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EVOLUTION OF PRE-SERVICE MATHEMATICS TEACHERS' SPATIAL VISUALISATION SKILLS DURING A COGNITIVE LOAD THEORY-BASED EDUCATION

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Abstract

This study explores how pre-service mathematics teachers' spatial visualisation skills evolved during a Cognitive Load Theory (CLT) based education. The study used the qualitative theory-testing case study method, which guided the identification of participants, the design of technology-supported education, and the data collection and analysis process. The four participants meeting specific criteria were selected as the study sample. A CLT-based education equipped with technology was provided to help participants overcome difficulties in spatial visualisation problems, improve their existing schemas, and build higher-order schemas. Various teaching approaches (e.g., worked examples) were applied to optimise participants' learning in CLT-based education. The study data (e.g., transcripts of interviews) were analysed using the pattern-matching technique, in which the observed patterns were compared with the derived hypotheses from the theoretic models regarding the problem-solving process and novice-expert schemas. The study achieved remarkable results: In CLT-based education, where teaching approaches have an important role, the improvement in their spatial visualisation skills happened as the participants overcame their challenges in problem-solving steps throughout their cyclic problem-solving processes and gained more knowledge and skills. The participants' acquisition of expertise in spatial visualisation skills went through various developmental stages. They strengthened their initial spatial problem-solving schemas by completing the deficiencies in their prior knowledge. They gained practicality in same-category tasks and constructed higher-order problem-solving schemas when dealing with high-category tasks by activating their assimilation and adaptation processes.
Keywords: *Cognitive Load Theory, the development of spatial visualisation skills, theory-testing method, acquiring an expert spatial problem-solving schema*

Introduction

Spatial and geometric (spatial-geometric) knowledge and skills of mathematics teachers directly and profoundly impact the quality of their students' knowledge and skills which they acquire. The term spatial-geometric refers to the intertwined bond between geometry and spatial thinking that is impossible to separate from each other fully. Learners' spatial thinking abilities significantly impact their proficiency in comprehending, articulating, and utilising geometry (Hawes & Ansari, 2020). On the other hand, geometry assists learners in comprehending spatial concepts and procedures and allows them to solve real-life spatial problems (Susilawati et al., 2017). Therefore, it is essential to highlight that the term "spatial visualisation" used in this study is directly connected to spatial-geometric knowledge, thinking processes, and competencies. The evolution of mathematics teachers' spatial visualisation skills provides a comprehensive and in-depth understanding of how their students gain those skills.

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Spatial visualisation plays a critical role in mathematical performance, so it has been the subject of much research over the years. Spatial visualisation includes (a) perceiving and interpreting visual and spatial information (Rafi et al., 2006), (b) understanding exactly three-dimensional (3D) objects from their two-dimensional (2D) representations (Korakakis et al., 2009), (c) imagining moving parts of 2D and 3D objects (Burnett & Lane, 1980; Lowrie & Logan, 2023), and (d) comprehending and performing complex and multi-step transformations/ rotations (Linn & Petersen, 1985; Miyake et al., 2001). As in this study, tasks that require visualising and reasoning about the surface nets of solids can be used to assess individuals' spatial skills.

Spatial visualisation, essential for problem-solving, planning, and decision-making, is a skill that can be honed with practice, and research has established that technology-based education can significantly enhance spatial performance. This study focuses on how pre-service mathematics teachers' spatial visualisation skills evolve with technology-supported education. Dynamic geometry environments (DGEs) are commonly used technologies to develop learners' spatial-geometric knowledge and skills in mathematics education. They are highly preferred due to their interactive nature and the ability to visualise mathematical concepts dynamically. They support acquiring conceptual and procedural knowledge and improving sophisticated cognitive skills, such as spatial visualisation, geometrical and spatial reasoning, and creative thinking. DGEs provide opportunities to visualise 2D or 3D geometric figures, investigate their properties (Adelabu et al., 2019; Sarracco, 2007), develop and implement problem-solving strategies (Koyuncu et al., 2015; Kuzle, 2017), and formulate and test hypotheses within the problem-solving context (Mevarech & Kramarski, 2014). Considering the supportive power of DGEs in learning, Cabri3D was used to shape the technological dimension of this study.

DGEs have limited potential to support the learning process on their own. Therefore, research suggests educational materials should be designed by considering the cognitive and developmental levels of the learners and the limitations of human cognition to significantly enhance the quality, effectiveness, and efficiency of learning. Cognitive Load Theory (CLT) presents a guideline for educators to optimise technology-based education through a multidirectional framework comprising the most effective teaching approaches. Multidirectionality implies deciding whether learners need guidance, what activities are necessary for effective learning, how teaching materials should be designed, and in which order tasks should be presented. This study provides pre-service teachers with a technology-supported learning environment based on CLT, enabling them to develop their spatial problem-solving processes and schemas in multiple directions.

CLT and Its Relationship with Problem-solving Processes and Schemas

The CLT, used as a reference framework in designing the teaching process of this study, was formulated by John Sweller and his colleagues (See Sweller, 1988; Sweller et al., 2019; Sweller & Levine, 1982). The theory offers a broad perspective on structuring teaching based on how the human brain processes and retains information. By leveraging insights into the brain's cognitive architecture, educators can design teaching optimised for the learners' cognitive capacities. This type of teaching promotes the assimilation and retention of new information while preventing cognitive overload. CLT was used in designing and conducting this study's teaching process to help pre-service teachers acquire advanced spatial visualisation skills by managing their cognitive load.

CLT was formulated based on the functions of working- and long-term memory. Working memory, a cognitive system that temporarily holds a limited amount of information, is responsible for processing information and managing complex cognitive tasks such as learning mathematical or spatial-geometric concepts and procedures and solving mathematical or spatial-geometric problems (See Cowan, 2014). Long-term memory is the storage with unlimited capacity, which holds information for a long time or permanently in mental structures called schemas (Paas & van Merriënboer, 2020). The information comprising concepts, procedures, problems, and corresponding solutions is systematically arranged and categorised hierarchically within these schemas.

A hierarchical approach was adopted in designing this study's teaching process to enable pre-service teachers to construct high-order spatial visualisation schemas. The creation of increasingly complex schemas results from integrating lower-level schemas into higher-level ones, leading to the development of expertise (Paas & van Merriënboer, 2020). New information is integrated into the existing knowledge to generate more sophisticated and nuanced schemas. The high-level schemas' construction and their automatic processing reduce cognitive load (Paas & van Merriënboer, 2020). Learners can perform tasks or solve problems without conscious effort as they become proficient in employing schemas. Novices tend to process information with a more cautious and controlled strategy due to their limited working memory capacity, so their performance on various problem-solving tasks is often suboptimal.

When confronted with a problem, learners tend to compare it with the problems in their schemas, which are categorised and organised based on the similarities of their solution processes (See Sweller, 1988). If the problem matches the one in their schemas, they can solve it by recalling the relevant solution process from their long-term to working memory (See Nokes et al., 2010). For example, asking pre-service teachers to perform spatial visualisation tasks they are familiar with can activate problem-solving processes stored in their long-term memory. Suppose an appropriate problem-solving schema does not exist. In that case, they consciously process the information in the problem within their working memory and resort to more general problem-solving strategies (e.g., working backwards, breaking the problem into sub-problems) to arrive at a solution. The absence of an appropriate problem-solving schema will likely lead to suboptimal performance outcomes. Those who exhibit perseverance in problem-solving think deeply about their solution process; that is, they rehandle and evaluate their solution process, which compasses problem-solving steps – the comprehension of the problem, the appropriateness of the selected strategy, and the accurate execution of the solution procedures without any errors (See Nokes et al., 2010). Pre-service teachers may face challenges in problem-solving steps due to excessive cognitive load in their working memory caused by unfamiliar tasks or their primitive schemas.

Components of Technology-supported Teaching Designed Based on Problem-solving Activities Where Cognitive Load is Managed

The characteristics of learners and tasks play a crucial role in guiding a course design that facilitates the effective and efficient use of working memory resources while promoting schemas' formation in long-term memory. Cognitive load theory draws attention to different teaching approaches to improve the problem-solving skills of novices by managing their cognitive load and turning them into experts who own advanced problem-solving schemas and use them automatically. These teaching approaches can be used either independently or in conjunction with each other. This adaptability allows customising the teaching process to suit learners' needs and preferences. This study utilised multiple CLT teaching approaches simultaneously to design an optimal learning environment for pre-service teachers, in which they can overcome challenges while solving spatial problems, increase and deepen their conceptual and procedural knowledge, develop their geometric and spatial reasoning skills, acquire flexible thinking skills, and develop high-order spatial visualisation schemas. Firstly, the approaches that support novice learners in reaching the correct solution and developing a deep understanding of problem-solving are presented below. Then, teaching approaches that facilitate novices in constructing problem-solving schemes and attaining automaticity in them are included. It should be remembered that the teaching approaches employed in problem-solving significantly impact the creation and development of schemas.

Teaching Approaches Concerning Problem-Solving Process

When spatial tasks place a heavy cognitive load on pre-service teachers' working memory, leading to difficulties in different solution steps of these tasks, various teaching approaches can be applied to manage their cognitive load and overcome these challenges. These approaches, which can improve participants' cognitive abilities, support them in increasing their knowledge and skills and help them complete those tasks successfully, are:

- *Goal-free problem:* A conventional problem is characterised by a starting point, a specified endpoint, and a set of processes that transform the starting point into the endpoint, but a goal-free problem or no-goal problem does not have a predetermined endpoint or final destination (See Paas & Kirschner, 2012).
- *Problem-solving first:* Learners are given an opportunity to solve problems using their prior knowledge before providing any instruction (González-Cabañes vd., 2021, 2023).
- *Guided learning:* Learners are encouraged to identify the problem, determine its objective and constraints, generate solutions, determine the solution, analyse, synthesise, and evaluate the solution (Stonewater, 1980, as cited in Masuhara, 1983).
- *Worked example:* Instead of requiring learners to solve a problem themselves, they are provided with a step-by-step solution to study and learn from (Ayres, 2012).
- *Self-explanation:* Learners explain the learning material by generating information not provided in the given material (Salle, 2020). They tend to create their understanding by connecting their existing knowledge with the new information presented in the material.
- *Imagination:* Learners can imagine a procedure or concept instead of studying worked examples (Ignatova et al., 2020). Conversely, after studying the worked example, they can imagine the concept or the steps involved in the procedure (Sweller et al., 2011).

