Optimal Value Determination Using Traditional and Newly Developed Method Using Initial Basic Feasible Solution of North West & Russell Method

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Abstract- This paper deals with a study used to determine optimal value using the traditional simplex method and the newly developed Putcha-Bhuiyan method. The methods involved used the initial basic feasible solution obtained from the North-West and Russell Method. Using the traditional simplex method one can determine an optimal solution for transportation problems through a series of iterations until optimality is obtained. The newly developed Putcha-Bhuiyan Method eliminates having to go through many iterations to determine optimality for a transportation problem. Both methods are useful for determining optimality and determining the minimal/maximum costs for a transportation problem.

Keywords

Transportation Problem, Optimal Solution, Initial Basic Feasible solution, Cost Estimation, North West Method, Russell Method, Putcha-Bhuiyan Method

INTRODUCTION

The first step in most transportation problems is to determine the initial basic feasible (IBF) solution. The IBF can be determined by using the North West Method or the Russell Method [1]. The North West method is much simpler than the Russell method, however the north west method does not produce an accurate IBF solution. Using the Russell method, a more accurate IBF can be obtained however with a smaller number of entries to begin the simplex method needed to determine optimality. If the IBF proves to not be optimal then a chain reaction must be formed using the traditional simplex method. The traditional simplex method can provide an optimal solution after a series of iterations [2, 3]. Depending on the nature of the problem many iterations could be needed to find the optimal solution. Therefore, the Putcha-Bhuiyan is used for simplification purposes. The Putcha-Bhuiyan method eliminates having to use many iterations and instead determines an optimal solution by entering basic variables into the cells containing negative values that lead to a solution not being optimal. A transportation problem between shipping costs and railroad costs was used to analyze and compare the two methods to determine optimality. The transportation problem was studied to determine the optimal solution for the method yieldingthe highest "Z' cost value.

EXISTING METHODOLOGIES

The general methods for obtaining the initial basic feasible solution are shown below:

A. *Northwest Corner Method*

In this method the procedure begins by beginning the optimization in the uppermost (northwest) left corner. The first entry/cell is filled based on the demand of the model provided. Next, the column to the right is selected and filled based on the supply and demand needed. After the demand and supply have been allocated the next entry filled is the one below. The procedure is repeated until all supply and demand values are satisfied. This creates a right-down procedure until the IBF solution is obtained.

B. Russell Method

In this method the procedure begins by calculating the u and v values corresponding to every row and column respectively. The formula $\boldsymbol{c_{iiii}} = \boldsymbol{u_{ii}} + \boldsymbol{v_{ii}}$ (using basic cells only) is used to determine the **u** and values. The **c**ij value refers to the cost of that specific entry. After calculating all the basic cells, the following formula $\Delta\Delta_{iiii} = \boldsymbol{c_{iiii}} - \boldsymbol{u_{ii}} - \boldsymbol{v_{ii}}$ is used to determine the non-basic cells using several iterations until the IBF is formed.

PROPOSED METHODS FOR OPTIMAL SOLUTION

The first step of an optimization problem is to determine the initial basic feasible solution. The IBF solution can be determined using any of the two methods mentioned above. The number of entries in an IBF solution is primarily $m + n - 1$, where m is the number of columns and n is the number of rows. After obtaining the IBF solution one can determine the u and v values to determine if the IBF solution is optimal. If a solution is not optimal then the traditional simplex method or the newly developed Putcha-Bhuiyan method can be used as shown below.

TRANSPORTATION OPTIMIZATION PROBLEM SOLUTION

The proposed methods were implemented to a transportation problem trying to determine optimality after obtaining the initial basic feasible solution using the North West and Russell method. Below is the procedure for determining an optimal solution using the traditional simplex method.

1. Arrive at the initial basic feasible solution which gives enough entries to start the iteration for optimal solution.
2. Calculate all the necessary ui and vj values for the transportation problem using the allocation for basic cells known from IBF.
3. Calculate the values of all the non-basic cells and see if all values are greater than zero. If yes, declare the solution as optimal.
4. If not, allocate necessary values to the cells (having negative) and adjust allocation to other cells based on the values of supply and demand. This is the key element of the new method.

