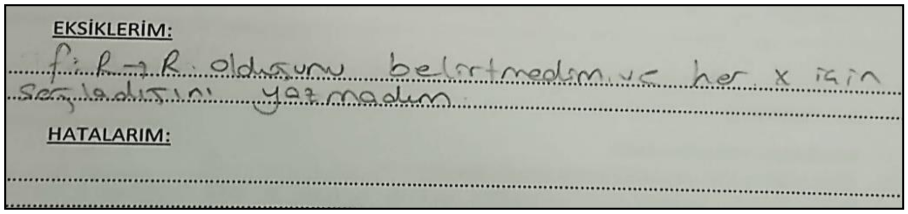
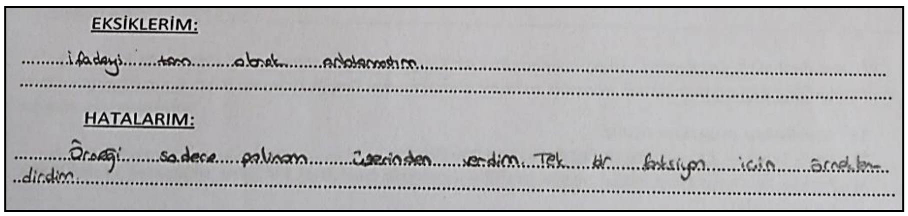
My mistakes / my shortcomings template response	
Example 2) Teacher candidate's first answer	
My mistakes / my shortcomings template response	

Table 3. mentions the following:

Example 1) Teacher candidate's first answer	*Odd function: f(x)= - f(-x) is symmetrical with respect to the origin.* $f(x)=x^3$ *Even function: f(x)= f(-x) is symmetrical with respect to the y-axis. It is dual function since it is* $f(x)=x^2+1$, $f(-x)^2+1=x^2+1$
My mistakes / my shortcomings template response	*MY SHORTCOMINGS:* *I didn't specify F: R → R and didn't write what it provides for every x.* *MY MISTAKES:* -
Example 2) Teacher candidate's first answer	*Odd function:* $\forall x \in R$ *için* $f(x)=ax^n+b$ *If n is odd, the function is odd.* *Even function:* $\forall x \in R$ *için* $f(x)=ax^n+b$ *If n is even then the function is odd.*
My mistakes / my shortcomings template response	*MY SHORTCOMINGS:* *I could not fully explain the expression.* *MY MISTAKES:* *I just gave the example over the polynomial. I exemplified for one example function.*

3.2 *Sample GeoGebra application from the second worksheet*

In the lesson where the second worksheet was applied, the GeoGebra activity in Figure 3 was applied. The purpose of this application is to create an environment where prospective teachers will be able to obtain the $y = f(k.x)$ transformation rule using GeoGebra.

UYGULAMA 4) $y = f(k.x)$ **dönüşümü**

➢ $f(x) = x^2$ olsun.

1) GeoGebra programını çalıştırınız sağ tarafta açılan pencereden Grafik Çizme seçeneğini seçiniz.
2) **Sürgü** aracını seçiniz. Açılan pencerede **k** isimli bir sürgü oluşturup minimum değeri **-10**, maksimum değeri **10** ve artış miktarını **0.1** olarak tanımlayınız.
3) **Giriş** kısmına x**^2** yazıp **Enter** tuşuna basınız.
4) **Giriş** kısmına ($k.x$)**^2** yazıp **Enter** tuşuna basınız.
5) Oluşturduğunuz sürgüdeki noktayı sağa ve sola hareket ettirerek fonksiyonun grafiğindeki değişimleri inceleyiniz.
6) Özel olarak $k = 2$ ve $k = 1/2$ seçiniz ve $y = x$^2 , $y = (2x)$^2 ve $y = (\frac{1}{2}x)$^2 grafiklerini çizdiriniz.
7) $A(-4,4), B(-2,4), C(-1,4), D(1,4), E(2,4)$ ve $F(4,4)$ noktalarının yerlerini işaretleyerek gözlemleyiniz.

Figure 3. Sample GeoGebra application from the second worksheet

Figure 3. mentions the following GeoGebra application:

APPLICATION 4) $y = f(k.x)$ *transformation*

Let f(x)= x^2

1) *Run the GeoGebra program, select the Graphic Drawing option from the window that opens on the right.*

2) *Select the slider tool. In the window that opens, create a slider named k and its minimum value is -10, define the maximum value as 10 and the increment as 0.1.*

3) *Type x ^ 2 in the input field and press Enter.*

4) *Type (k.x) ^ 2 in the input field and press Enter.*

5) *Examine the changes in the graph of the function by moving the point on the slider you have created to the right and left.*

6) *Especially choose k=2 ve k=1/2 and draw the graphics of* $y=x^2$, $y=(2x)^2$ *and y=1/2.x*

7) *Observe by marking the locations of the points A(-4,4), B(-2,4), C(-1,4), D(1,4), E(2,4) and F(4,4.).*

The GeoGebra application in Figure 3 is designed for interpretation, analysis, inference, evaluation and analysis skills from the critical thinking sub-skills determined by Facione. Pre-service teachers were given enough time to complete the application and get a result. The preservice teachers wrote their results in the section in Figure 4 below the application.

