SOI: 1.1/TAS DOI: 10.15863/TAS

International Scientific Journal

Theoretical & Applied Science

p-ISSN: 2308-4944 (print) **e-ISSN:** 2409-0085 (online)

Year: 2022 **Issue:** 08 **Volume:** 112

Published: 10.08.2022 **http://T-Science.org**

Issue Article

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ANALYTICAL AND COMPUTER APPROACHES TO SOLVING QUADRATIC EQUATIONS IN A RURAL GENERAL EDUCATION SCHOOL (FOR EXAMPLE ASKINO SECONDARY SCHOOL №1 REPUBLIC OF BASHKORTOSTAN)

Abstract: *A detailed analysis of the topic "Quadratics Equations" of author's algebra textbooks for a high school course was carried out. All mathematical concepts related to the further preparation of 9th grade graduates for the final exam in the subject "Algebra" are considered in detail. Methods and techniques for solving quadratic equations are given, including well-known transformation formulas. Students are offered various options for solving quadratic equations, including Vieta's theorem. For the possible solution of quadratic equations of any degree of complexity within the general education school, a universal program has been developed in the programming language Pascal.ABC.NET. A computer experiment was carried out to find the roots of a quadratic equation using known coefficients and the formula for finding the discriminant. The program allows students to get acquainted with the listing and launch protocols. As a result, when preparing for the final exam in algebra, graduates of rural general education schools can apply not only analytical, but also computer-based approaches.*

Key words: *equation, middle school, algebra, mathematics, rural school, basic state exam, quadratic equation, discriminant, Vieta's theorem, Pascal programming language, biquadratic equation, PascalABC, algorithmization, programming, education.*

Language: *English*

Citation: Miniakhmetov, A. A., & Khamzina, E. A. (2022). Analytical and computer approaches to solving quadratic equations in a rural general education school (for example Askino secondary school №1 Republic of Bashkortostan). *ISJ Theoretical & Applied Science, 08 (112)*, 271-277.

Soi: http://s-o-i.org/1.1/TAS-08-112-24 ***Doi***: crossref https://dx.doi.org/10.15863/TAS.2022.08.112.24

Scopus ASCC: *3304.*

Introduction

UDC 332.12

Equations in the school course of algebra occupy one of the leading places in any educational and methodological set. Much more time is devoted to their study than to any other topic of the school mathematics course, no matter where the training takes place, in a large city or a rural secondary school [1].

Quadratic equations begin to be studied already in the second half of the 8th grade, according to the chosen school curriculum and the calendar-thematic planning of the subject teacher based on the author's textbook.

Students begin to study this topic, having already accumulated some experience, as well as owning a fairly large stock of algebraic, geometric and general mathematical representations, concepts and skills.

When studying and further consolidating the material on this topic, the ability to solve quadratic equations [2] and the simplest rational equations [3] by a certain method, as well as apply them to solving problems, is developed.

This contributes to the understanding of such concepts as the quadratic equation, the incomplete quadratic equation [4], the reduced quadratic equation [5], the derivation of the discriminant formula [6] and the roots of the quadratic equation [7], the Vieta theorem [8], and the inverse her, acquaintance with fractional rational equations [9], as well as with various methods of solution.

During the study, there is an understanding that the quadratic equation is an apparatus for solving many different problems of mathematics, as well as all kinds of related fields of knowledge and interdisciplinary disciplines, for example, computer science and ICT [10].

However, students of rural general education schools still experience difficulties in studying this topic in the course of a secondary general education school.

2. The purpose of the study.

In this regard, the purpose of this work is to analyze and compare algebra textbooks by different authors on the topic *"Quadratic Equations"*, as well as to conduct a computer experiment to find the roots of the reduced quadratic equation in the environment of the school programming language Pascal on the basis of the rural secondary school *"Askino Secondary School №1 Republic of Bashkortostan"* [11]. The topic is relevant not only for students of middle and high school education, but also for subject teachers when preparing lessons on this topic, as well as for the final exam in algebra.

3. A brief analysis of author's textbooks and methods for solving the given quadratic equation with the indicated coefficients.

3.1. Analysis of the author's textbook №1. Let us consider some features of solving quadratic equations according to the algebra textbook for grade 8, edited by the authors [12], and provided for by the Federal State Educational Standard [13].

The study of the topic *"Quadric Equations"* begins with the item *"Incomplete Quadratic Equations"*. Here the concepts of a quadratic equation, a reduced quadratic equation, a non-complete quadratic equation are introduced, the names of the coefficients *a*, *b* and *c* in an equation of the form $ax^2+bx+c=0$, where $a\neq 0$, are given, and are indicated in various quadratic equations.

The tasks systematize the methods known to students for solving incomplete quadratic equations, which are directly applied in practice when solving relevant examples, as well as when faced with similar problems in subjects of the natural science cycle (geometry, physics, chemistry, biology, astronomy).

We have found that a fundamentally new step for students is the solution of word problems using incomplete quadratic equations. When performing exercises, they are convinced of the importance of the formed skills. Here, for the first time, students encounter a situation where one of the found roots of the equation does not correspond to the meaning of the problem. Of course, for many students it is absurd and strange.