The difference between the traditional simplex method and the Putcha-Bhuiyan method is that the Putcha-Bhuiyan method eliminates having to go through many iterations by adjusting the allocations until there are no negative values in the cells.

RESULTS AND DISCUSSION

EXAMPLE PROBLEM

Consider the transportation problem having the following parameters in Table 1 and Table 2.

Table 1. Problem Data Set 1 (Railroad Costs)

X	1	2	3	4	5	Supply
1	61	72	45	55	66	30
2	69	78	60	49	56	40
3	59	66	63	61	47	30
Demand	22	24	18	20	16	**Z**

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Table 2. Problem Data Set 2 (Shipping Costs)

X	1	2	3	4	5	Supply
1	31	38	24		35	30
2	36	43	28	24	31	40
3		33	36	32	26	30
Demand	22	24	18	20	16	**Z**

Table 3. IBF Solution Using NW Method for Transportation Problem Data Set 1

X	1	2	3	4	5	Supply	U
1	61 **22**	72 **8**	45 -9	55 12	66 37	30	$U_1 = -6$
2	69 2	78 **16**	60 **18**	49 **6**	56 21	40	$U_2 = 0$
3	59 -20	66 -24	63 -9	61 **14**	47 **16**	30	$U_3 = -12$
Demand	22	24	18	20	16	**Z = 6146**	
V	$V_1 = 67$	$V_2 = 78$	$V_3 = 60$	$V_4 = 49$	$V_5 = 59$		

Sample Calculation of $\mathbf{u}_i$ and $\mathbf{v}_j$ values are shown below:Assume $U_2 = 0$

Now, $c_{iiii} = u_{ii} + v_{ii}$

Now, $C_{22} = U_2+V_2$

Implies, V_2=78C_{23}=U_2+V_3

Implies, $V_3 = 60$

Similarly, other values of *u* and *v* are calculated as shown in the table above

CHECK FOR OPTIMALITY

For all non-basic cells: (1,3), (3,1), (3,2), and (3,3) the values of $\boldsymbol{c_{iiii}} - \boldsymbol{u_{ii}} - \boldsymbol{v_{ii}}$ are less than zero therefore the solution is not optimal. The simplex method or Putcha-Bhuiyan method can beused to determine optimality.

$$Z = 61(22) + 72(8) + 78(16) + 60(18) + 49(6) + 61(14) + 47(16) = 6146$$

Table 4. IBF Solution Using Russell Method for Transportation Problem Data Set 1

X	1	2	3	4	5	Supply
1	61 **12**	72	45 **18**	55	66	30
2	69 **4**	78	60	49 **20**	56 **16**	40
3	59 **6**	66 **24**	63	61	47	30
Demand	22	24	18	20	16	**Z = 5632**

Table 5. IBF Solution Using NW Method Problem Data Set 2

X	1	2	3	4	5	Supply
1	31 **22**	38 **8**	24		35	30
2	36	43 **16**	28 **18**	24 **6**	31	40
3		33	36	32 **14**	26 **16**	30
Demand	22	24	18	20	16	**Z = 3186**

Table 6. IBF Solution Using North West Method for Problem Data Set 2

X	1	2	3	4	5	Supply
1	31	38 **10**	24	 **20**	35	30
2	36	43 **6**	28 **18**	24	31 **16**	40
3	 **22**	33 **8**	36	32	26	30
Demand	22	24	18	20	16	**Z= 1902**

The tables above show the results obtained for the initial basic feasible solution for both problemdata sets. In this study the traditional simplex method and the Putcha-Bhuiyan method were usedfor the IBF solution having the highest “Z” value which is Table 3. As one can see the IBF solution in table 3 is not optimal due to the negative values therefore the simplex method and Putcha-Bhuiyan method are needed.

Table 7. Optimal IBF Solution After Using Traditional Simplex Method

X	1	2	3	4	5	Supply	U
1	61 **12**	72	45 **18**	55	66	30	$U_1 = -8$
2	69 **4**	78	60	49 **20**	56 **16**	40	$U_2 = 0$
3	59 **6**	66 **24**	63	61	47	30	$U_3 = -10$
Demand	22	24	18	20	16	**Z = 5632**	
V	$V_1 = 69$	$V_2 = 76$	$V_3 = 53$	$V_4 = 49$	$V_5 = 56$		

A total of 5 iterations were needed to obtain an optimal solution using the traditional simplex method.