Uygulama sonunda elde ettiğim sonuçlar ....
....
....
....
.... şeklindedir.

Figure 4. Sample GeoGebra application from the second worksheet (continued)

Figure 4 mentions the following:

The results I got at the end of the application are; ....
....

After obtaining their own results, the formal mathematical definition was shared with the prospective teachers:

"The graph of the function $y = f(k.x)$ *is the graph of the function* $y = f(x)$ *that contracts k times the horizontal direction along the x-axis."*

When the results obtained from the application were examined, 8 pre-service teachers reached the formal result. 22 teacher candidates provided incomplete information in their results. This situation was considered as an incomplete result. 9 teacher candidates could not reach valid results and obtained wrong results.

After this stage, pre-service teachers were asked to compare their own results with the formal result and express any deficiencies/mistakes, if any, in the section of the worksheet in Figure 5.

Varsa;

EKSİKLERİM:

....

....

HATALARIM:

....

....

Figure 5. Sample GeoGebra application from the second worksheet (continued)

The section in Figure 5 is designed for the self-regulation skill, one of the critical thinking sub-skills determined by Facione. All of the pre-service teachers who achieved incorrect (9 teacher candidates) and incomplete (22 teacher candidates) results wrote their deficiencies/mistakes in the section given in Figure 5. Table 4. includes the results of the teacher candidates and the examples they wrote in the section of my mistakes/shortcomings.

Table 4. Examples of results obtained on the second worksheet

Example 1) Result of the teacher candidate	Uygulama sonunda elde ettiğim sonuçlar k değeri mutlak değer olarak arttıkça f(x) fonksiyonunun kolları birbirine yaklaşmaktadır yani kollar arasındaki açıklık daralmaktadır. $f(x)=x^2$ grafiği k sürgüsü hareket ettirildikçe genişleyerek veya daralarak bu fonksiyonların üçüyle de çakışıyor şeklindedir.
My mistakes / my shortcomings template	Varsa; EKSİKLERİM: X ekseni boyunca yatay yönde ve daralma HATALARIM: —
Example 2) Result of the teacher candidate	Uygulama sonunda elde ettiğim sonuçlar $y=f(kx)$ fonksiyonu $y=f(x)$'in k katıdır şeklindedir.
My mistakes / my shortcomings template	EKSİKLERİM: Hangi yönde daraldığını yazmadım. HATALARIM: k katı olarak belirtmek yerine mutlak değer olarak anladım.

Table 4 mentions the following:

Example 1) Result of the teacher candidate	*The results I got at the end of the application are as follows; As the value of k increases in absolute value, the arms of the function f (x) converge. In other words, the span between the arms gets narrower. The graph f (x) = x2 overlaps all three of these functions, expanding or contracting when the slider k is moved.*
My mistakes / my shortcomings template	*<u>MY SHORTCOMINGS:</u>* *Along the X-axis, in the horizontal direction and contraction* *<u>MY MISTAKES:</u>* -
Example 2) Result of the teacher candidate	*The results I got at the end of the application are the function y=f(k.x) is k times of y=f(x)*
My mistakes / my shortcomings template	*<u>MY SHORTCOMINGS:</u>* *I didn't write in which direction it contracted.* *<u>MY MISTAKES:</u>* *I did not specify k as a multiple. Instead, I understood it as absolute value.*

3.3 *Sample question and analysis from the third worksheet*

In this worksheet, teacher candidates were asked the question in Figure 6 and enough time was given to answer them. In this question, pre-service teachers were asked to establish a connection between the translation transformation and the limit issue. In this way, it was aimed to direct the teacher candidates to think from a different perspective in the solution of the problem. Figure 6. aims to increase the skills of interpretation, analysis, evaluation, inference and explanation among the critical thinking sub-skills determined by Facione.