Approaches Concerning Constructing and Developing Problem-Solving Schemes

There are distinct CLT teaching approaches which can help pre-service teachers gain expertise in spatial tasks by building higher-order problem-solving schemas. These are:

- *Categorisation:* Becoming an expert requires categorising problems based on their fundamental characteristics, such as domain-specific knowledge, problem states, strategies employed, and associated moves for solutions (See Sweller, 1988). Novices tend to categorise problems based on their superficial characteristics, such as specific geometric shapes presented within them.
- *Variability:* To create new tasks, one can make either superficial or structural changes to existing tasks (Likourezos et al., 2019). The tasks derived from superficial changes can be solved by relying on learners' existing knowledge and recalling previously learned solutions. The tasks with structural changes require learners to improve their knowledge and skills in order to find solutions.
- *Prior knowledge and sequencing tasks from simple to complex:* The tasks are arranged in order of increasing complexity because learners build complex knowledge and skills onto simple ones (See van Merriënboer & Sweller, 2005). However, when learners possess a considerable amount of prior knowledge, the task at hand tends to become less complex (Endres et al., 2023)

This study aims to investigate how the spatial visualisation skills of pre-service teachers evolve through a technology-supported education based on CLT. This education consisted of teaching sessions utilising the teaching approaches defined above. An investigation into the evolution of spatial visualisation skills was handled from diverse perspectives, including problem-solving steps and cycles, the process of gaining expertise, the knowledge and skills required for expertise, and the construction of problem-solving schemes. In line with these perspectives, answers were sought to the following research questions:

- How does the given CLT-based education affect eliminating pre-service teachers' challenges in spatial visualisation tasks' solution steps?
- How do the participants' spatial visualisation problem-solving cycles work during the CLT-based education?
- What stages of expertise do pre-service teachers go through during their CLT-based education process?
- What types of knowledge and skills are pre-service teachers equipped with throughout their CLT-based education to gain expertise in spatial visualisation tasks?
- How are pre-service teachers' spatial visualisation problem-solving schemas constructed throughout their CLT-based education?

Research Methodology

An Overview of the Methodology

The study, which addressed the evolution of participants' spatial visualisation skills with technology-supported education based on CLT, was formed from three parts. In the first phase, participants completed pre-drawing tasks, and task-based interviews were conducted with them. In the second part, each participant completed CLT-based education, which consisted of six teaching sessions lasting approximately one hour each. In the third part, after completing their education, the participants completed post-drawing tasks. Following this, task-based and non-task-based interviews were conducted with them.

The qualitative theory-testing case study method guided the identification of this study's sample (four participants who met specific criteria), designing technology-supported teaching, and collecting and analysing data. The qualitative theory testing method determines whether the evidence collected supports the propositions of a relevant theory in a case or group of cases (Dul & Hak, 2007). The teaching and the tasks used in this study were designed according to the CLT teaching approaches regarding the problem-solving process and the acquisition of automatic problem-solving schemas. The data collected from task-based and non-task-based interviews and the learning process were analysed considering CLT's pre-identified structures, propositions, and assumptions. When using case studies for theory testing, it is essential to identify testable propositions and translate abstract concepts into observable ones (Bhattacherjee, 2012; Chukwudi vd., 2019). In the study, it was tested whether participants' spatial visualisation skills evolved in parallel with the principles defined in CLT about the process of problem-solving and the acquisition of automatic problem-solving schemas. However, the evolution of the participants' spatial visualisation skills was only considered in the context of creating surface nets for solids.

Participants

The sample was selected among second-year lower secondary school mathematics teacher education program students. The purposeful sampling method, preferred in theory testing, was used, so four participants who met specific criteria constituted the study's sample.

The sample, meeting five specific criteria, was determined among students who participated in the spatial visualisation test and the spatial visualisation drawing tasks consisting of low (5 tasks) and high (5 tasks) complexity tasks. The aim of using criteria in sampling is to reveal all cases that satisfy predetermined criteria and then subject these cases to review and analysis (Patton, 1990). These were: (a) having poor or moderate performance on the spatial visualisation test; (b) having challenges in different solution steps of spatial visualisation drawing tasks; (c) demonstrating novice behaviour in high-complexity tasks while in low-complexity tasks behaving like a novice or someone in the transition from novice to expert; (d) being described by their lecturers as someone who can articulate their thoughts and provide strong verbal explanations; and (e) willing to participate in the CLT-based education. Studying learners with low- and moderate-level spatial visualisation skills would provide more data about various cases related to the evolution process of their spatial visualisation skills. Purposive sampling is an effective method for identifying a sample that contains high-quality and information-rich data on situations that are of central importance (Coyne, 1997; Emmel, 2013). That sample provides accurate, meaningful answers to the research questions (Emmel, 2013). The study participants' names were replaced with pseudonyms, and the research was conducted with all necessary permissions. Maral's success rate was average at 58% in the spatial visualisation test. Onat's success rate was below average at 44%, while Tara and Pera had a low success rate of only 30%. All four participants were in the novice stage (NS) in high-complexity drawing tasks. Tara and Maral were in the transition stage from novice to expert (NtoES) in low-complexity drawing tasks, while Pera and Onat were in the NS in two low-complexity drawing tasks and were in the NtoES for the other two.

Data Collection Process and Procedures

The study employed a variety of data collection tools, including pre/post-drawing tasks, pre/post-task-based interviews, non-task-based interviews carried out after CLT-based education, observations, educator guidelines, and learners' explanations. Data collection tools were shaped to reveal participants' problem-solving processes, schemas, and their automaticity in using these schemas. Spatial visualisation tasks of making net drawings of solids were crucial in collecting data. In each category, a minimum of one task necessitated the participants to construct multiple nets of the given solid using geometric tools such as a ruler and a protractor. For accurate task performance, one had to consider the geometric properties of the solid and its faces. The solid's surface had to be unfolded towards its base to reveal its inner surface entirely.

The participants' responses to low/high complexity pre/post-drawing tasks, the records of interviews conducted based on these tasks and the records of six teaching sessions were the data collected for analysing participants' spatial visualisation problem-solving processes, including problem-solving cycles and stages, and the changes on their knowledge and skills throughout the CLT-based education. The recorded teaching sessions included data concerning how participants created drawings of solids' nets, the educator's guidelines, and participants' self-explanations. Five different cases were focused on the collected data: The first was in which problem-solving step the participants experienced challenges in each task and how these difficulties were resolved through CLT-based teaching approaches. The second was the participants' problem-solving cycles completed by one or more accurate net drawings, error-free (optimal) solution/s for each task. Throughout one or more problem-solving cycle/s, participants' challenges in various steps of the problem-solving process and/or their attempts to tackle the problem using alternative strategies were handled. The third relates to the knowledge (conceptual and procedural) and skill (chains of reasoning, spatial skills, and flexible thinking) types participants acquired and/or increased to gain expertise in the tasks by CLT approaches. The fourth addressed determining the stages of expertise (NS, NtoES and expert stage (ES)) in each one of the participants' task answers. The last was identifying the cognitive processes involved in constructing automatic problem-solving schemas.

Upon completion of the CLT-based education, participants were interviewed to comment on and express their opinions about the technology-supported learning they experienced, including the evolution of their spatial visualisation skills in each session, as well as the whole CLT-teaching process. During the interviews, the participants articulated their viewpoints regarding the influence of the CLT-teaching approaches on their development and competence in solving spatial visualisation tasks and performance in accomplishing each task specific to a given category. These interviews aimed to gather data about the participants' problem-solving processes, schema acquisition, and proficiency in performing spatial visualisation tasks without conscious effort (automatically).

Designing the CLT-based Education

The following six subheadings explain how CLT teaching approaches were used to design this study's technology-supported learning environment to develop participants' spatial visualisation skills.

- *Problem-solving-first approach and cyclic problem-solving with guided learning:* The term problem-solving activity describes a cyclic process of constructing at least one correct net drawing of a solid. That process likely began with the participant's suboptimal solution, which was made without guidance. It continued with the educator's guidance, in which the participant was directed to become aware of and overcome his/her challenges in any of the problem-solving steps to construct an accurate net drawing or to support them in resolving the task through other strategies.
- *Goal-free problems:* The tasks of making net drawings of solids presented in Cabri3D were used in this study. A net drawing task is a goal-free problem that can be solved without a specific strategy or following specific procedures. A participant could generate various strategies to construct the nets of any solid, select a less demanding one and construct one of the nets of the given solid by applying immediately applicable procedures without trying to recall the specific procedures within a particular order.
- *Worked examples:* At least two Cabri3D models, each embedding a different solution process, were designed to serve as worked examples for each task. The models were intended to help participants overcome challenges that could have emerged during any step of the problem-solving process. (a) Understanding the solid's configuration: Using Cabri3D to manipulate, move, rotate, and measure edges and angles of solids to identify their geometrical properties. (b) Understanding the task requirements and developing a strategy: Unfolding a solid to reveal its entire inner surface by satisfying two specific conditions, which were to prevent the simultaneous movement of faces not in the same plane and to rotate a face along the common fold line to bring it to the plane of its neighbouring face. If the conditions were complied with, any strategy modelled in Cabri3D could be grasped as a pattern of actions consisting of the rotational movements of each one of the solid's faces, except its base. (c) Implementing the strategy: Consider the spatial and geometric relationships both in the solid and its net. (d) Evaluating the solution: comparing two nets (both or one of these were of Cabri3D) belonging to the same solid or examining which edge pairs will join when the net of Cabri3D is folded.
- *Categorizing the tasks and sequencing them from simple to complex:* The six teaching sessions focused on six category-specific tasks. The first category included prisms, second pyramids, third concave prisms, fourth truncated prisms, the fifth composite

solids formed by two solids and the sixth composite solids formed by three solids. The complexity of the tasks increased gradually from the first to the last category and within each category.