Assume $U_2 = 0$

Now, $c_{iiii} = u_{ii} + v_{ii}$

Now, $C_{21} = U_2 + V_1$

Implies,

$V_1 = 69$

$C_{22} = U_2 + V_2$

Implies,

$V_2 = 76$

Optimality Check

Using $c_{iiii} - u_{ii} - v_{ii} > 0$

$C_{12} - U_1 - V_2 = 72 - (-8) - 6 = 4$

$C_{14} - U_1 - V_4 = 14$

$C_{15} - U_1 - V_5 = 18$

$C_{22} - U_2 - V_2 = 2$

$C_{23} - U_2 - V_3 = 7$

$C_{33} - U_3 - V_3 = 20$

$C_{34} - U_3 - V_4 = 22$

$C_{35} - U_3 - V_5 = 1$

Since all the values of $\mathbf{c}_{ij} - \mathbf{u}_i - \mathbf{v}_j > 0$ for the non-basic cells, the solution is optimal.

$$Z = 61(12) + 45(18) + 69(4) + 49(20) + 56(16) + 59(6) + 66(24) = 5632$$

Similarly, an optimal solution can be obtained using the Putcha-Bhuiyan method. As previously stated, this method eliminates having to go through many iterations and instead adjusts the entries having a negative value by moving a basic variable over to the cell containing the negative value. It is important to note that this method goes beyond the number of entries presented in $m + n - 1$ as shown in the table below.

Table 8. Optimal Solution Using Putcha-Bhuiyan Method

X	1	2	3	4	5	Supply	U
1	61 **16**	72	45 **14**	55	66	30	U_1 = -8
2	69 **2**	78	60 **2**	49 **20**	56 **16**	40	U_2 = 0
3	59 **4**	66 **24**	63 **2**	61	47	30	U_3 = -10
Demand	22	24	18	20	16	**Z = 5686**	
V	V_1 = 69	V_2 = 76	V_3 = 60	V_4 = 49	V_5 = 56		

Assume $U_2 = 0$

Now, $c_{iiii} = u_{ii} + v_{ii}$

Now, $C_{21} = U_2 + V_1$

Implies,

$V_1 = 69$

$C_{22} = U_2 + V_2$

Implies,

$V_2 = 76$

Optimality Check:

Using $c_{iiii} - u_{ii} - v_{ii} > 0$

$C_{12} - U_1 - V_2 = 72 - (-8) - 6 = 4$

$C_{14} - U_1 - V_4 = 14$

$C_{15} - U_1 - V_5 = 18$

$C_{21} - U_2 - V_1 = 4$

$C_{34} - U_3 - V_4 = 22$

$C_{35} - U_3 - V_5 = 1$

Since all the values of $\mathbf{c}_{ij} - \mathbf{u}_i - \mathbf{v}_j > 0$ for the non-basic cells, the solution is optimal.

Z = 61(12) + 45(18) + 69(4) + 78(16) + 49(20) + 56(16) + 59(6) + 66(24) = 5686

After obtaining the optimal solution for table 3 using the simplex method and Putcha-Bhuiyan method, one can see that the “Z” value differs slightly. The Putcha-Bhuiyan method produced a total of 9 allocations while the simplex method only used 7 allocations. However, the simplex method needed 5 iterations to produce the optimal solution. Therefore, the Putcha-Bhuiyan is more effective for simplification purposes because there is no requirement that the optimal solution have entries based on $m + n - 1$.

CONCLUSION

The Putcha-Bhuiyan method is a very fast method for arriving at the optimal solution of a transportation system, when compared to the traditional simplex method. The Putcha-Bhuiyan method can determine optimality for an IBF solution in fewer steps than the traditional simplex method because it requires less iterations and begins to find an optimized solution as the IBF is solved. Based on the transportation cost, transportation method one would be the better transportation option since it has the greatest transportation cost. While the Putcha-Bhuiyan is the fastest method for finding the optimal solution, it is not the only method for finding the optimal solution as seen by the traditional simplex method.

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