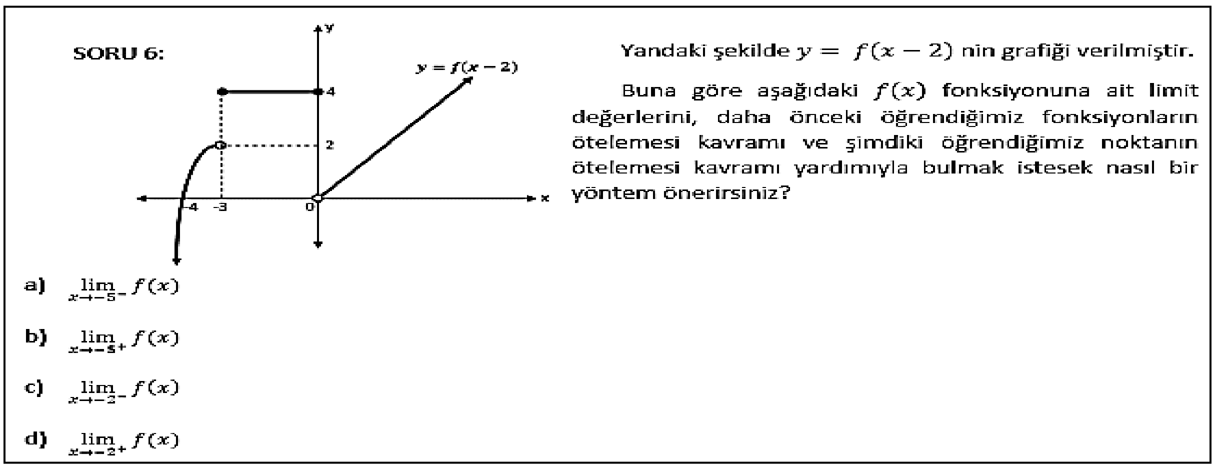


Figure 6. Sample question from the third worksheet

Figure 6 mentions the following question:

QUESTION 6: The graph of y = f (x-2) is given in the following figure. Accordingly, if we wanted to find the limit values of the function f (x) below with the help of the concept of translation of the functions we learned earlier and the concept of translation of the point we learned now, what method would you suggest?

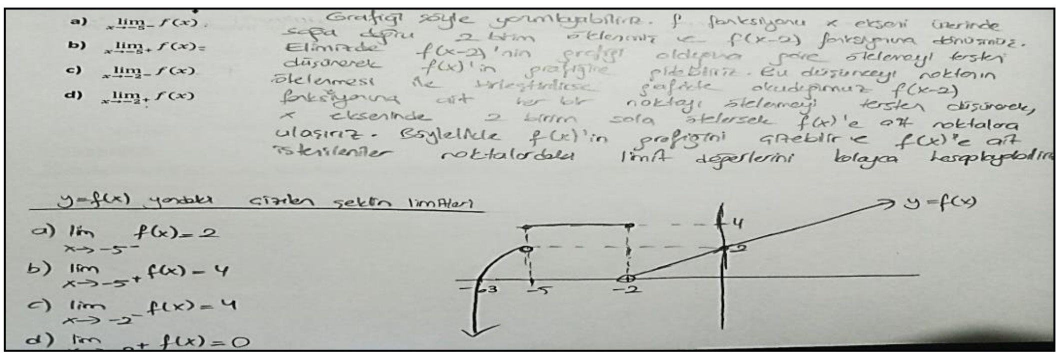
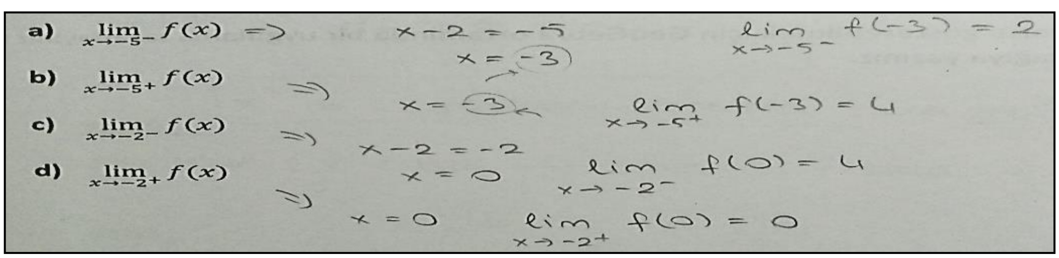
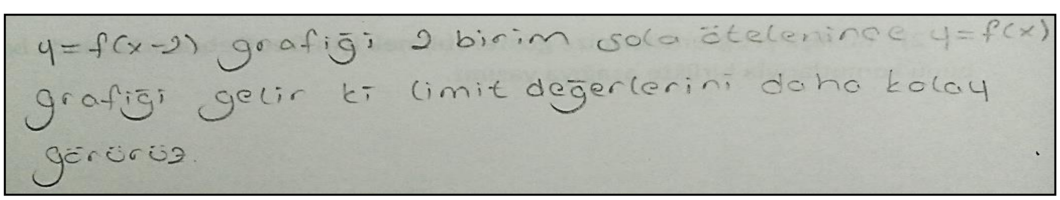
The limit values asked in the options a, b, c and d of the question in Figure 6 were answered correctly by all teacher candidates. However, as stated in the main text of the problem, 27 pre-service teachers proposed a method for how to solve it using the concept of translation. Table 5 includes content analysis of teacher candidates' solutions.