- *Using low or high variability tasks:* Same-category tasks, meaning low variability tasks, could be solved by taking advantage of their notable similarities with the solution process of the previously solved task (e.g., the hexagonal prism's net can be drawn by making relatively minor changes in the solution process of the triangular prism whose net was previously made). In the study, tasks other than the initial tasks of the same category were referred to as low variability tasks. The first tasks of each category, except the first category, were called high variability tasks because they could be solved by significantly modifying the solution processes of previously solved same-category tasks.
- *Self-explanation and imagination:* Participants were guided to imagine and make self-explanations throughout their cyclical problem-solving processes, dependent on or independent of the Cabri3D models. The educator's directions included (a) imagining and explaining the properties of the solid and its faces and the strategies used to unfold the surface of the solid, (b) explaining how they applied the strategy which they developed and how they constructed the solid net composed of face shapes,(c) describing the unfolding and/or folding process embedded in Cabri3D model, (d) explaining the similarities and differences between net drawings of themselves and the net embedded in Cabri3D model, or between the two nets they produced for the same solid, or between the two Cabri3D solid's nets, and (e) imagining and explaining which edge pairs will join when the net of Cabri3D is folded or when the net drawings of themselves are folded.

Data Analysis

The study data were analysed using the pattern-matching technique, a fundamental procedure applied in theory-testing case studies described in Campbell's work (1975). This technique compares the expected patterns from the theory to be tested with the observed patterns highlighted in the study data. Before the comparison, the expected patterns should be formulated by drawing insights from the theoretical model that had been previously reported, as well as from the researcher's ideas and experiences. Then, the initial observations, namely the identified patterns, were subjected to interpretation, classification, and clustering based on their distinguishing characteristics. Thereafter, the patterns were compared against the anticipated or predicted observations outlined by the theoretic model. Lastly, an assessment was made to determine how well the observed patterns aligned with the theoretical patterns.

Through analysis of data obtained from six teaching sessions, responses to low- and high-complexity drawing tasks and interviews conducted based on these tasks, five different cases were identified regarding the participants' spatial visualisation problem-solving skills matched with CLT:

Case 1. Four discernible patterns were identified concerning the CLT approach's effect on the challenges in spatial visualisation problem-solving steps: understanding the problem, developing a strategy, implementing the chosen strategy, and evaluating the solution. *Case 2.* A rigorous analysis revealed participants' thoughts about their problem-solving processes until achieving at least one optimal outcome. These cyclical processes provided data about participants' challenges in problem-solving steps and/or whether the same task was solved with different strategies. *Case 3.* The analysis concerning gaining expertise through CLT-based education revealed five observable patterns related to a change in knowledge or skills: the deepened conceptual and procedural knowledge and the increased reasoning, spatial thinking, and flexible thinking skills. *Case 4.* Three patterns, closely aligned with the principles of CLT, were uncovered regarding the participants' stages of expertise in each task: NS, NtoES and ES.

Case 5. The interview analysis concerning evaluating the effect of technology-supported teaching conducted at the end of the CLT-based education and analysis of six teaching sessions brought out that arranging teaching tasks categorically from simple to complex by considering the participants' prior knowledge and including low and high-variability tasks concretised that a two-step iterative process consistent with CLT principles was activated in the acquisition of problem-solving schemas. These processes, treated as observable patterns that helped participants develop their spatial visualisation problem-solving skills, were assimilation and accommodation. The process of assimilation refers to the ability to gain proficiency and automaticity in same-category tasks. Accommodation involves the modification of participants' problem-solving schemas to deal with higher-category tasks.

The collected data was analysed using five cases, presented in Table 1. Data, investigator, and theory triangulation were utilised to increase the study's validity (Flick, 2004). Data triangulation includes combining data drawn from different sources (e.g., interviews, drawings, teaching sessions) and different participants. Investigator triangulation refers to using more than one investigator or data analyst in the study. In parallel with this, the data collected in this study was analysed by a second experienced mathematics educator. Lastly, theory triangulation addresses approaching data with multiple perspectives (e.g., problem-solving steps, gaining expertise).

Table 1
Codes Used in Analysing Data Addressing the Observable Patterns Compatible with CLT

Case 1's observable patterns compatible with CLT				
The effect of the CLT-based education on eliminating the participants' challenges in the problem-solving steps				
The effect of the CLT-based education on eliminating	the challenges in understanding the problem [CU] the challenges in developing a strategy [CS] the challenges in implementing the chosen strategy [CI] the challenges in evaluating the solution [CE]			
Case 2's observable patterns compatible with CLT				
Problem-Solving Cycles				
Problem-solving cycles with or without challenges in solution steps and problem-solving cycles including the implementation of different strategies: Problem-solving cycle [PSC] An accurate drawing [AD] Challenges experienced in any solution step (CU, CS and CI) throughout the problem-solving cycle Implementation of a different strategy [IDS] **Coding examples and explanations:** PSC1->AD The correct drawing was made after the first problem-solving cycle PSC1->CU->PSC2->AD->IDS1->AD In the first problem-solving cycle, the challenge of understanding the task was experienced The accurate drawing was made after the second problem-solving cycle The accurate drawing was again made when the task was solved by implementing a different strategy.				
Case 3's observable patterns compatible with CLT				
Improvement of participants' knowledge and skills for gaining expertise in the tasks through the CLT teaching approaches				
Conceptual Knowledge [CK]	Procedural Knowledge [PK]	Chains of Reasoning [CR]	Spatial Skills [SS]	Flexible Thinking [FT]
Case 4's observable patterns compatible with CLT				
The stages of gaining expertise in the tasks				
Novice Stage [NS]	The Stage of Transition from novice to expert [NtoES]	Expert Stage [ES]		
Case 5's observable patterns compatible with CLT				
The processes activated during the construction and development of automated problem-solving schemas				
Assimilation: Adopting a solution of a task in one category and using this to solve another task in the same category [ASSI]	**Accommodation:** Making modifications in the solutions of the known tasks to solve higher-category tasks [ACCO]			

Research Results

The study achieved remarkable results about how participants' spatial visualisation skills evolved through technology-supported education based on CLT. These results were: The improvement in spatial skills happened as the participants overcame their challenges in problem-solving steps throughout cyclic problem-solving processes and gained more knowledge and skills. During the CLT-based education, the participants' acquisition of expertise in spatial visualisation skills went through various developmental stages. By activating assimilation and adaptation processes, practicality in same-category tasks was gained, and higher-order problem-solving schemas were developed when dealing with high-category tasks. Furthermore, each CLT-based teaching approach played a significant role in improving the participants' spatial visualisation skills.

The CLT-based Education's Effect on Eliminating Challenges Experienced in Problem-Solving Steps

The study demonstrated that spatial visualisation tasks could overload participants' working memory and cause them to have challenges in problem-solving steps, but CLT-teaching approaches could be used as a way to cope with these challenges.

Challenges in understanding the task. The two main factors that hindered achieving the desired output in this phase were not comprehending the provided visual or textual information and the task requirements. Inability to process visual information based on geometric concept knowledge and mathematical thinking processes caused (a) the creation of the solid's internal representation mismatched with its external representation; (b) the misperceptions concerning some faces' geometric properties (e.g., side lengths, angle measures) and (c) the failure to visualise or the omission of drawing the solid's particular faces (especially its unseen faces). The inadequate background knowledge regarding a solid's surface (e.g., a solid's inner and outer surface) and faces and the limited experiences regarding how to unfold a solid to obtain its nets were the two reasons behind not understanding what was asked in the tasks. As a result, participants divided particular faces of the solid while unfolding it, did not take the solid's base as the reference face to unfold it, and did not unfold the solid so that its entire inner surface was visible (See Table 2, 3).

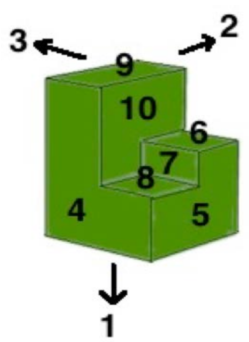
The effect of CLT-based education on eliminating challenges in understanding the task. Viewing the solid with different coloured faces from different perspectives by Cabri3D and measuring its faces' side lengths and angles supported participants in handling the solid and its properties from a conceptual and mathematical viewpoint, which helped them understand the presented visual information in the task. Increasing mental images about the solids and their surfaces and the solids' unfolding/folding processes enabled participants to expand their experiences about how to unfold/fold the solids and to develop their knowledge about what the surfaces and faces of solids were. Self-explanations, dependent on or independent of Cabri3D, assisted participants in understanding the problem in depth (See Table 2).

Table 2
Interviews with Pera, Tara, Maral and Onat Regarding Understanding the Task

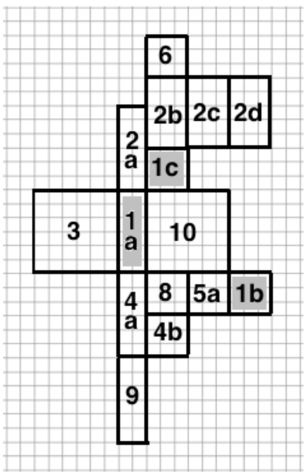
Participant	The interview sections concerning challenges in understanding the task and the effect of CLT-based education on eliminating challenges in understanding the task
Pera:	When I was looking at a solid having oblique faces, I was not able to imagine what kind of shapes these faces were. Are they trapezoid? Are they parallelograms? Which types of quadrilaterals they were? …. For instance, I predicted some faces as parallelograms because I saw them as parallelograms. But when I manipulated these solids in Cabri 3D, I understood that these faces were rectangular.
Tara:	I could not imagine some parts of the solids, because they were not visible from my viewpoint. I could not predict their geometrical properties…. I had difficulty imagining the back faces of the combined solids. I was trying to predict what types of geometric shapes they were or what properties they had. ... But, with Cabri 3D, I was able to see the parts where I could not see. When I rotated the solid left or right, I saw these invisible faces.
Maral:	For example, in this solid, I was thinking as if the surfaces of these three different solids were combined. I did not think that it had a single surface. Thus, I was drawing a lot of faces in my nets. I was drawing more faces than the solid's current number of faces. Now, I know that there is only a single L-shaped face here, but before the teaching, I thought there was one square and one rectangle face here. Now, I know that this square and this rectangular are joined. Before the sessions, I was thinking of them as separate shapes. I ought to have drawn them as only one face. I have understood that in the teaching sessions.
Onat:	Initially, I did not think which was the bottom face, or which were the side faces of the solid, while I was making my drawing…. I made my drawing without thinking what was asked in the task. I unfolded the solid directly. In the sessions, I realized that I was asked to make my drawing by unfolding the given solid to its base-plane.

