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TWO-DIMENSIONAL MODELING OF CAR RELIABILITY DURING WARRANTY PERIOD

Summary. The paper focuses on presenting the concept of two-dimensional modeling of passenger car reliability during the warranty period. The main objective of this paper is to detect the regularity in the intensity of the number of first failure reports during the warranty period. The two-dimensional distribution of the time and mileage of failure-free exploitation is estimated. The period from the date of purchase to the first warranty repair is analysed. The concept presented incorporates the existing state of knowledge on two-dimensional warranties, expanding it through the use of a nonparametric approach and probability smoothing with the use of P-splines. The estimation involved censored data, i.e., data on vehicles that were not submitted for warranty repair within the warranty limits of time and mileage. The originality of this paper entails the combination of a nonparametric approach with probability smoothing. The statistical analyses presented in the paper were carried out on a population of 1005 vehicles of two car brands sold and serviced in 2011-2021 at the Authorized Service Station (Dealership). There were sales, repair, and warranty claim databases.

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Keywords: two-dimensional warranty, car warranty, warrant claims, reliability analysis, survival analysis, nonparametric estimation, two-dimensional smoothing, P-splines

1. INTRODUCTION

The subject of warranty has been increasingly often addressed by researchers of diverse scientific disciplines, as expressed by the increasing number of scientific publications, which can also be seen in review papers [10, 15, 24]. Warranty defines the contractual obligations of the guarantor and the rights of the consumer regarding the quality of the goods sold. In the automotive sector, the guarantor (car manufacturer or importer) is obliged to rectify a diagnosed failure by repairing and/or replacing parts, as well as provide mobility to the user, in the form of transportation cost reimbursement or a replacement car rental. The warranty is intended to protect the consumer in the event of vehicle failure or non-conformity with the contract. It is worth noting that the scope of the warranty is determined by the warranty policy of the car manufacturer as well as the legal regulations in the country of purchase.

The automobile manufacturer's warranty is an important marketing tool used for the promotion of both product reliability and quality [1]. According to Z.S. Ye and D.N. Pra Murthy, manufacturers consider warranties as an element of market competitiveness, assuming that the so-called friendly warranties can increase customer satisfaction and, as a result, increase the manufacturer's market share [25]. As a result, product warranties are becoming an increasingly important component of sales offerings, which customers do take note of [24].

The paper focuses on two issues: the data mining of the warranty data acquired in after-sales processes and the modeling of car reliability during the warranty period. In the analysed automotive sector, the warranty claims data are collected in a warranty multiprocess (an interorganizational warranty service process), in which activities are undertaken by multiple organizations, including the car manufacturer, parts manufacturer, importer, authorized service stations and third-party companies. Warranty data provides useful information for product reliability analysis [2]. The monitoring and analysis of automobile warranty claim data is important from several perspectives. The first concerns the manufacturer's ability to provide high-quality products with concern for the safety of the users as well as the workers performing the repairs and maintenance. The second perspective concerns the dealership network and the need to implement training for after-sales service process implementers. The third, and most relevant from a management perspective, concerns the levels of customer retention and satisfaction with both the car and the services carried out in the repair and warranty processes.

Manufacturers offer various types of warranties to promote their products [12]. From the manufacturers' perspective, the choice between one-dimensional and two-dimensional warranties is a particularly significant aspect. As Y. Wang et al. note, “one-dimensional warranty policies are usually characterized by a calendar time interval based on the age of the item, called the warranty period. In contrast, a two-dimensional warranty policy is characterized by a two-dimensional region, with one axis representing item age and the other one representing item usage” [21].

The indexed Web of Science Core Collection resource database search, using the keywords ("two-dimensional" AND "warranty") for the ‘title’ and ‘topic’ criteria, yielded, respectively, 102 and 174 documents addressing the issue of two-dimensional warranty. The relevance of this issue is evidenced by the fact that the number of publications in the period of 2002-2022 (as of November 2022) is characterized by an upward trend. It is also worth underlining that the issue of two-dimensional warranties is interdisciplinary in nature. The identified set of publications is indexed under such Web of Science categories as, among other things, engineering industrial, operation research management science, engineering multidisciplinary, and computer science. In the identified group of scientific publications (102 under the search criterion ‘title’), such issues as optimization of warranty policy [8, 14, 19, 20], customer segmentation [8, 25, 26], estimation of expected warranty cost [4, 15, 16, 17, 18] have been mainly considered. The commonly cited areas of the proposed solutions were often addressed to the automotive industry, but most of the studies were based entirely, or at least in part, on the assumptions made, numerical examples [8, 9, 17, 27], or simulation studies [6, 7]. There is a shortage of studies based on large data sets that allow the identification of empirical distributions of the analysed variables.

For the purpose of this study, a unique database of 1005 passenger cars has been collected. The main paper objective was to detect the regularity in the intensity of the number of first failure reports during the warranty period. The two-dimensional distribution of failure-free time and mileage of passenger car, from purchase to first warranty repair was estimated. There were applied such research methods as bibliometric analysis, literature review, unstructured interview, data mining and statistical methods of reliability analysis, survival analysis, supplemented by smoothing methods. The calculations involved the R programming language, with the use of survival, survMisc, MortalitySmooth packages.

2. RESEARCH DATA AND METHODOLOGY

2.1. Stage of scientific and research process

The study was carried out in 2021-2022. The scientific and research process was divided into 7 stages, described in Table 1.