Table 5. Sample question solution analysis from the third worksheet

Theme	Code	f
6th question	*1. Two unit left translation and method of plotting f (x)*	17
	2. Solution only without method specified	13
	3. Two unit left translation method (f (x) is not graphed)	10

For the answer to the sixth question in Figure 6 above, which was examined as the third worksheet sample question, the pre-service teachers suggested as a method to shift the function f (x-2) to the left by 2 units at most, and then plot the f (x) formed by this displacement. (17 teacher candidates). Thirteen pre-service teachers, on the other hand, solved the limit values of the problem without specifying the method that the problem actually wanted. Examples of the solutions of the pre-service teachers for the codes of the sample question from the third worksheet are given in Table 6.

Table 6. Sample question solutions from the third worksheet

Code	Pre-service teacher solution
Code1	
Code2	
Code3	

a) $\lim_{x\to -5^-} f(x)$ b) $\lim_{x\to -5^+} f(x)$ c) $\lim_{x\to -2^-} f(x)$ d) $\lim_{x\to -2^+} f(x)$

Grafiği şöyle yorumlayabiliriz. f fonksiyonu x ekseni üzerinde sağa doğru 2 birim ötelenmiş ve f(x-2) fonksiyonuna dönüşmüş. Elimizde f(x-2)'nin grafiği olduğuna göre ötelemeyi tersten düşünerek f(x)'in grafiğine gidebiliriz. Bu düşünceyi noktanın ötelenmesi ile birleştirirsek grafikte okuduğumuz f(x-2) fonksiyonuna ait her bir noktayı ötelemeyi tersten düşünerek, x ekseninde 2 birim sola ötelersek f(x)'e ait noktalara ulaşırız. Böylelikle f(x)'in grafiğini çizebilir ve f(x)'e ait istenilen noktalardaki limit değerlerini kolayca hesaplayabiliriz

y=f(x) yandaki çizilen şekle limitleri

a) $\lim_{x\to -5^-} f(x)=2$

b) $\lim_{x\to -5^+} f(x)=4$

c) $\lim_{x\to -2^-} f(x)=4$

d) lim f(x)=0

a) $\lim_{x\to -5^-} f(x)$ ⟹ $x-2=-5$, $x=-3$, $\lim_{x\to -5^-} f(-3)=2$

b) $\lim_{x\to -5^+} f(x)$ ⟹ $x=-3$, $\lim_{x\to -5^+} f(-3)=4$

c) $\lim_{x\to -2^-} f(x)$ ⟹ $x-2=-2$, $x=0$, $\lim_{x\to -2^-} f(0)=4$

d) $\lim_{x\to -2^+} f(x)$ ⟹ $x=0$, $\lim_{x\to -2^+} f(0)=0$

y=f(x-2) grafiği 2 birim sola ötelenince y=f(x) grafiği gelir ki limit değerlerini daha kolay görürüz.

Table 6 mentions the following solutions:

Code1	*We can interpret the graph as follows. The f function has been shifted 2 units to the right on the x-axis and transformed into f (x-2). Since we have the graph of f (x-2), we can go to the graph of f (x) by thinking the translation in reverse. We can connect this thought with the translation of the point. If we think of shifting each point belonging to the function f (x-2) that we have read in the graph in reverse and shifting 2 units to the left on the x axis, we reach the points of f (x). Thus, we can draw the graph of f (x) and easily calculate the limit value of f (x) at the desired points.*

Code2	-
Code3	*When the graph of y = f (x-2) is shifted 2 units to the left, y = f (x) graph comes up. So, we can see the limit value more easily.*

3.4 *Sample question and analysis from the fourth worksheet*

Starting with the question given in Figure 7 to the lesson in which the fourth worksheet was applied, prospective teachers were asked to find and prove the rule that gives the coordinates of the P' point, which is formed by rotating the *P* point around *O* (origin) in a positive direction by α angle.

Yandaki şekilde olduğu gibi P noktasının, O (orijin) etrafında pozitif yönde α açısı kadar döndürülmesiyle oluşan P' noktasının koordinatları $P'(x', y')$ olsun. Bu bilgiler ışığında P' noktasının apsis ve ordinatını x, y ve α cinsinden bulunuz.