Table 3
Maral's Response Showing that She did not Understand the Pre-Drawing Task

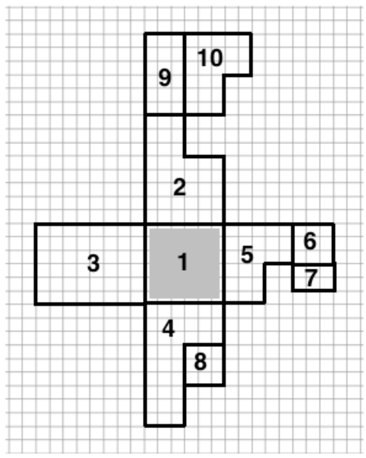
From the responses of spatial visualisation drawing tasks before CLT-based education



Composite Solid



Maral's Drawing



Accurate Drawing

Maral thought of the composite solid as the combination of three separate solids. These are the square prism with base 1a (its net includes 1a, 2a, 3, 4a, 9 and 10), the square prism with base 1c (its net includes 1c, 2b, 2c, 2d and 6) and the cube with base 1b (its net includes 1b, 5a, 8, 4b).

Challenges in developing strategies. This phase's two challenges were to develop the wrong strategy to open the surface of the solid and the inability to develop any strategy. The challenges were due to the belief that only one strategy could open the solid, which was engraved in memory. In the early sessions, participants' self-questions, such as whether they had made a mistake despite developing a correct strategy after seeing Cabri's strategy, were compatible with that belief (See Table 4).

The effect of CLT-based education on eliminating challenges in developing strategies. Observations on unfolding/folding strategies embedded in Cabri models belonging to different categories of solids, self-explanations on various different strategies modelled in Cabri3D, opportunities to imagine, generate or implement various strategies to unfold a solid and chances to make comparisons between their own strategies and those embedded in Cabri3D models were effective on improving participants' strategy development skills (See Table 4).

Table 4

Interviews with Pera and Onat Regarding Developing Strategies

Participant	The interview sections concerning challenges in developing strategies and the effect of CLT-based education on eliminating challenges to develop strategies
Pera:	(for a composite solid) Assume that I try to unfold this square prism a little bit, but this time I could not predict how to unfold these triangular prisms, and I gave up. When I unfolded the square prism, I could not find how to unfold this triangular prism and where to place its net. I could unfold this solid (touching the square prism), but when combined with the triangular prism I could not know how to unfold it.
Onat:	I thought there was only one strategy to unfold a solid; there was no other strategy for unfolding it. But I learned that there are a lot of different strategies in the sessions. I can start to unfold a solid by opening the faces to the top face. Or, I can start by opening the side faces to the bottom base face. Or, I can start by opening the right face to the plane of the back face... I can unfold a solid in many ways. There exist many strategies. I learned these. Before the teaching, I had said that I could unfold a solid by using only one strategy. Before the sessions, I did not need another strategy to unfold it. I did not try other strategies... I learned that I can use many different strategies because if I can not think about a specific strategy, I can unfold it by using another strategy which came to my mind.

Challenges in implementing the chosen strategy. Even if participants understood the problem and developed a correct strategy, they made errors in implementing it. Their errors encountered while drawing the nets were disregard for the spatial and geometric relations between the solid's adjacent faces (e.g. What edges and angles are adjacent to the common edges of two faces?), non-application of the definitions and properties of geometric shapes (the two base angles of an isosceles triangle are equal to each other), and non-compliance with geometric construction rules (e.g., how I can construct an angle whose measure is 60 degrees on one of the endpoints of a line segment) (See Table 5, 6, 7).

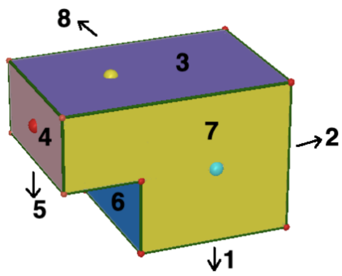
The effect of CLT-based education on eliminating challenges in implementing the chosen strategy. Participants were guided to correct their errors, including prompts to use Cabri3D models, imagine geometric and spatial relations and make self-explanations. With guidance, they focused on the spatial and geometrical relationships between the adjacent faces of the solid and its net. They articulated self-explanations concerning their conceptual knowledge of the definition and properties of geometric shapes, as well as their procedural knowledge of constructing such shapes. With these CLT-teaching approaches, the participants were able to overcome the challenges that emerged during the implementation phase of the strategy (See Table 5, 6, 7).

Table 5
Interviews with Pera and Maral Regarding Implementing the Chosen Strategy

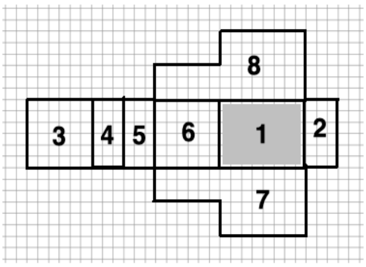
Participant	The interview sections concerning challenges in implementing the chosen strategy and the effect of CLT-based education on eliminating challenges in implementing the chosen strategy
Pera:	I knew the geometric shapes, but I could not reflect the shapes on my drawing by considering their geometric properties… I learned to draw the shapes by considering their edge lengths and angle measurements during teaching.
Maral:	For example, I had to draw a regular hexagon. I ought to have constructed its edges with equal lengths. I ought to have drawn the interior angles of the hexagon equal to 120 degrees. I experienced difficulty at first. When I drew the edges with equal lengths, I could not draw the angles in equal measurements. I had difficulty in constructing the shapes. I forgot to take into account some of the geometrical properties. But now, I can draw them easily.

Table 6
Onat's Response Shows her Disregard for the Spatial Relationships Between Specific Faces in the Pre-drawing Task and her Second Accurate Drawing

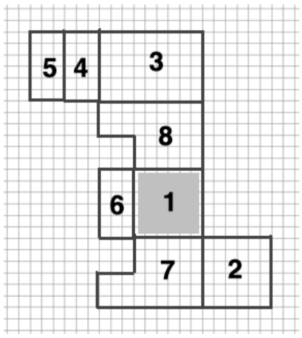
From the third teaching session



Concave Solid



Onat's First Drawing

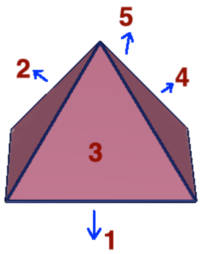
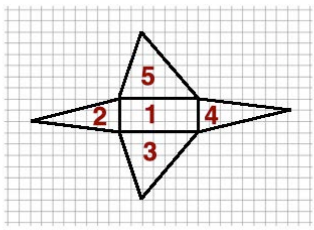
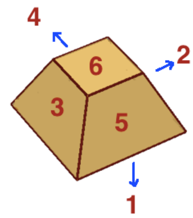
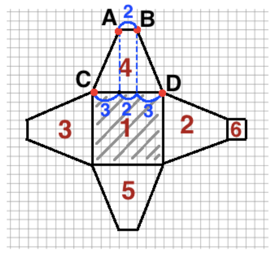


Onat's Second Drawing

Onat misplaced the faces labelled with 7 and 8 in her drawing in her first drawing, she recognized her misplacement errors while working with Cabri3D and corrected them in her second drawing.

Table 7

Onat's Response Showing That She could not Reflect the Properties of the Faces of the Solid in the Pre-drawing Tasks, and her Response Showing That She Reflected the Properties of the Faces of the Solid in the Post- drawing Task

From the responses in spatial visualisation drawing tasks before and after CLT-based education			
Pre-drawing Task		Post-drawing Task	
 Pyramid	 Onat's Drawing	 Truncated Pyramid	 Onat's Drawing
• Onat drew the side faces of the solid, which she saw as a rectangular pyramid, as a scalene triangle instead of an isosceles triangle. (Misrepresenting the geometric properties of triangles)		• Onat drew the side faces of the solid, which she saw as a truncated square pyramid, as congruent isosceles trapezoids. (Accurately representing the geometric properties of congruent isosceles trapezoids)	

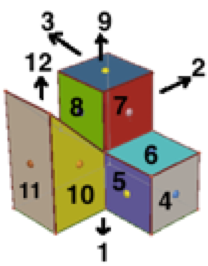
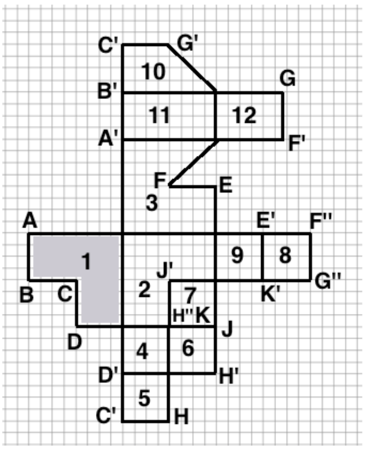
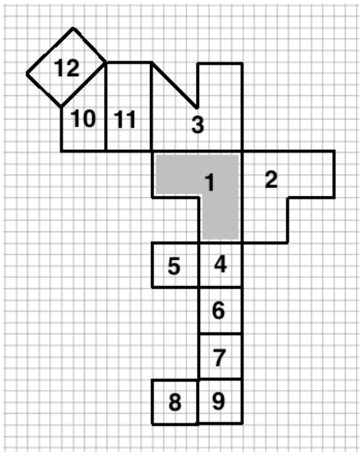
Challenges in evaluating the solution. Participants' general habits were checking without questioning where they might have made errors or not controlling or evaluating their net drawings before teaching sessions.

The effect of CLT-based education on eliminating challenges in evaluating the solution. Participants were guided to think about Cabri3D solutions, their own solutions, and alternative solutions through particular approaches: worked examples, imagination, and self-explanations. Guided by these approaches: (a) They checked their drawings to identify any errors in their solution steps. Once they found the error(s) and determined its(their) cause, they returned to the relevant step and made corrections as shown above. (b) They generated arguments, based on their imaginations, concerning the folding process of their nets to identify and correct any mismatches in the lengths of the edge pairs that would join. (c) They compared and explained the similarities and differences between their net drawings and the net modelled in Cabri 3D and reached some generalisations about obtainable nets through prototype, simple, and complex unfolding strategies. (d) They drew an alternative net for the same solid by utilising an inventive or more complex strategy, which differed from the one used in the initial drawing and required imagination and implementation. In short, participants were able to evaluate and handle their own solutions from an objective and reasonable viewpoint and develop alternative ways to solve the tasks (See Table 8, 9).