Tab. 1

Scientific and research process description

Stage	Research task/s	Research method/s and technique/s
1	Identification of cognitive gaps. Formulation of the research problem and study objective.	Bibliometric analysis and literature review
2	Selection of an organization that measures its sales and after-sales service megaprocesses and digitizes its sales, repair, and warranty service data. Selection of the car brands, as well as the years of sale and warranty services, to be included in the study	Semi-structured interview and participant observation to assess the degree of measurement and digitization of event data in the processes under study

	Analysis of the warranty policies and after-sales service policies in the chosen organization.	
3	Design and construction of a database, including sales, after-sales and warranty service. Data mining. Formal and substantive verification of the data.	Data mining using the CRISP-DM methodology
4	Preliminary statistical analysis of the data.	Analysis of the data structure, and analysis of the relationship between variables
5	Estimation of one-dimensional distribution of the variables ‘mileage’ and ‘time’.	Kaplan-Meier survival analysis, tests for two or more reliability curves.
6	Estimation of the two-dimensional distribution of the variables ‘mileage’ and ‘time’.	Estimation of conditional failure probability in a two-dimensional distribution, two-dimensional smoothing of probability using P-splines
7	Formulation of conclusions, practical implications, and directions for further research.	

The data accumulated in the RD (Research database) was explored in accordance with the assumptions of the Cross-Industry Standard Process for Data Mining (CRISP-DM) methodology (see [22]). The data mining phases using a scheme based on the CRISP-DM assumptions are shown in Figure 1.

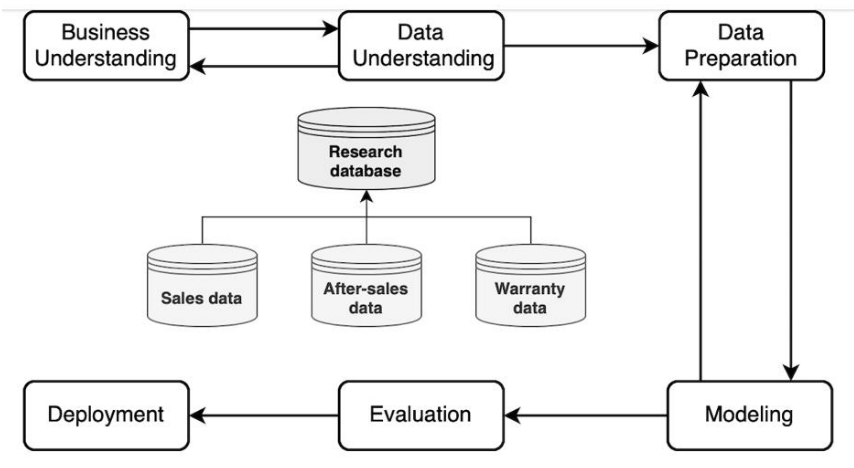

Fig. 1. Data mining scheme in the study implemented in 2021-2022
Source: own elaboration based on [22]

The data mining phases shown in Figure 1 involved the following activities:

- Business understanding – based on the studied literature and the interviews with three representatives of Authorized Service Stations (ASSs), the course of the processes under examination was reconstructed, and information on the generated and digitized data was obtained.
- Data understanding – in this phase, data stored in various ASS databases (sales data, after-sales data and warranty data) were first collected, followed by various steps taken to describe and assign classes to the database variables; data gaps and data quality issues were identified
- Data preparation – in this phase, activities were undertaken to build a relational database, primarily involving database cleaning and the definition of the criteria for survey entity selection. As a result, this phase involved a design of the database used in the subsequent data mining stages.
- Modeling - in this phase, using the database constructed, tasks employing statistical methods and reliability analysis techniques were undertaken.
- Evaluation – in this phase, nonparametric models and failure-free automobile exploitation time and mileage were estimated via a one- and two-dimensional approach.
- Deployment - substantive verification of the statistical model as well as formulation of conclusions, research limitations, and further research directions

2.2. Data source and structure

The data used in the empirical study were generated in 2021. Warranty repairs accounted for 36% of all repairs during the period under study. The data covered 1005 passenger cars (1005 unique VIN numbers) of brands A (27.5%) and B (72.5%), sold between 2011 and 2020 at the Dealership surveyed. The data collected was generated via identified and measured processes of new car Sales data (SD), After-sales data (AD) and Warranty data (WD) (Figure 2). The integration of the aforementioned databases allowed for reconstruction of the event sequence in the processes under study, including the date of service execution. This enabled not only to determine the time and car mileage up to the warranty order, but also to observe the censored units (cars) for which no after-sale warranty claims were recorded. It should be noted here that the approach involving the enlargement of Warranty Data (WD) with supplementary data has been presented in the literature (see: [23]). According to S. Wu [23], warranty claims and follow-up data contain a range of useful information not only on product quality but also on product reliability. The use of such data can positively affect the early identification of product irregularities. Figure 2 shows a reconstructed architecture of the processes, documents, and databases in the Dealership under survey. The scheme presented enabled data collection and integration thereof to design the Research database (RD).