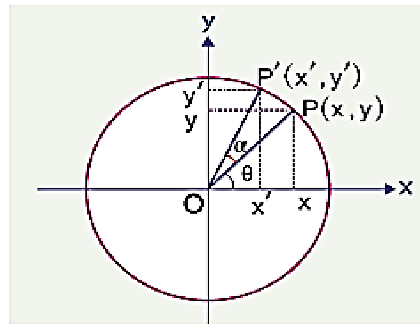


Figure 7. Sample question from the fourth worksheet

Figure 7 mentions the following question:

Let the coordinates of the P 'point, which is formed by rotating the P point around O (origin) about α angle in the positive direction, be P' (x ', y') as in the figure on the left. In the light of this information, find the abscissa and ordinate of the P 'point in terms of x, y and α.

Sufficient time was given for the prospective teachers to find the formula and prove it. The formula and its proof in the question is aimed at the skills of interpretation, analysis, evaluation, inference and explanation from the critical thinking sub-skills determined by Facione. When the proofs were examined, 12 teacher candidates were able to reach the correct proof. 1 teacher candidate made false proof and 23 teacher candidates made incomplete proof. 5 teacher candidates left the question blank. After the pre-service teachers made their own solutions, the formal mathematical formula and proof were shared with the pre-service teachers.

“ $\frac{x'}{r} = cos(\alpha + \theta) \rightarrow x' = r.cos(\alpha + \theta)$

Similarly, $y' = r.sin(\alpha + \theta)$

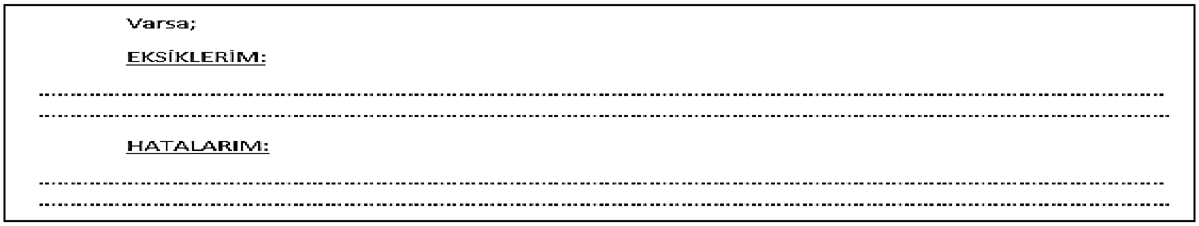
$$x' = r.cos(\alpha + \theta) = r(cos\alpha.cos\theta - sin\alpha.sin\theta) = r.cos\alpha.cos\theta - r.sin\alpha.sin\theta = x.cos\alpha - y.sin\alpha$$

$$y' = r.si\,n(\alpha + \theta) = r(sin\alpha.cos\theta + sin\theta.cos\alpha) = rsin\alpha.cos\theta + rsin\theta.cos\alpha = x.sin\alpha + y.cos\alpha$$

The P' point obtained by rotating the P point around the origin by the angle α in the positive direction;

$P'(x', y') = (x.cos\alpha - y.sin\alpha, x.sin\alpha + y.cos\alpha)$”

After this stage, pre-service teachers were asked to make self-evaluations for their own answers and they were asked to write down their deficiencies/errors, if any, in the section in Figure 8, according to the formal answer.


Varsa;

EKSİKLERİM:

....

....

HATALARIM:

....

....

Figure 8. Sample question from the fourth worksheet (continued)

The section in Figure 8. is designed for self-regulation skill, one of the critical thinking sub-skills determined by Facione. The pre-service teachers who gave the wrong answers wrote their mistakes in the section given in Figure 8. Of the 23 pre-service teachers who gave the missing answers, 21 stated their shortcomings by filling in Figure 8. Table 7 includes the first answers of the teacher candidates and the examples they wrote in the part of my mistakes/shortcomings.