Table 8
Interviews with Onat, Pera, and Tara Regarding Evaluating the Solution

Participant	The interview sections concerning challenges in evaluating the solution and the effect of the CLT-based education on eliminating challenges in evaluating the solution.
Onat:	I learned that I must draw the edges in equal lengths, which would join when I fold my drawing; otherwise, my net is not closed. I am not able to get the solid.
Pera:	I learned that the number of faces in my drawing must equal the number of faces of the solid…
Onat:	Sometimes, I had difficulty positioning a particular polygonal face in my drawing. I remembered that I could position it in another place. I could change my strategy, I could use another strategy to unfold the given solid.
Tara:	I started by opening the side faces around the L-shape base, but Cabri 3D started by opening nearly all faces to the left rectangular face.

Table 9
Tara's Second Net Drawing Referring to her Approach, Which She used to Solve the Same Task in a Different Way

Tara's two accurate net drawings for a composite solid taken from the last teaching session		
The composite solid	**Tugce's first drawing**	**Tugce's second drawing**
		

Problem Solving Cycles

The problem-solving process was initiated with the commencement of the first cycle and continued until at least one accurate net drawing of the solid was produced. The table summarises the systematically collected data on each participant's problem-solving process and performance across all tasks (Table 10). Based on the table, if a participant created a correct drawing at the end of the first problem-solving cycle, it can be inferred that either the problem-solving process was completed, or the cycle was repeated utilising another strategy/ies until the accurate drawing/s was/were made again. For example, in the first problem-solving cycle, Maral reached the accurate drawing for the 5th task in the prisms category (See Table 10). She was not asked to tackle the task using a different strategy in that problem-solving process. Even though Tara reached the correct solution in the first problem-solving cycle in the second task in the prisms category (Table 10), she was asked to tackle the task again with two different strategies. Tara made accurate drawings using these strategies as well. From the table, it is clear that participants experienced difficulty/ies with the same or different problem-solving steps until they reached the accurate drawings during their problem-solving process. For example, in the first problem-solving cycle of the third task in the prisms category, Pera faced difficulty understanding the task but made the accurate net drawing by the end of the second cycle. From the table, the participants' problem-solving cycles in each of the tasks of each of the categories can be interpreted in similar ways. Through the table, the participants' problem-solving cycles in each task found in all categories can be interpreted similarly.

Table 10
Problem-solving Cycles of Each Participant for Each Task throughout the CLT-based Education

Solid Type	Tasks	Participants			
		Maral	**Onat**	**Pera**	**Tara**
Prisms	1	PSC1->AD-> IDS1->AD-> IDS2->AD	PSC1->AD-> IDS1->AD	PSC1->AD-> IDS1->AD	PSC1->AD-> IDS1->AD
	2	PSC1->AD-> IDS1->AD	PSC1->CU-> PSC2->AD-> IDS1->AD	PSC1->CU-> PSC2->AD-> IDS1->AD	PSC1->AD-> IDS1->AD-> IDS2->AD
	3	PSC1->CI-> PSC2->AD-> IDS1->AD	PSC1->AD-> IDS1->CI-> PSC2->AD-> IDS2->AD	PSC1->CI-> PSC2->AD	PSC1->CU-> PSC2->CI-> PSC3->AD-> IDS1->AD-> IDS2->AD
	4	PSC1->AD-> IDS1->AD	PSC1->AD-> IDS1->AD	PSC1->CU-> PSC2->AD	PSC1->AD
	5	PSC1->AD	PSC1->CI-> PSC2->AD-> IDS1->AD-> IDS2->AD	PSC1->CI-> PSC2->CI-> PSC3->AD	PSC1->CI-> PSC2->AD
	6	PSC1->AD	PSC1->AD	PSC1->AD	PSC1->AD-> IDS1->AD-> IDS2->AD-> IDS3->AD
Pyramids	1	PSC1->CI-> PSC2->AD-> IDS->AD	PCS1->CU-> PSC2->AD	PSC1->AD-> IDS1->AD	PSC1->AD
	2	PSC1->AD	PSC1->AD	PSC1->CU-> PSC2->AD	PCS1->CI-> PSC2->AD-> ISD1->AD
	3	PSC1->AD	PSC1->AD-> IDS1->AD	PSC1->AD	PSC1->AD

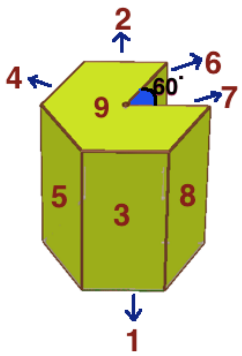
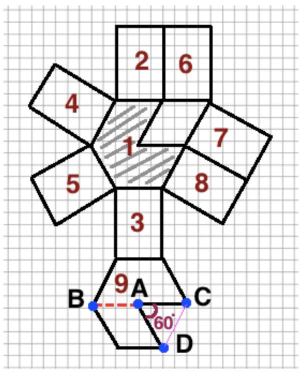
Concave Prisms	1	PSC1->CU-> PSC2->AD	PSC1->CI-> PSC2->AD	PSC1->CU-> PSC2->CI-> PSC3->AD	PSC1->CI-> PSC2->AD
	2	PSC1->AD	PSC1->CU-> PSC2->AD	PSC1->CI-> PSC2->AD	PSC1->CU-> PSC2->AD
	3	PSC1->CI-> PSC2->AD-> IDS1->AD	PSC1->CI-> PSC2->AD	PSC1->CI-> PSC2->AD	PSC1->AD-> IDS1->AD
	4	PSC1->AD	PSC1->AD-> IDS1->AD	PSC1->AD-> IDS1->AD	PSC1->CI-> PSC2->AD
Truncated Prisms	1	PSC1->CI-> PSC2->AD	PSC1->CS-> PSC2->AD	PSC1->CI-> PSC2->AD	PSC1->AD
	2	PSC1->AD-> IDS1->AD	PSC1->AD-> IDS1->AD	PSC1->AD-> IDS1->AD	PSC1->CI-> PSC2->AD
	3	PSC1->CI-> PSC2->AD	PSC1->CI-> PSC2->AD	PSC1->AD	PSC1->AD-> IDS1->AD
Composite Solids formed by two solids	1	PSC1->CU-> PSC2->AD	PSC1->CU-> PSC2->CI-> PSC3->AD	PSC1->CU-> PSC2->CI-> PSC3->AD	PSC1->AD
	2	PSC1->CI-> PSC2->AD-> IDS1->AD	PSC1->CU-> PSC2->AD	PSC1->AD-> ISD1->AD	PSC1->AD-> ISD1->AD
	3	PSC1->AD	PSC1->CI-> PSC2->AD-> IDS1->AD	PSC1->CI-> PSC2->AD	PSC1->AD
Composite Solids formed by three solids	1	PSC1->CS-> PSC2->CI-> PSC3->AD	PSC1->CI-> PSC2->AD	PSC1->CU-> PSC2->CI-> PSC3->AD	PSC1->AD
	2	PSC1->CI-> PSC2->AD-> IDS1->AD	PSC1->CI-> PSC2->AD	PSC1->CI-> PSC2->CI-> PSC3->CI-> PSC4->AD	PSC1->CI-> PSC2->AD
	3	PSC1->CI-> PSC2->AD	PSC1->CI-> PSC2->AD-> IDS1->AD	PSC1->AD-> IDS1-AD	PSC1->AD-> ISD1->AD-> ISD2->AD

Improvement of Knowledge and Skills for Expertise

It seemed like the goal-free task, worked example, self-explanation, imagination, and guided learning activities were helpful in improving the participants' thinking habits. Their thinking approaches were brought closer to those of an expert problem solver who possesses and applies conceptual and procedural knowledge, can build a chain of reasoning, has advanced spatial skills, and exhibits flexible thinking. The examples of how participants applied each of the components that make up the expert's knowledge network when drawing the solids' nets were:

Conceptual knowledge. It refers to knowledge about the definition and properties of solids and 2D geometric shapes (See Table 11).

Table 11
Applying Conceptual Knowledge

<table>
<tr><th colspan="3">From the responses in spatial visualisation drawing tasks after the CLT-based education</th></tr>
<tr><td colspan="3">Pera's net drawing and her self-explanations that contain her conceptual knowledge about the properties of a regular hexagon</td></tr>
<tr><th>Concave Solid</th><th>Pera's Drawing</th><th>Interview with Pera</th></tr>
<tr><td>
</td><td>
</td><td>Pera's self-explanations containing her conceptual knowledge of the regular hexagon:

First, I thought that the solid's base was a regular hexagon. I thought that one of the angles of the hexagon is 120 degrees. I drew a hexagon whose edges were 2 units long. I ensured that the angles between these adjacent edges were 120 degrees…</td></tr>
</table>

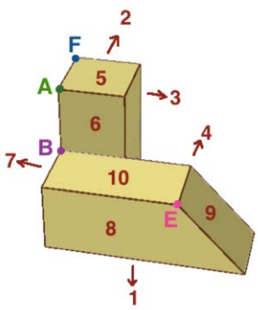
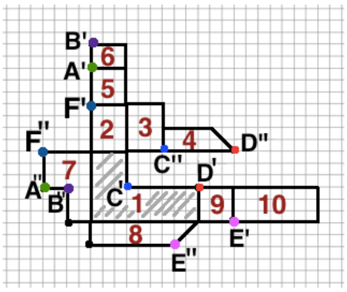
Procedural knowledge. It refers to knowing the construction rules and procedures necessary to draw geometric shapes and being able to use geometric tools as necessary (See Table 12).

Table 12
Applying Procedural Knowledge

Participant	Tara's self-explanations including her procedural knowledge about the construction of a regular hexagon during the CLT-based education
Tara:	I will take one side of the hexagon as 3 cm, find 120 degrees with the help of the protractor, and draw a 3 cm length side. Then, I will retake the protractor, try to find 120 degrees, and it will be 3 cm long. Then again, I will put it like this (She showed how to hold the protractor and measured the 120-degree angle). I will draw 3 cm again. (She determined 120 degrees and drew the 3 cm length side). (She combined her initial point with her endpoint by using a ruler to close the hexagon). ..It is completely closed. Angles are 120 degrees.

Chains of reasoning. It refers to a series of connected premises (assumptions) and conclusion (inferences or rebuttals) pairs. It includes determining pairs of edges that would join or not join by folding the faces in the net and then identifying which edges and corners of the solid would result from joining these edges (See Table 13).