In broad terms, the subject of the study was an Authorized Service Station (ASS), i.e., a premium passenger car Dealership, operating within the area of three provinces in Poland. In narrow-scope terms, the subject of the empirical investigation entailed the first warranty repairs in the examined vehicle population of car brands A and B. The research material used in the investigation described was the sales, service and warranty documentation generated in the megaprocesses of car sales (sales process, vehicle storage process) and after-sales servicing (processes: after-sales customer reception, diagnosis and verification, repair, quality control, warranty repair support and warranty claim settlement) (see Figure 2). The Dealership database covered a total of 1211 cars. After substantive verification, 206 vehicles whose mileage or pre-sale history indicated operation prior to the date of sale were rejected. Ultimately, a population of 1005 cars, sold as new vehicles, was subject to examination.

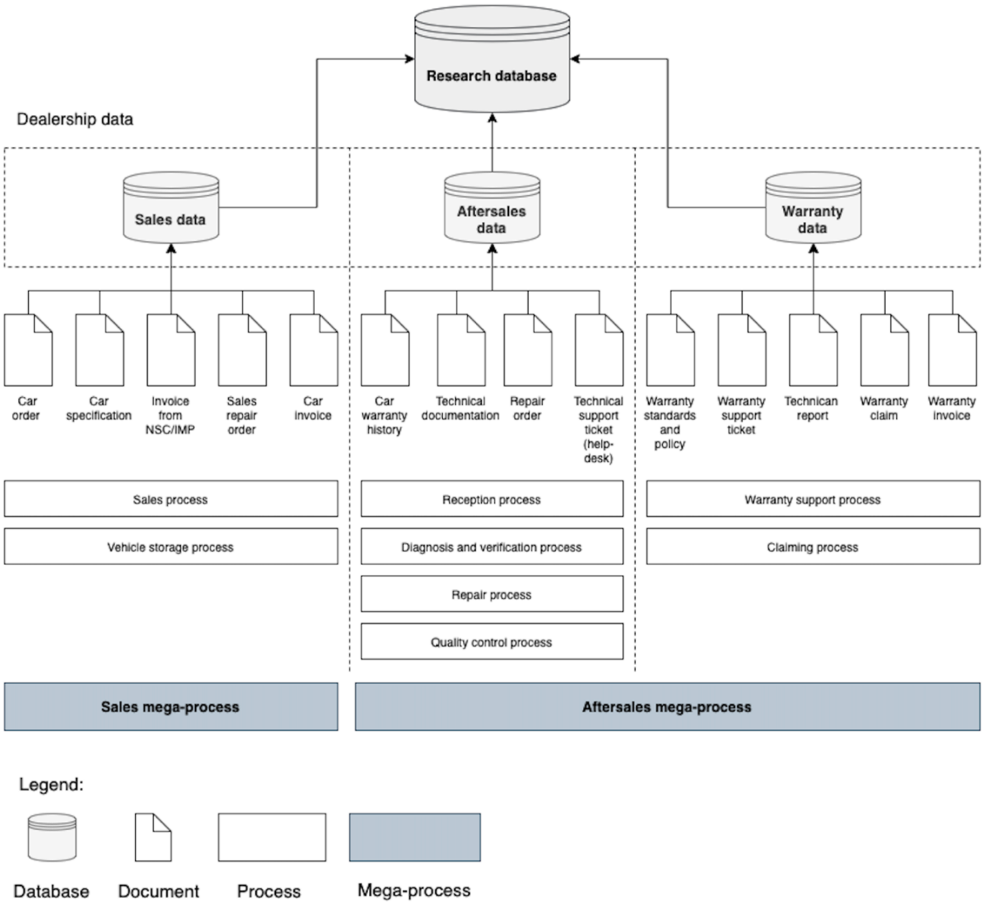

Fig. 2. Sales and after-sales data architecture in the Dealership surveyed

The unit of observation in the reliability analysis was a vehicle whose Dealership order history was followed from the date of sale until the occurrence of the event under study, defined as the first post-sale warranty repair order, excluding regular vehicle servicing. Table 2 presents a description of the variables occurring in the relational database used in the statistical analyses carried out in the following sections of this paper.

Tab. 2

Variables and their characteristics

Variable	Description
Time	Time from the date of sale to the occurrence of the first failure (complete data). If the first failure did not occur after the sale (censored data), the time from the date of sale to any last order recorded after the sale (the last moment when the unit was in the region of observation).
Mileage	The mileage from the time of car production to the occurrence of the first failure (complete data). If the failure was not recorded after the sale (censored data), the mileage recorded at the time of any last order after the sale (the last moment when the unit was in the region of observation).

Mileage per week	Mileage per week (usage rate), provided that the failure occurred after the first week; otherwise, the mileage at the time of the event.
Car Brand	The study involved two car brands, A and B, manufactured by the same car concern with different warranty rules.
Car Model	Car model: 7 models of brand A and 12 models of brand B.
Car segment	Car segment: D - Large cars, E - Executive cars, F - Luxury cars, J - Sport utility Cars, S - Sports coupe.
Car class	Car class: Small SUV, Standard SUV, Executive, Large family car, Luxury, Roadster/Sports car.

The observation of the units under study was carried out in two dimensions: time of use (no more than 3 years) and mileage (no more than 100 000 km). Units for which the event under study did not occur in the two-dimensional region of observation were treated as censored data (18.8% of the units were censored).