Table 7. Solution examples from the fourth worksheet

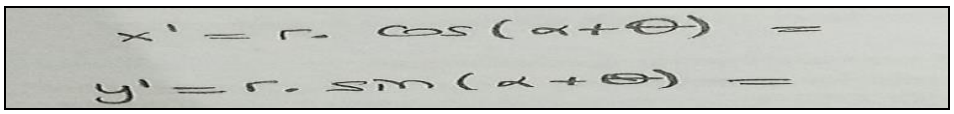
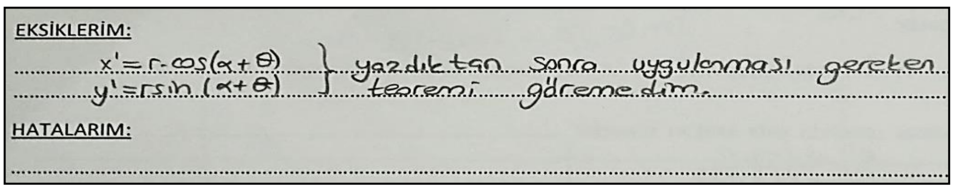
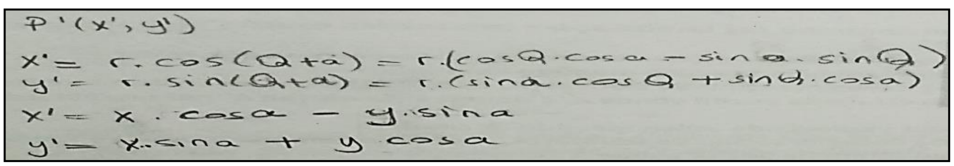
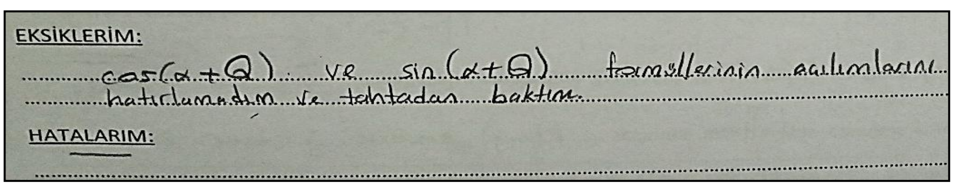
Example 1) Pre-service teacher response	 $x' = r. \cos(\alpha+\theta) =$ $y' = r. \sin(\alpha+\theta) =$
My mistakes /my shortcomings template response	 EKSİKLERİM: $x'=r.\cos(\alpha+\theta)$, $y'=r.\sin(\alpha+\theta)$ } yazdıktan sonra uygulanması gereken teoremi göremedim. HATALARIM:
Example 2) Pre-service teacher response	 $P'(x', y')$ $x' = r.\cos(Q+a) = r.(\cos Q.\cos a - \sin a.\sin Q)$ $y' = r.\sin(Q+a) = r.(\sin a.\cos Q + \sin Q.\cos a)$ $x' = x.\cos a - y.\sin a$ $y' = x.\sin a + y \cos a$
My mistakes /my shortcomings template response	 EKSİKLERİM: $\cos(\alpha+Q)$ ve $\sin(\alpha+Q)$ formüllerinin açılımlarını hatırlamadım ve tahtadan baktım. HATALARIM:

Table 7 mentions the following question:

Example 1) Pre-service teacher response	-
My mistakes / my shortcomings template response	*MY SHORTCOMINGS:* $x' = r.cos(\alpha + \theta)$ $y' = r.sin(\alpha + \theta)$ } *I could not see the theorem that should be applied after I wrote that.* *MY MISTAKES:* -
Example 2) Pre-service teacher response	-

My mistakes / my shortcomings template response	*MY SHORTCOMINGS:* $cos(\alpha + \theta)$ *ve* $sin(\alpha + \theta)$ *I could not remember the expansions of the formulas and looked from the board.* *MY MISTAKES:* -

3.5 *Sample question and analysis from the fifth worksheet*

In the fifth worksheet, teacher candidates were asked the question in Figure 9 and enough time was given to answer them. This question was prepared for the sub-skills of interpretation, analysis, inference, evaluation and explanation, which are among the critical thinking sub-skills determined by Facione. In the question, pre-service teachers were asked to think about how to apply a method to find the symmetry of a point with respect to the line.

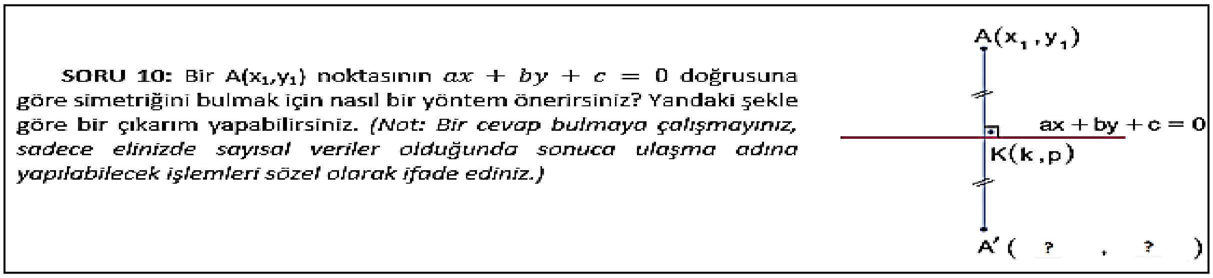


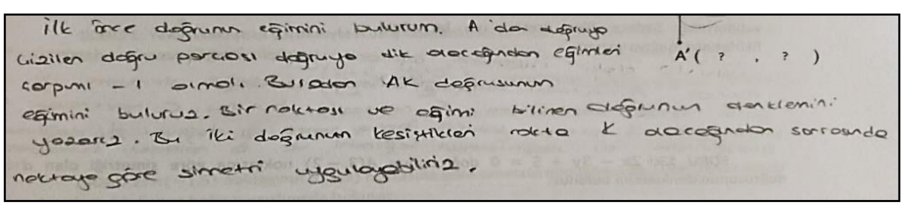
Figure 9. Sample Question from the Fifth Worksheet