Table 13
Creating Chains of Reasoning

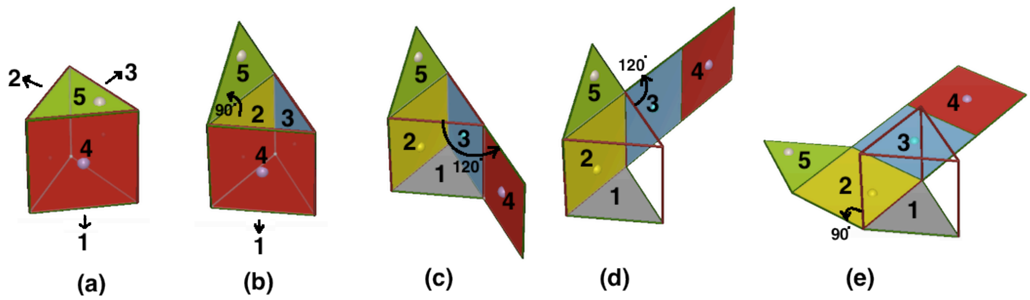
From the responses in spatial visualisation drawing tasks after the CLT-based education		
Tara's net drawing and her self-explanations that contain her chains of reasoning about how to fold that net		
Composite solid	**Tara's drawing**	**Interview with Tara**
		Tara's self-explanations concerning the corners meeting when she closes the net of the solid which she has drawn: As I close it now, I raise this back face 90 degrees. Similarly, I raise this left face 90 degrees up. When I raised these two faces 90 degrees up, the corner point (F') of the back face and this corner point (F'') of the left face overlapped. They overlap because this edge and this one are equal lengths. When they overlapped, I obtained this corner point of the solid (F).

Spatial visualisation skills. These include visually representing the non-visible faces of the solid, mentally comprehending the geometric properties of the faces of the solid and recognising the spatial relationships between them. It also involves imagining the rotation of the faces around the fold lines to obtain the net from the solid or the solid from its net (See Table 14, Figure 1).

Table 14
Applying Spatial Skills

Participant	Onat's self-explanations, taken from the CLT-based education, including the application of her spatial visualisation skills
Onat:	I rotated the green top face (5) 90 degrees around the fold line common with the solid's yellow face (2). The green face (5) was carried to the plane of the yellow face (2)… I rotated this square red face (4) 120 degrees and carried it to the plane of the blue face (3). I rotated the blue face (3) 120 degrees, and the red (4) and blue (3) faces were carried to the plane of the yellow face (2). Eventually, these green, red, blue and yellow faces were in the same plane. When I rotated the yellow face (2) 90 degrees around the common fold line with the base (1), all faces were carried to the base plane of the solid.

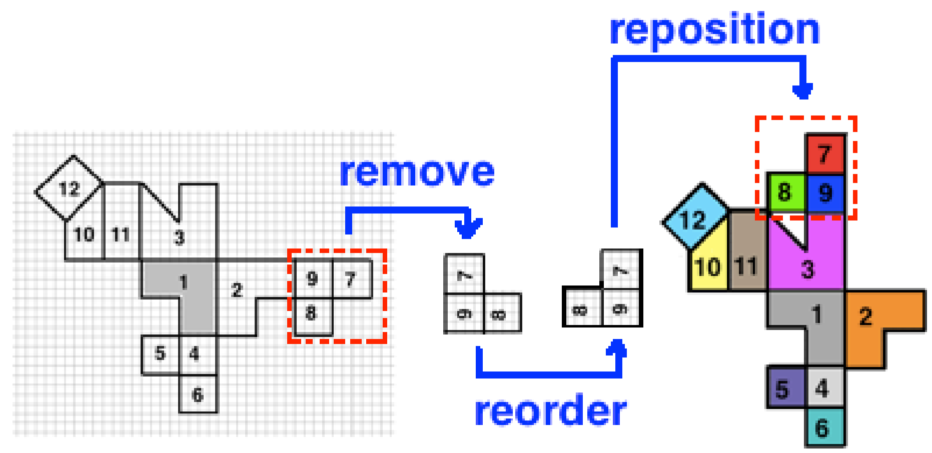
Figure 1
Representation of Onat's Imagination, Reflecting her Self-explanations about How to Open Triangular Prism by Using Her Spatial Skill



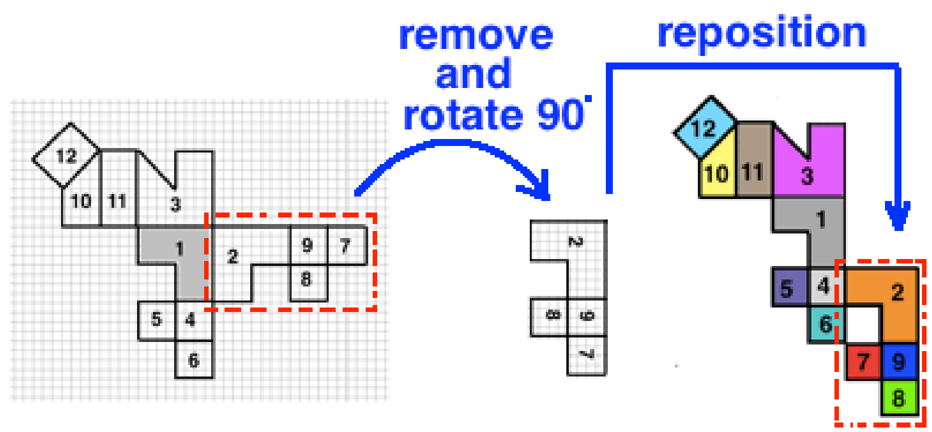
Flexibility. There are multiple ways to develop strategies to unfold a solid. For example, changing the position of one or more than one of the faces of the particular net of the solid to obtain another net of the solid (See Table 15).

Table 15
Gaining Flexibility

Maral's flexible thinking skill in drawing the nets of the composite solid
Maral's self-explanations included how she could obtain another net of the same solid by removing faces 7, 8 and 9 from face 2, reordering them, and repositioning them on face 3.



Maral's self-explanations included how she could obtain another net of the same solid by removing faces 7, 8, 9 and 2 from face 1, rotating them 90 degrees clockwise, and repositioning them on face 4.



The Stages of Gaining Expertise in the Tasks

The problem-solving first approach showed that participants' problem-solving skills in each category went through three stages: NS, NtoES and ES. Characteristics of the NS were: inability to form an accurate mental representation of the solid, failure to comprehend the properties of the given tasks, limited mental imagery of how to open the solid or not having any imagery, developing and applying wrong strategies, and having misunderstandings and misperceptions about the surface, nets, and properties of the solids. The NtoES was where participants faced challenges in implementing the strategy and evaluating their solution, despite their understanding of the solid's configuration and task requirements and developing an appropriate strategy. However, they still had minor misperceptions, misunderstandings, misconceptions, or incomplete and immature procedural knowledge compared to the NS. In ES, participants reached a correct solution and had a knowledge web similar to an expert problem solver. At that stage, the participants could produce multiple accurate net drawings of the given solid, indicating that they had become more flexible in performing tasks (See Table 16). Additionally, they could easily develop and visualise different unfolding strategies in their minds, meaning automaticity. Table 17 demonstrates the effect of CLT-based education on participants' spatial visualisation skills development by comparing their developmental stages (NS, NtoES and ES) for each drawing task before and after the teaching sessions.

Table 16

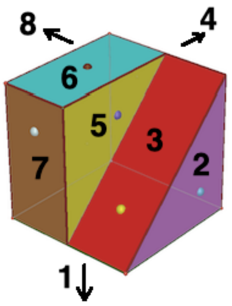
The Participants' Expertise Stages for Low - and High-complexity Drawing Tasks Before and After the CLT-based Education

Participants/ Tasks		Low-complexity Drawing Tasks					High-complexity Drawing Tasks				
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Pre-drawing Tasks	Tara	ES	ES	NtoES	NtoES	NtoES	NS	NS	NS	NS	NS
	Maral	ES	NtoES	NtoES	NtoES	NtoES	NS	NS	NS	NS	NS
	Pera	ES	NS	NtoES	NtoES	NS	NS	NS	NS	NS	NS
	Onat	ES	NtoES	NtoES	NS	NS	NS	NS	NS	NS	NS
Participants/ Tasks		**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Post-drawing Tasks	Tara	ES	ES	ES	ES	NtoES	NtoES	ES	NtoES	ES	NtoES
	Maral	ES	ES	ES	ES	ES	NtoES	ES	NtoES	ES	NtoES
	Pera	ES	ES	ES	ES	NS	NS	NtoES	NtoES	NtoES	NtoES
	Onat	ES	ES	NtoES	ES	NtoES	NS	ES	NS	ES	NtoES

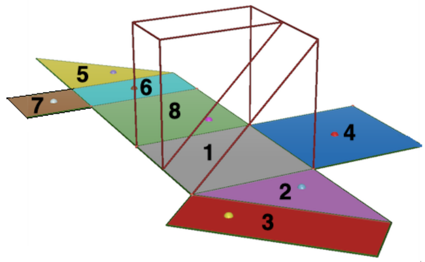
Table 17
The Three Stages of Gaining Expertise in the Tasks

The participants' net drawings, constructed in the CLT-based education, indicating their stages of gaining expertise —NS, NtoES and ES– for the composite solid

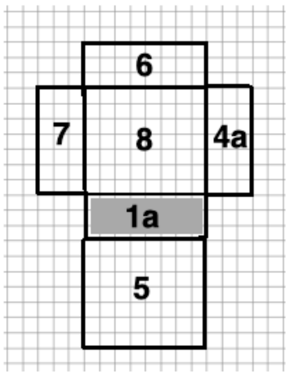
The composite solid



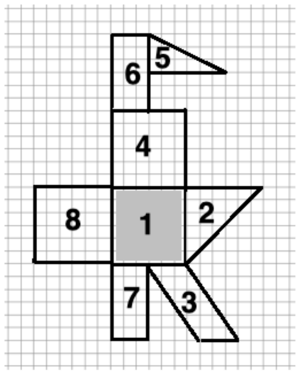
One of nets of the composite solid



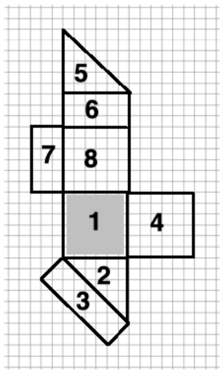
Maral, Onat and Tara's first problem-solving cycle:
NS: Maral had misunderstandings about the composite solid's nets so she thought that she could obtain the net of the composite solid by drawing the nets of the square prism and triangular prism separately.
NtoES: Although Onat developed an accurate strategy to unfold the composite solid, she was not able to reflect geometric features of the face 3 and misplaced the face 5 in her drawing.
ES: Tara accurately made the composite solid's net drawing.