3. RELIABILITY ANALYSIS RESULTS

3.1. One-dimensional reliability analysis

In the first step, the reliability functions (survival functions) [5] up to the first failure were estimated using the Kaplan-Meier (KM) estimator [11]. The one-dimensional Kaplan-Meier analyses were performed for the time and mileage distribution separately. Figure 3 shows the estimation results for both reliability functions, along with 95% confidence intervals (CI), while Table 3 shows the quartile values for both distributions. The KM curves show the probability of the first warranty claim non-occurrence up to a specific point in time (left panel in Figure 3) or a specific vehicle mileage (right panel in Figure 3). The KM curves take on the value of one at a time zero (the moment of sale and mileage are 0). The value decreases at times when subsequent vehicles experience their first failures. In the set of vehicles examined, the largest decreases in the KM curve occurred in week one (as many as 12.5% of cars required warranty repair in week one), after one year (week 53), and after two years (week 105) (Figure 3, left panel). At 3 years, almost all vehicles (98.5%) were brought in for warranty repair. The median time to first warranty repair was 29 weeks (95% CI 26-34) (Table 3).

The analysis of the vehicle mileage at the time of the first warranty failure showed that the largest drops in the KM curve occurred up to 50 km of mileage (right panel in Figure 3). In the first 50 km of mileage, as many as 13.0% of the vehicles required warranty repair. The median mileage up to the first warranty repair was 12 000 km (95% CI 10.342-13.639) (Table 3). Within the range of up to 100 000 km, the first warranty repair was reported for almost all cars (98.8%).

In the following step, the car brand, model, segment, and class (see Table 2) were associated with the distributions of time or mileage up to the first failure. To verify the hypothesis that reliability, as a function of time or vehicle mileage to the occurrence of the first warranty repair, differs in the vehicle groups under examination, Wilcoxon-type tests were used [11]. When the test showed statistically significant differences between KM reliability curves for multiple samples in the groups identified by attributes of a given characteristic, tests for pairwise curves were carried out. In an instance of intersecting pairs of KM curves, Renyi-type tests were used [11].

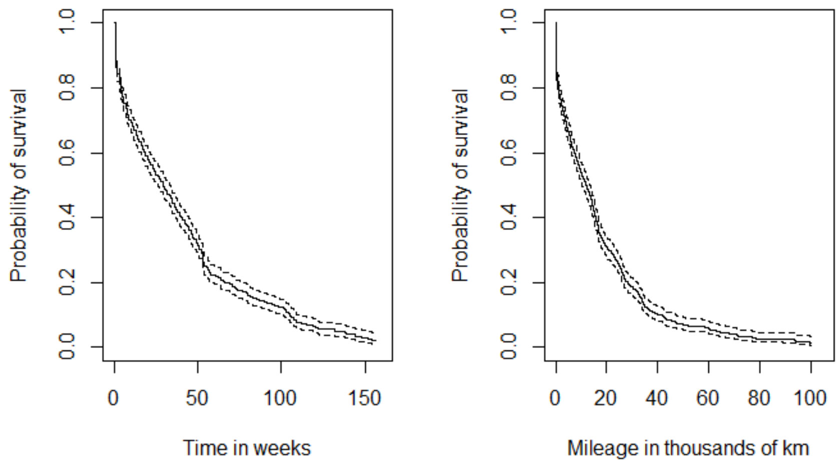

Fig. 3. Kaplan-Meier reliability curves (probability of survival without any warranty repair) with confidence intervals (dashed lines)

Tab. 3

Quartiles in the distributions of time and mileage without any warranty repair, with confidence intervals (in parentheses)

Variable	Quartile of 0.25	Quartile of 0.50 / Median	Quartile of 0.75
Time in weeks	7 (5 - 8)	29 (26 - 34)	55 (53 - 61)
Mileage in thousan d km	2.240 (1.500 – 3.223)	12.000 (10.342 – 13.639)	25.264 (23.028 – 26.583)

Tab. 4

Wilcoxon tests with log-ranks (p-value) for KM reliability curves distinguished by variants of vehicle features

Variable	Time	Mileage
Car Brand	0.236	0.251
Car Model	0.003	0.056
Car segment	0.911	0.694
Car class	0.522	0.833

According to the tests, only the KM reliability curves of time to first failure for car models were significantly different (Table 4). Due to the small size of some car model groups, only 13 of the 18 models were considered. Finally, the 78 pairs of KM curves of time to first failure were tested. In 18 of them, differences were identified. Model B12 was identified as one with the highest reliability, for which the KM curve differed from 10 other KM curves. The reliability of this model did not differ from models A1 and A2. Models B7 and B10 were identified as ones with the lowest reliability, differing from the reliability of models A1, A2, B4, B8, and B1. The tests for KM curves as functions of car mileage showed no statistically significant differences (at a significance level of 0.05).

The lack of statistically significant differences in the reliability function for brands A and B is an interesting result since the brands differed in their warranty policies. Brand A's warranty covered 3 years with no mileage limit, while brand B's warranty covered 3 years, with a 100 000 km mileage limit. To illustrate this, Figure 4 shows the reliability functions for the vehicle brands, and Table 5 shows the quartiles in the estimated distributions. The median time up to the first warranty repair for brand A was 30 weeks (95% CI 22-40), and for brand B - 29 weeks (95% CI 25-34), whereas the median mileage up to the first warranty repair was 13 000 km (95% CI 10715-15642) for brand A and 11405 km (95% CI 9622-13639) for brand B (Table 5).