Figure 9 mentions the following question 10:

QUESTION 10: *What method would you suggest to find the symmetry of the point A (x1, y1) with respect to the line ax + by + c = 0? You can make an inference according to the figure on the right. (Note: Do not try to find an answer, only verbally express the actions that can be taken to reach the result when you have numerical data.)*

For the question in Figure 9, 29 teacher candidates suggested a correct solution method and 6 of them suggested a wrong solution method. 7 teacher candidates left the question blank. The prospective teachers who gave correct answers found the slope of the given equation and found the line equation that is perpendicular to it and passing through point A. They calculated the point K (k, p) by solving the equation together with the obtained equation. Finally, they stated that the desired point to be found can be calculated by taking the symmetry of the point A with respect to K (k, p). All of those who gave the wrong answer stated that the answer can be found by calculating the distance from the point to the line. In Table 8, examples for the solutions of teacher candidates are given.

Table 8. Solution examples from the fifth worksheet

The right solution	

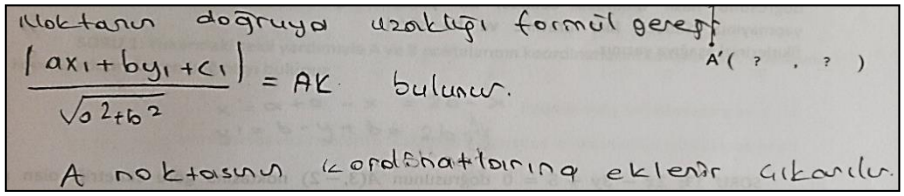
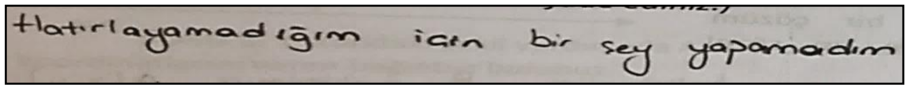
The wrong solution	
Blank	

Table 8 mentions the following solution:

The right solution	*First I find the slope of the line. Since the line segment drawn from A to the line will be perpendicular to the line, the product of its trends should be -1. Here we find the slope of the AK line. We write the equation for a line whose point and slope are known. Since the intersection of these two lines will be K, we can apply symmetry with respect to the point afterwards.*
The wrong solution	*The distance from the point to the line is* $\frac{\lvert ax_1+by_1+c_1\rvert}{\sqrt{a^2+b^2}} = AK$ *according to the formula.* *Is added to the coordinates of point A and subtracted.*
Blank	*I couldn't do anything because we couldn't remember.*

4. Discussion and conclusion

When the first worksheet sample question was analyzed, the results showed 4 trainee teachers gave correct answers, 30 pre-service teachers gave incomplete answers, and 4 prospective teachers gave wrong answers. When the continuation of the question was examined, 26 of the 30 candidates that did not give complete answers, and only one of 4 candidates who did not give the right answers completed my shortcomings/mistakes section. 27 (79%) of 34 teacher candidates who should complete this section stated their shortcomings or mistakes. In the fourth worksheet applied three weeks later, prospective teachers were asked to find and prove the rule that gives the coordinates of the *P'* point, which is formed by rotating the *P* point around *O* (origin) by the angle α in the positive direction. While 12 pre-service teachers reached the correct proof, 23 pre-service teachers made missing and 1 teacher candidate made false proofs. In the continuation of the worksheet, it was determined that 22 (92%) of a total of 24 pre-service teachers who made incomplete or false proofs made self-regulation by completing my shortcomings / mistakes section, and were able to examine their own ideas by using their own inferences, analysis and evaluation skills. During the period between the first worksheet and the fourth worksheet applications, the applications of the designed lesson plans continued. In line with the findings, it was concluded that the implementation of lesson plans improved the self-regulation skill, which is one of the critical thinking capabilities necessary to be a teacher.