Maral's Drawing
NS



Onat's Drawing
NtoES



Tara's Drawing
ES

The Processes Activated during the Construction and Development of Automated Problem-solving Schemas

Despite their prior knowledge and experiences, participants had weak problem-solving schemas for the first category tasks, while the other categories were new to them. The first session helped participants fix their prior knowledge, which involved misunderstandings, misperceptions, and conceptual and procedural knowledge deficiencies. It also allowed them to identify and restructure their faulty or distorted problem-solving habits and update their problem-solving schemes, which were necessary for constructing basic solids' nets (See Table 18).

High variability tasks enabled participants to accommodate their current category-specific problem-solving skills to the next higher new category-specific tasks. The accommodation process started with cognitive imbalance/disequilibrium, which was the moment when they realised that focusing on both similarities and differences between the solutions of the new category task and the previous category-specific tasks was necessary to regain a state of balance/equilibrium. After comparing the similarities and differences of potential solutions, they constructed new category-specific problem-solving schemas by making comprehensive changes in their existing schemes. These changes included acquiring new conceptual knowledge, producing new chains of reasoning, and creating new category-specific strategies (Table 20).

The low variability tasks allowed participants to improve and strengthen their problem-solving schemes by adapting the new learnings they gained after overcoming difficulties encountered in solution steps of the same-category tasks. The new learnings were assimilated and transferred into the new situations encountered while practising with the other tasks in the same category. Performing similar tasks helped participants become automatic, meaning they developed expertise in the same-category tasks (Table 18, 19).

Table 18
Interviews with Pera, Tara and Maral

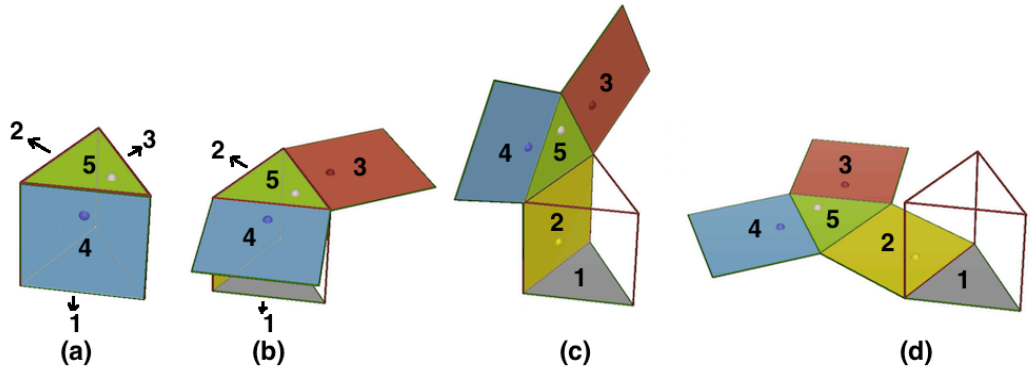
Participant	The self-explanations of Pera, Tara and Maral, taken from the interviews made after the CLT-based education, reflecting their activated learning processes during the construction and development of automated problem-solving schemas
Pera:	In the first session, we drew nets of the solids like cube, rectangular prism, triangular prism, etc.... We had seen these types of solids in mathematics lessons, and we had made drawings of their nets. I had already known these types of solids and their nets. But if I asked myself, was the first session unnecessary? None. It was necessary for us. In the first session, I learned that I had to make my drawing by considering the lengths of the edges or the measurements of the angles of the given solid.
Tara:	I have progressed by taking lessons throughout the sessions. In the first session, we started with simple tasks…(third section) I learned how the measurements of the edges were changed when the solid was cut. I had difficulty positioning the faces, which were cut, in my net while drawing… I learned that I needed to consider these types of things. I progressed by learning new knowledge in each session… When I got the sessions, I felt I had passed through the specific teaching stages…
Tara:	With Cabri, I could see the reflection of what I had in mind...This improved my mind even more. So, my thinking process speeded up a little more. Cabri speeded up my thinking because I saw, my mind immediately perceived, and recorded. When I encountered it again, I did it more easily...I opened or closed the faces one by one... I was turning the faces...While opening or closing the solid, all rotations were recorded in my mind.
Maral:	I gained practicality; that is, I learned something by practising and doing it, and I gained practicality, which speeded up my thinking speed; for example, I was able to perform drawing tasks more easily.

Table 19

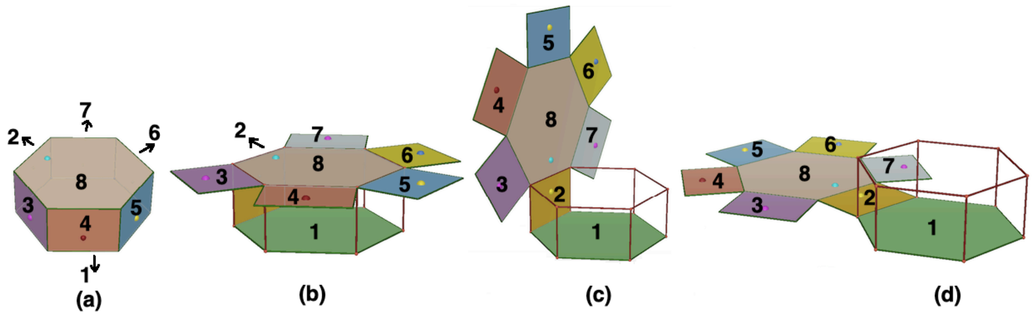
Integrating Existing Strategy to Deal with the Newly Encountered Task

Assimilation

Pera assimilated the unfolding strategy modelled in Cabri3D and then used this strategy to unfold the surface of the hexagonal prism.



Pera's self-explanations describing the strategy modelled in Cabri3D: When I pull my blue face (4), it comes to the same plane as my top face (5). When I pull my red face (3), it comes to the same plane as my top face (5). If I move the top face (5), all three faces (3, 4 and 5) come to the same plane as my yellow face (2). When I lay my four faces (2, 3, 4 and 5) to the back like this, they are in the same plane as the base.



Pera's self-explanations describe the strategy she imagined to unfold the hexagon prism: If I lift my red face (4), it comes to the top face (8). If I lift my blue (5) the same way, yellow (6) the same way, light blue (7) the same way, and purple (3) the same way, these five faces come to the same plane as my top face (8). If I unfold all of these faces (3, 4, 5, 6, and 7), including the top face (8), to the face on the left (2), they will all be on the same plane as the left face (2). All will be on the base (1) plane if I tilt them back.

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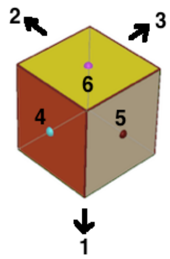
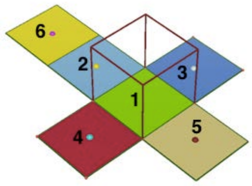
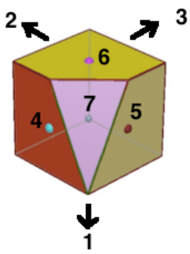
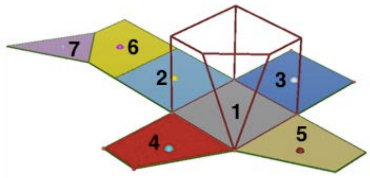
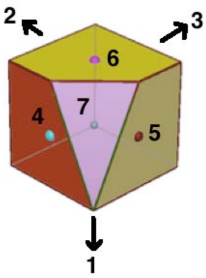
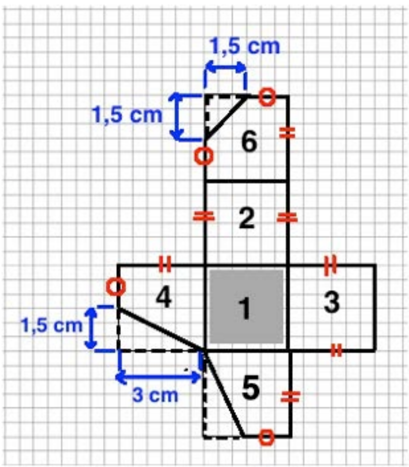
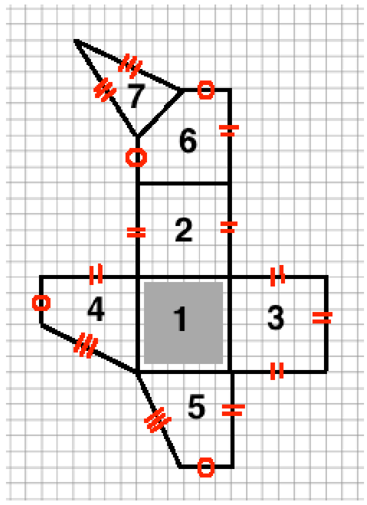
Table 20
Amending Existing Knowledge and Skills to Cope with a Newly Encountered High-Variability Task

Accommodation

Tara's self-explanations, indicating how she modified or expanded her existing knowledge and skills about constructing cube nets to be able to construct the net of a truncated cube.

Cube	The net of the solid	Truncated cube	The net of the truncated cube
			

Tara's self-explanations reflecting how she modified existing knowledge and skills: When I saw this solid, I imagined that a triangular pyramid was removed from one of the corners of a cube. I will start by drawing the uncut faces of this solid (1, 2 and 3). Next, I will draw these truncated faces (3, 4, 5 and 6) and add this triangular face (7) formed after cutting the cube's corner…. Firstly, I will draw the square bottom face (1) and the other two squares (2 and 3) that are not seen from my viewpoint. Next, I will draw the truncated top face (6) of this solid. For this, I will draw a square (4) whose sides are 3 cm. I will remove this triangle with 1.5 and 3 cm sides from this square. Then I will draw this face like a square, but I will take away a triangular part whose orthogonal sides are 1.5 and 3 cm. Lastly, I will draw this pink triangular face…

Truncated cube	Tara's drawing process for the net of the truncated cube	Tara's drawing for the net of the truncated cube
		

Discussion

This study revealed the importance of using CLT as an effective teaching technique to help learners acquire and refine new knowledge and skills, especially in constructing and developing their problem-solving schemas concerning spatial visualisation tasks. The evolution of the participants' spatial visualisation skills was discussed in six aspects in the light of the research results.