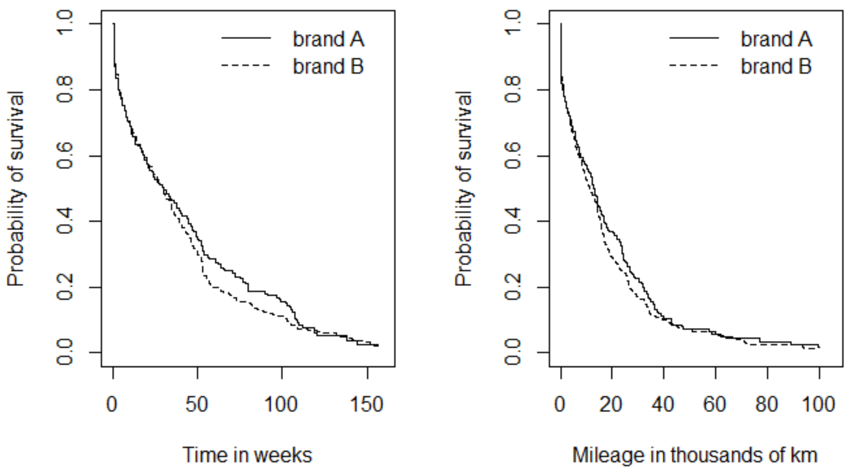

Fig. 4. Kaplan-Meier reliability curves
(probability of survival without any warranty repair) by car brand

Tab. 5

Quartiles in the distributions of time and mileage without any warranty repair, with confidence intervals (in parentheses), by car brand

Variable	Car brand	Quartile of 0.25	Quartile of 0.50 (Median)	Quartile of 0.75
Time in weeks	A	7 (4 - 11)	30 (22 - 40)	66 (54 - 80)
	B	7 (5 - 8)	29 (25 - 34)	53 (52 - 57)
Mileage in thousand km	A	2.141 (1.024 – 4.541)	13.000 (10.715 – 15.642)	26.978 (24.000 – 32.533)
	B	2.274 (1.451 – 3.278)	11.405 (9.622 – 13.639)	24.257 (20.283 – 26.176)

3.2. Reliability variability by mileage per week

It is to be expected that the timing of the first failure report depends on the intensity of vehicle exploitation. To test this regularity, the average weekly vehicle mileage, from the date of purchase to the first warranty order, was calculated. Figure 5 (left panel) shows the median weekly mileage for the groups of vehicles subjected to their first warranty repair in the same month. The group of cars reported for warranty repair in the first month had the lowest median of weekly mileage (the lowest point in the left panel in Figure 5). In the remaining months, the warranty submissions were later, on average, due to the lower intensity of car exploitation (right panel in Figure 5). This has been confirmed by the significance test of the slope in the regression line for the number of months, up to the first failure reporting, as a function of weekly mileage. The slope was not statistically significantly different from zero (p-value = 0.807) prior to the withdrawal of the vehicles with a failure reported in the first month, whereas after the withdrawal, it differed statistically significantly from zero (p-value = 0.004).

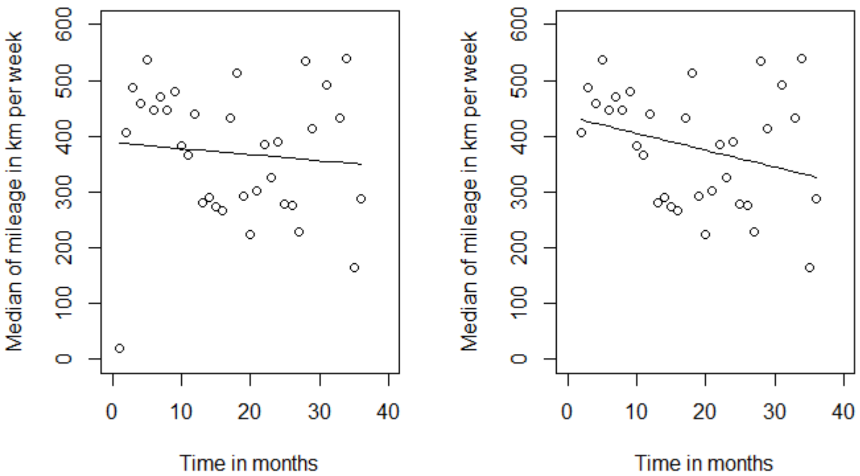

Fig. 5. Weekly mileage medians (in km) by number of months up to the first failure report, with a linear regression function (left panel – prior to withdrawal and right panel – after withdrawal of vehicles with failure reported in the first month)

In order to analyze the KM reliability curve as a function of time by weekly mileage, the vehicles were divided into three groups, according to their weekly mileage terciles. The first group were cars with an average weekly mileage, up to the first failure, of no more than 201 km; the second group – between 201 and 441 km; and the third group – over 441 km. The KM curves in these groups are statistically significantly different (Table 6), as shown by the tests for the three groups and the tests for pairwise curves. The higher the intensity of vehicle exploitation, the steeper the curves, i.e., the shorter the time to the first failure report, except for the failures of low-mileage cars reported in the first weeks, as illustrated by the large decrease in the reliability function in the first weeks, in the group with the lowest weekly mileage (Figure 6).