When the sample question in the third worksheet was examined, the limit values required to be calculated were found by all teacher candidates. However, 13 pre-service teachers did not give an idea about how to arrive at a solution by relying on the concept of translation. Therefore, the results suggest that teacher candidates (32%) who did not express an opinion had difficulty in revealing the similarities between the two different approaches that were given as a solution to the problem requiring analysis and explanation skills. However, in the fifth worksheet applied two weeks later, the prospective teachers were expected to formulate a way to apply a method to find the symmetry of a point with respect to the line, and 29 pre-service teachers suggested a correct solution method and 6 pre-service teachers suggested a wrong solution method. 7 teacher candidates left the question blank. Therefore, we inferred that 70% of the prospective teachers described the methods and presented the reason in the form of convincing and consistent results, that is, they could use their analysis and explanation skills. Finally, it was concluded that the ongoing lesson plans practices contributed towards the advancement of student teachers' analysis and explanation skills.

Basri and As'ari (2018) designed a teaching model with specially designed tasks (questions) for enhancing high school students' critical thinking abilities. Using this study, we intend to determine how students' critical thinking skills change before and after they've learned. In the study, critical thinking skills were analyzed in terms of assessment, inference, explanation and self-regulation skills. At the end of the analysis, the findings indicated that students' ability to think critically had improved. In order to be able to do critical thinking, they stated that students should be given a task that requires them to employ their critical reasoning skills.

Our study yielded results that were similar to the results of the Basri and As'ari (2018)'s research. It was concluded that the implementation of lesson plans from the first worksheet to the fifth worksheet enhanced the critical thinking abilities of future teachers.

Similarly, the results of this study corroborate the results of the following studies; the study in which Suh (2010) determined that suitable environments were created for the advancement of critical thinking skills within the mathematics discipline with the support of technology; the study in which Peter (2012) pointed out that instructors should demonstrate how to think, use reasonable questioning strategies, and encourage students' critical thinking efforts so as to strengthen students' ability to think critically; Obay's (2009) qualitative research using problem solving as a method in teaching critical thinking.

Although the study shows similarities, there are also studies in which there are differences. As part of the fourth worksheet of the study, the candidates were required to discover and prove the rule that gives the coordinates of the P' point, which is formed by rotating the P point around O (origin) by the angle α in the positive direction. When the answers were examined, 12 pre-service teachers made the proofs true and 1 teacher candidate made a false proof. 23 teacher candidates gave incomplete answers and 5 pre-service teachers left the question blank. Considering that 29% of the pre-service teachers can make a complete proof, it can be concluded that the success of the teacher candidates in making proof is low. This result is inconsistent with the result that Ceylan's (2012) GeoGebra software helps pre-service teachers make assumptions and motivates them to prove. The fact that there are no practices for proving with GeoGebra in the teaching process is thought to cause the pre-service teachers not to show the progress in Ceylan's (2012) study.

In the research, observations regarding the in-class activities were made during the teaching. Teacher candidates actively participated in the lesson. They carefully solved the questions on the worksheets distributed to them and completed the GeoGebra applications to the end. When faced with situations that they could not understand or remember, they did not hesitate to ask questions to the researcher in order to resolve this situation.

In the worksheet, the results, formulas and definitions of the related outcome were not given ready-made, and the pre-service teachers were directed to discover them themselves. With the questions given in the worksheet and GeoGebra applications, they made an effort to reach these results and expressed their results mathematically in their own words. They compared these results with the formal results shared by the researcher at the end of the application and corrected any deficiencies or errors, if any.

Consistent with the observation findings, when the designed lesson plans were evaluated in terms of pre-service teacher response findings and in-class teacher candidate dialogue findings on the worksheets, it was concluded that the lesson plans included activities aimed at using the critical thinking skills of the pre-service teachers. Accordingly, considering the expressions on the worksheets and the interactions within the lesson, it can be said that the teaching process contributes to the development of pre-service teachers' critical thinking skills.

In addition, although they were not directed to work as a group, it was observed that in-class practices led the teacher candidates to communicate, ask their opinion and discuss with their desk mates, their friends in front of them or behind them. From here, it can be concluded that the lesson plans designed are more suitable for group work.

Based on the conclusion that pre-service teachers have low proof levels, lesson plans designed for research can be revised by adding proof problems/applications using GeoGebra to its content.

In lesson plans designed to develop critical thinking, it may be recommended that the implementation period should be well planned, spread over time and include a large number of applications.

Considering that the designed lesson plans are effective on high-level thinking skills such as critical thinking, studies can be conducted on the effects of these and similar applications on different higher-order thinking skills such as problem solving and creative thinking in the field of mathematics.

The application of the worksheets of this research to high school students and reporting the results are thought to be important in terms of comparison. Because it is important to reveal how much critical thinking skills have progressed with a lesson plan that is above or close to the knowledge level of the student.

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