Challenges Encountered in Problem-solving Steps and Ways to Overcome These Challenges

Participants encountered challenges in the problem-solving steps of the tasks requiring drawing the surface nets of solids. However, CLT-based education, which included guided learning activities aimed at helping participants overcome such challenges, supported them in expanding and deepening their knowledge and skills and developing and constructing their problem-solving schemas. In line with these findings, other studies (Patton, 1990) report that novice learners may struggle to represent a problem accurately and develop practical solutions. In such cases, guiding and encouraging learners to invest more time and effort in defining the problem and finding solutions allows them to reflect on the quality of their own representations and solutions (Law et al., 2021). Various studies (Azimah et al., 2020; Dhlamini, 2016; Zahara et al., 2020) also stated that CLT-based education could effectively develop and strengthen learners' problem-solving skills and schemas and their learnings, as demonstrated by research focusing on problem-solving first approach (Likourezos & Kalyuga, 2017), goal-free tasks (Purnama & Retnowati, 2020), worked examples (Sentz & Stefaniak, 2019), self-explanations (Joo et al., 2020), imagination (Leopold & Mayer, 2015) and guided learning (Simamora et al., 2019).

Cyclistic Problem-solving Process

By using the problem-solving-first approach, the participants identified their challenges in any step of the solution and recognised the need to focus on more profound aspects for a clearer understanding of the problem. This research finding aligns with other studies (González-Cabañes et al., 2021; Loibl & Rummel, 2014; Sinha & Kapur, 2021)suggesting that problem-solving-first can aid learners in recognising their knowledge gaps by exposing them to challenging aspects of the problem. After the first cycle of problem-solving, participants cyclically handled and evaluated their problem-solving processes (re-interpreting the external problem situation, re-representing the problem, developing and choosing a new strategy, making minor changes to the implemented strategy, making changes or corrections to the implementation or implementing a new strategy and re/evaluating the solution) until not only arriving to a satisfactory solution but also generating multiple solutions for the same task and/or implementing at least one additional solution. This teaching approach was approved by other researchers. Suppose novice learners cannot represent a problem correctly and devise a reasonable solution. In that case, researchers recommend encouraging them to invest more time and effort in refining their representation of and solutions to their mental problems. This enables them to evaluate the quality of their problem representations and solutions critically.

Network of Knowledge and Skills Required for Problem Solving

A problem-solving schema is a comprehensive structure encompassing a web of diverse knowledge and skills; all interconnected with the central problem and its solution (Atit vd., 2020; Mayer, 2006). Regarding tasks that involve drawing the nets of solids, several interconnected skills and knowledge are required: conceptual and procedural knowledge, chains of reasoning, spatial skills, and flexible thinking. Participants could create, expand, and deepen their mentioned knowledge and skills through goal-free problems, worked examples, self-explanations, imagination, and guided learning activities. Other studies investigating teaching approaches in cognitive load theory support these findings. These studies reported the following. Goal-free problems help students develop their flexible thinking skills (Maulidya et al., 2017; Purnama & Retnowati, 2020; Youssef-Shalala et al., 2014). Learners can acquire a comprehensive explanation of the required concepts and procedures by worked examples with guidance (Kirschner et al., 2006; Sweller et al., 2007). Worked examples and self-explanation have a positive effect on learning conceptual and procedural knowledge (Atkinson et al., 2000; Renkl, 2017; Wittwer & Renkl, 2010; Wynder et al., 2017) and on acquiring complex skills, including spatial skills (Chen et al., 2015; Pillay, 1994**).** Self-explanations enable learners to generate inferences that are not provided in the given material (Chi, 2000; Salle, 2020). Learners make inferences during their self-explanations to organise given or new information to make sense of the material, which results in learning (Fiorella & Mayer, 2016; Salle, 2020). Imagining activities can help to learn procedures, recall facts, and make connections between words and images, which aid in understanding concepts. (Leopold & Mayer, 2015).

The Process of Becoming an Expert

Problem-solving first and guided learning cycle, based on learning from one's own failures, helped participants bridge the gap between their current domain knowledge and skills and those of an expert (Mazziotti et al., 2017). Practising with tasks in the same category enabled participants to acquire expert problem-solving schemas in the category-specific tasks. The process leading to expertise in each category occurred in three stages: NS, NtoES and ES. In NS, participants had challenges understanding the problem situation (e.g., the configuration of a given solid and what was asked in the task) and developing a correct strategy. In NtoES, they experienced either no or minimal challenges in the mentioned stages but had challenges implementing the chosen strategy and evaluating their solutions. They reached an accurate solution and evaluated their solutions, including developing different strategies and implementing one or two of those strategies. These stages are associated with what other studies state (See Chen & Kalyuga, 2020; Donovan et al., 1999; Lane, 2012; Stieff et al., 2020). Learners in NS try to solve problems without making sense of them and tend to answer even if they are insolvable (Donovan et al., 1999). They may also experience difficulty perceiving visuospatial information (Klein & Hoffman, 1993). NtoES is similar to Cascade's impasse, repair, and reflection cycle. Impasse emerges with being aware that the existing knowledge and current approach are insufficient to understand and solve the problem (Chen & Kalyuga, 2020; Lane, 2012). Repairing impasses includes modifying the existing knowledge, adding new conceptual and procedural knowledge and finding new strategies and solutions (Lane, 2012). Whether the proposed solution is correct or the developed strategy is proper is judged in the reflection (Lane, 2012; Lodge et al., 2018). The problem situation, including a large amount of complex spatial information, is easily understood, and the correct solution is reached by organising and applying procedures in the ES (See Donovan et al., 1999; Stieff et al., 2020).

The Importance of Prior Knowledge

Commencing the teaching with tasks that the participants were already familiar with helped them correct any misunderstandings and misconceptions, fill in any gaps in their conceptual and procedural knowledge, and acquire and increase the necessary knowledge and skills to gain expertise on the first category tasks. This approach also prepared them to tackle more complex tasks in other categories. According to Hailikari (2009), it is essential to consider the learners' previous knowledge and skills to prevent cognitive overload and encourage more profound understanding. The teaching should strengthen students' prior knowledge by diagnosing and correcting misperceptions and misconceptions. This enables them to use their knowledge actively and functionally during the problem-solving process (Diaz, 2017; Hailikari, 2009), which can help them better retain and apply the information in new situations. The amount and quality of prior knowledge significantly affect new knowledge acquisition (Hailikari, 2009).

Construction and Development of Problem-solving Schemas by Category-specific Tasks

Implementing hierarchical CLT-based education, which involved category-specific tasks, supported participants in constructing problem-solving schemes. Participants could apply the knowledge gained from one problem situation to a similar one (assimilation), thanks to the same-category low variability tasks presented to them. Practice with tasks within the same category led to assimilation so participants could develop and strengthen their category-specific problem-solving schemas and acquire a systematic problem-solving approach for the same-category tasks, which became automatic over time. Different from this, participants acquired newer and more advanced knowledge and skills to tackle high-variability tasks (accommodation), as the solution approaches and procedures they had previously learned needed to be revised. Through accommodation, participants built problem-solving schemas tailored to the specific nature of these tasks to tackle higher-category tasks. This finding is supported by other studies (Atkinson vd., 2000; Merrill, 2013; Tang et al., 2020). In order to optimise learning, it is crucial to consider the hierarchy of tasks and the prerequisite learning sequence between them (Merrill, 2013; Tang et al., 2020). First, students must master low-variability tasks before moving on to high-variability tasks, as higher-level knowledge and skills build upon lower-level knowledge and skills (Tang et al., 2020). Learners' schemas strengthen and become automatic through practising, i.e. focusing on the similarities of patterns in plenty of problems' solutions (Atkinson et al., 2000).

Conclusions and Implications

This study investigated the evolution of pre-service teachers' spatial visualisation skills through technology-supported education based on CLT. The study recorded remarkable outcomes, establishing the connections between the efficacy of CLT-pedagogical practices and the nature of spatial visualisation skills' development processes. The learning environment for the study participants, who had poor or moderate performance on the spatial visualisation test and faced challenges in different solution steps of spatial visualisation drawing tasks, was designed considering when and how their cognitive loads can change. The technology-supported CLT-based education, including different teaching approaches, was provided to develop participants' spatial visualisation problem-solving skills and to enable their acquisition of higher-order schemas. These approaches were problem-solving-first, cyclic problem-solving with guided learning, goal-free problems, worked examples, categorising the tasks, sequencing the tasks from simple to complex, using low or high variability tasks, self-explanation and imagination, all of which were supported by technology. This study, grounded on the theory testing method, reached the following results regarding the evolution process of participants' spatial visualisation skills throughout CLT-based education: Participants who made errors in different solution steps of spatial visualisation tasks when exposed to cognitive overload learned to overcome those challenges. Participants cyclically handled and evaluated their problem-solving processes, which were able to close the gap between their knowledge and skills and those who specialised in the task. They increased their conceptual and procedural knowledge, improved their reasoning and spatial skills, and acquired flexible thinking skills. They strengthened their initial spatial problem-solving schemas by completing the deficiencies in their prior knowledge. Participants adjusted their solutions to accommodate the new demands when faced with tasks in different categories. The assimilation process was initiated for similar tasks, thereby resulting in automaticity being achieved in tasks falling under the same category. Participants' acquisition of expertise in spatial visualisation tasks followed specific developmental stages.

Understanding the evolution of participants' spatial visualisation skills during CLT-based education offers valuable insights into how teaching approaches can be tailored to optimise

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the learning experience. It also provides a comprehensive, multidirectional perspective on how learners attain spatial generalisations, including overcoming challenges in solution steps throughout problem-solving cycles, enhancing knowledge and skills, activating assimilation and accommodation processes, correcting misconceptions and misunderstandings, completing gaps in prior knowledge, and progressing through different developmental stages to gain expertise. Although this study is a functional resource regarding the evolution of learners' spatial visualisation skills and how CLT-based education can be designed, it possesses some limitations (e.g. a small sample size of four participants). For this reason, further research should be conducted with a large sample size (e.g., classroom environment), using different spatial visualisation tasks, incorporating both qualitative and quantitative aspects in the research design and using additional measurement tools to evaluate the effectiveness of each CLT instructional approach. It is also recommended that experimental studies be conducted to compare the effectiveness of CLT-based education with traditional education.

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