Tab. 6

Wilcoxon tests of logarithmic ranks for KM reliability curves as functions of time, distinguished by tertile groups of average weekly mileage up to the first failure

Case	Groups	p-value
All cases	1, 2, 3	<0.001
	1, 2	<0.001
	2, 3	<0.001
	1, 3	<0.001 [<0.001]*
Cases excluding vehicles with failure reported in the first month	1, 2, 3	<0.001
	1, 2	0.077 [0.126]*
	2, 3	<0.001
	1, 3	<0.001

* Renyi-type test for intersecting curves

After eliminating the vehicles with failures reported in the first 4 weeks, the KM curves for the tercile groups of weekly mileage (Figure 7) indicate that the distribution of time up to the first failure, in the first two groups, are similar until week 48 approximately, after which the probability of survival without failure in the first group increases. The reliability function for the third group is characterized by a much faster rate of decline, i.e., up to week 52 approximately, compared to the other groups. The three KM curves converge after the second year, when the survival probability drops to about 10%, i.e., almost all cars have undergone their first warranty repair by then. The tests for the KM curves showed that the reliability functions for the first and second groups do not differ statistically significantly, while statistically significant differences occur between the first and third groups as well as the second and third groups (Table 6).

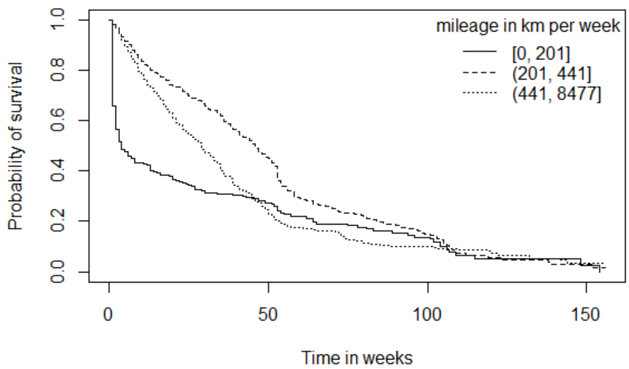

Fig. 6. Kaplan-Maier reliability curves (probability of survival without any warranty repair) by tertile groups of weekly mileage

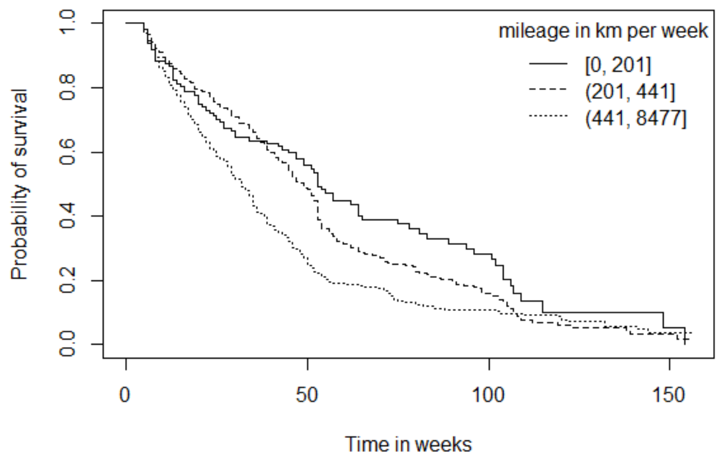

Fig. 7. Kaplan-Maier reliability curves (probability of survival without any warranty repair) by tertile groups of weekly mileage, excluding vehicles with failures reported in the first month

3.3. Two-dimensional reliability analysis

To determine the probability of the first warranty claim by time and mileage simultaneously, a two-dimensional random variable (X, T) was defined, where X denotes mileage in thousands of kilometers and T is time in months since the date of sale. The conditional probability of the first warranty claim, in a 1-month interval of car use and the interval of 1 thousand kilometers driven (provided there were no warranty claims prior to the time and mileage intervals considered) was estimated as the ratio of the number of occurrences of the event investigated in month t at mileage, from x to x+1 thousand km (where t and x are integers), and the number of vehicles exposed to the failure (the number of exposures), i.e., cars that were not recorded for warranty repair before "entering" the time and mileage intervals considered and remained in the region of observation (not censored). This method of estimation in survival analysis (reliability analysis) is called the reduced sample method [11].

Due to the random realization of failure events, the smaller the sample analyzed and the narrower the time and mileage intervals, the greater the irregularities in the failure probabilities estimated and the more difficult the detection of regularities. The grouping of the failure events into bins of 1 month per 1 000 km, with a realization of 816 events per 1005 cars, did not allow for the detection of clear regularities. An increase in the size of the bins would result in a loss of the nonparametric model’s practical significance; therefore, in order to eliminate random fluctuations and detect the regularities in the intensity of the number of first warranty repair claims, two-dimensional smoothing (by time and mileage simultaneously) of the conditional probabilities of the first warranty claim was applied. The calculations were performed in R with the MortalitySmooth package [3], using P-splines and GLAM (generalized linear array models), assuming the number of failure events, in a given time and at given mileage, at a Poisson distribution. The MortalitySmooth package was created to mainly smooth death rates by age and calendar year. However, the algorithms are more general and suitable for smoothing the so-called count data, i.e., data that is based on counting the occurrences of the event [6, 9]. P-splines of degree 3, with knots equidistant every 5 months and every 5 000 km, were used for the smoothing.

The results of the probability smoothing are shown in the below three-dimensional graph (Figure 8), with the axes representing time and mileage, and the shades of gray representing the decile groups of probability (for clarity purposes, the graph uses a logarithmic scale for probability, which does not alter the decile groups). The black color marks the 10% highest probability of first warranty repair claims, while the white color indicates the 10% lowest probability of such claims.

The timing of the first warranty claim is related to vehicle mileage. The highest probabilities of the first failure event were distributed around the diagonal rectangle connecting two points with coordinates: time and mileage equal to zero, as well as 3 years and 70 000 km. The 10% highest probabilities mostly involved failure reports up to the 20th month and a mileage increasing proportionally to, 35000km, as well as the last 5 months of the warranty period and a mileage of 65 000 – 80 000 km (Figure 8). This indicates that the 3-year warranty limit was of greater significance in the case of first failure events than the 100 000 km limit.

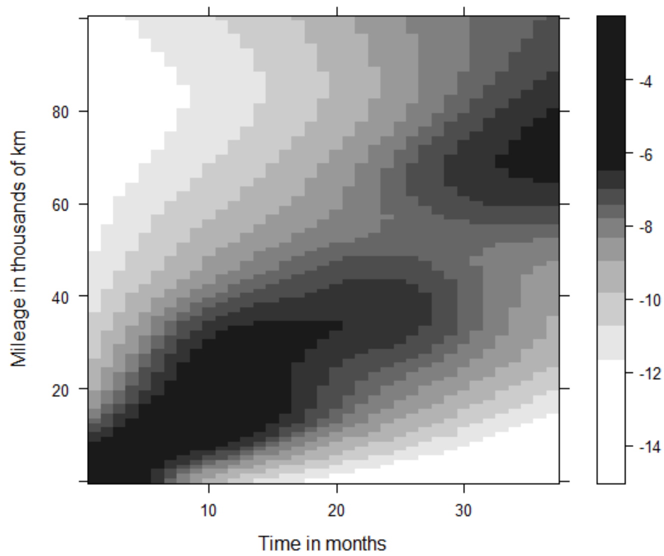

Fig. 8. Two-dimensionally smoothed conditional probabilities of the first warranty claim (logarithmic scale) in subsequent months and consecutive thousands of kilometers of mileage (gray shades indicate the decile groups)

The results shown in Figure 8 are also presented in two-dimensional graphs, i.e., Figure 9 illustrates the formation of the probability logarithm level for a fixed mileage interval in distribution by the month of vehicle exploitation, and Figure 10 illustrates the formation of the probability logarithm level for a fixed month by thousands of kilometers driven. A high probability of first failure reporting occurs in the first months at relatively low mileage (Figures 9 A and 10 A). For vehicles that have survived the first period without warranty repair, the maximum probability of a failure event shifts over time with higher mileage (Figures 9 B-D and 10 B-D). The research population included some vehicles with high reliability. 10% of the cars survived to the third year without warranty repair (KM curve, Figure 3, left panel). At the end of the 3-year warranty period, the probability of a first warranty claim increased for this group of cars (Figures 9 D and 10 D).

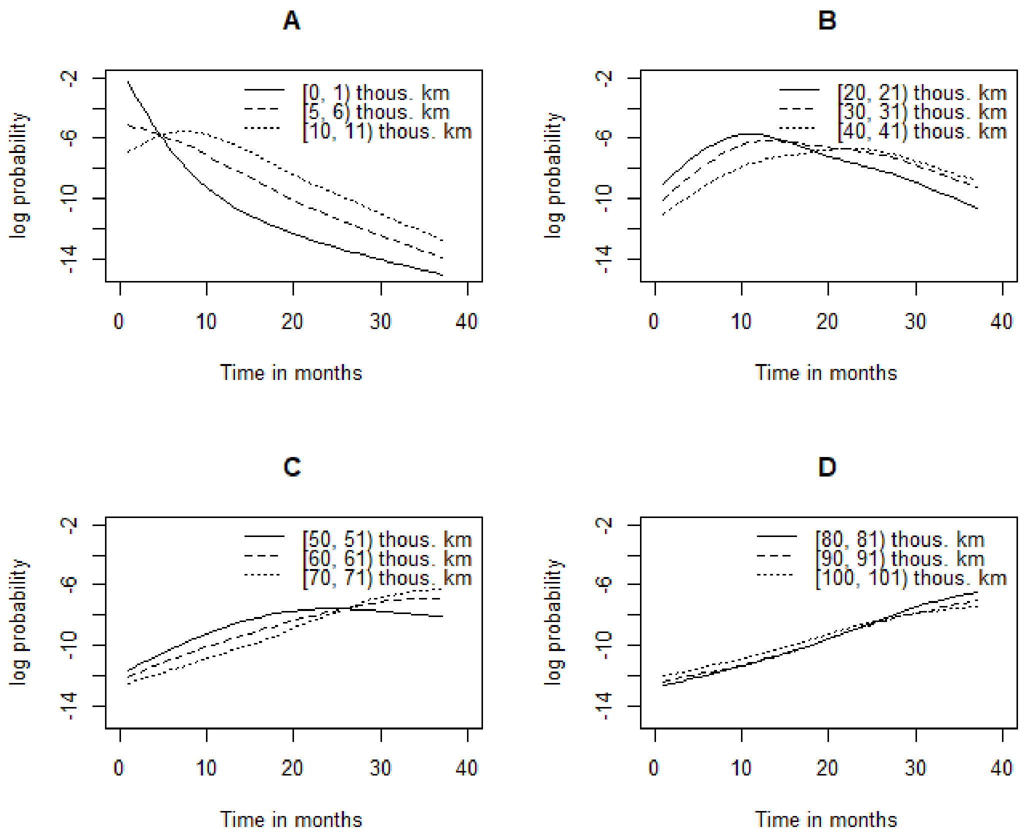

Fig. 9. Two-dimensionally smoothed conditional probabilities of the first warranty claim (logarithmic scale) in consecutive months at fixed mileage

4. CONLUSIONS

As a result of the study carried out, the following generalizing conclusions can be drawn:

1. It has been demonstrated that the reliability functions for vehicles with one-dimensional and two-dimensional warranties do not differ in the vehicle population examined.
2. The non-parametric approach to reliability modeling enabled the identification of vehicle groups with different reliability.
3. The two-dimensional probability analysis of the first warranty claim indicated two regions of the highest failure claim probability (Figure 8).
4. The smoothing methodology applied to the data grouped by time and mileage facilitated the identification of regularities in the two-dimensional distribution.
5. In the one-dimensional and two-dimensional analyses, warranty claims in the first weeks of vehicle exploitation with very low mileage are distinguished.
6. After excluding the above group from the analysis, a significant relationship between the intensity of exploitation and the time of failure was identified.

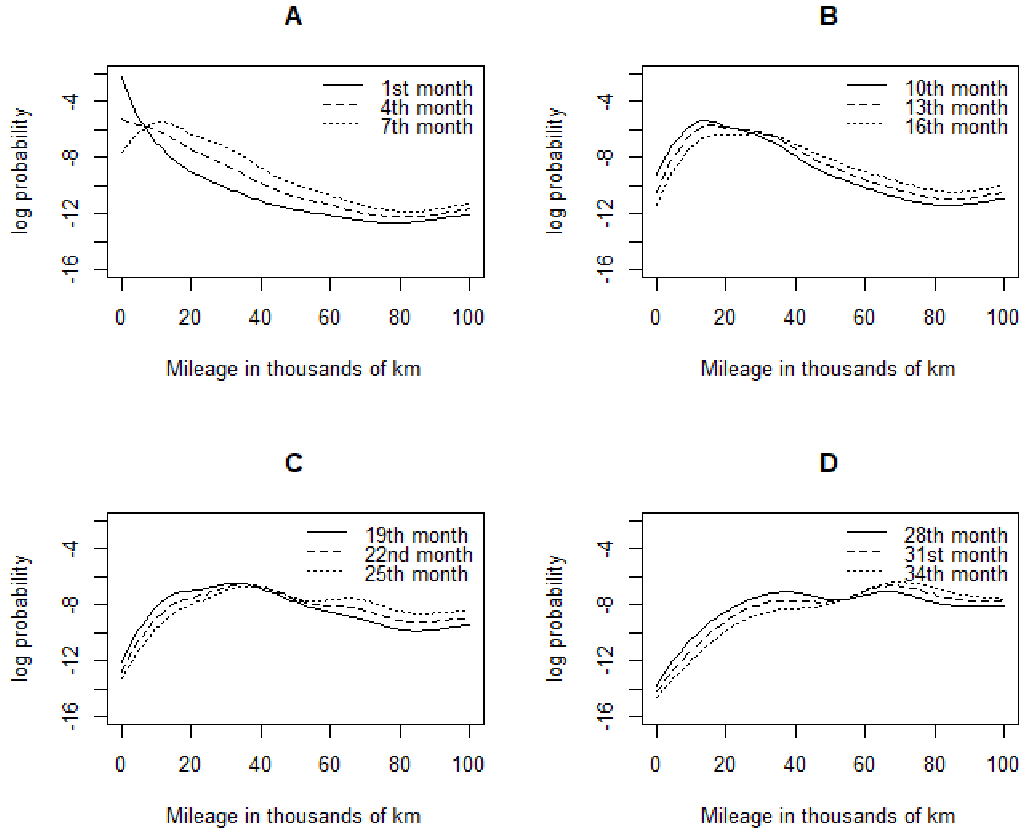

Fig. 10. Two-dimensionally smoothed conditional probabilities of first warranty claim (logarithmic scale) in consecutive thousand kilometers of mileage for a given month (after consecutive quarters of exploitation)

Based on the results of the study carried out, it is recommended to perform a separate reliability analysis for vehicles with low mileage and warranty claims in the first weeks of operation. When this group is eliminated from the analysis, the reporting time is dependent on the intensity of vehicle exploitation, expressed in weekly mileage.

As with any such empirical investigation, this study too has its research burdens and limitations, which signal the direction for further research in the area of the subject matter described in the article. The two-dimensional methods used require large datasets. The approach proposed in the paper cannot be replicated with small samples. With regard to research limitations, it should be emphasized that although the research proceedings presented, unlike other studies, involved empirical data on the entire population of the vehicles sold and after-sale serviced, the analysis covered only three provinces in Poland. It is the authors' intention to extend the study to the remaining provinces and to implement the proceedings on the entire population of the given brand's vehicle sales and service network in Poland. Such a study will allow empirical verification of the concept proposed in the paper on a population of tens of thousands of vehicles sold in Poland. Moreover, it should be emphasized that the study described in the paper covers the vehicle history up to the first warranty repair exclusively, without analyzing subsequent failure claims in and outside the warranty period. This also delineates the direction for further research, i.e., the study period should be extended to the post-warranty period, taking, inter alia, the manufacturer's goodwill participation in post-warranty repairs into account.

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