The next paragraph of the textbook *"Formula of the roots of a quadratic equation"* is devoted to general quadratic equations. First, the solution of the complete quadratic equation with numerical coefficients is considered. Further, all calculations are repeated for an equation of the form $ax^2+bx+c=0$ with letter coefficients *a*, *b* and *c* that are different from zero, and a general formula for the roots is derived. Students master the ability to solve quadratic equations using the appropriate sequence of steps:

(1) calculate the discriminant of the quadratic equation by the formula $D=b^2-4ac$;

(2) when $D>0$ $x_1=\frac{-b-\sqrt{D}}{2a}$, $x_2=\frac{-b+\sqrt{D}}{2a}$;

(3) when $D=0$ $x_{1,2}=-\frac{b}{2a}$;

(4) when it is concluded that there are no roots.

From the general formula for the roots of a quadratic equation, we derive the formula for the roots of the quadratic equation $ax^2+2kx+c=0$ with an even second coefficient.

It should be noted that the ability to solve quadratic equations using the general formula of the roots or the formula of the roots of a quadratic equation with an even second coefficient is one of the most important skills formed in the course of algebra.

The attention of students is drawn to the fact that when solving quadratic equations using the general formula for the roots of a quadratic equation or the formula for the roots of a quadratic equation with an even second coefficient, it is often convenient to simplify the equation by performing some transformations: *(1)* multiply all terms of the equation by *–1* if the first coefficient is negative; *(2)* get rid of fractional coefficients by multiplying all terms of the equation by a common denominator of fractions; *(3)* reduce all terms of the equation by the same number, if possible. Much attention is paid to the Vieta

theorem and its inverse when studying the reduced quadratic equations.

We believe that the positive aspects of this author's textbook can be called a detailed presentation of the theoretical part, competent presentation of the material, analysis of rather complex examples of tasks.

Consider an example of solving a quadratic equation of the form $8x^2+2x-1=0$ with coefficients $a = 8$, $b = 2$, $c = -1$ from this textbook.

According to the method given in the textbook, we calculate the discriminant of the quadratic equation D.

$$D = b^2 - 4 \cdot a \cdot c = 2^2 - 4 \cdot 8 \cdot (-1) = 4 + 32 = 36.$$

According to the analysis of the discriminant, we see that $D > 0$. Therefore, we can apply the formula for the roots of the quadratic equation and calculate the numerical values.

$$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a},$$

$$x_1 = \frac{-2+\sqrt{36}}{2 \cdot 8} = \frac{-2+6}{16} = \frac{1}{4} = 0,25,$$

$$x_2 = \frac{-2-\sqrt{36}}{2 \cdot 8} = \frac{-2-6}{16} = \frac{-8}{16} = -\frac{8}{16} = -\frac{1}{2} = -0,5.$$

Answer: $x_1 = 0.25$, $x_2 = -0.5$.

3.2. Analysis of the author's textbook №2. In the following author's textbook [14], the topic of quadratic equations is divided into the following main blocks in accordance with the theoretical material: *(1)* Practicing basic concepts related to quadratic equations; *(2)* Incomplete quadratic equations; *(3)* Complete quadratic equations (an algorithm for solving a quadratic equation using discriminant formulas, finding roots, studying and using Vieta's theorem); *(4)* Rational equations (algorithm for solving a rational equation; solving rational equations by introducing a new variable, biquadratic equations); *(5)* Text problems solved with the help of rational equations.

We found that when presenting the material, the following are used: *(1)* definitions and theorems; *(2)* examples, comments on solving examples and problems, as well as algorithms for solving equations are analyzed in detail; *(3)* there are questions for self-examination; *(4)* as well as at the end of each chapter, the main results and topics of research work are spelled out.

In our opinion, the most positive qualities of the textbook are as follows: *(1)* accessible and understandable style of presentation; *(2)* classification of tasks according to three levels of complexity; *(3)* the presence of drawings and drawings for a visual presentation of theoretical and task material; (4) tasks for repetition. Among the shortcomings, the most significant can be singled out: the material is presented in two books.

It is noteworthy that in the last two author's textbooks [13] and [14] the solution of systems of quadratic parametric equations [15] is considered, which is not observed in the textbooks of other authors.

We solve the reduced quadratic equation of the form $x^2 + 16x + 63 = 0$ using the theorem converse to Vieta's theorem. Equation coefficient $a = 1$, $b = 16$, $c = 63$.

We see that the coefficient $p = 16$, the free term $q = 63$. So, according to the theorem, the sum of the roots is –16, and their product is 63. We select such roots $x_1 = -7$, $x_2 = -9$. Indeed, the sum $(x_1 + x_2) = -7 + (-9) = -7 - 9 = -16$, and the product $(x_1 \cdot x_2) = (-7) \cdot (-9) = 63$. Thus, we can say that the roots were chosen correctly.

Answer: $x_1 = -7$, $x_2 = -9$.

3.3. Analysis of the author's textbook №3. Finally, in the author's textbook [16], provided for by the Federal State Educational Standard for Basic General Education, the study of the topic begins with the definition of a quadratic equation, a reduced quadratic equation, which are solved by isolating the square of the binomial [17]. Further, the formula of the roots of the quadratic equation, the concept of discriminant, the number of roots of the quadratic equation are proposed.

This manual contains a large number of examples for solving equations by changing the variable [18], that is, reducing to quadratic equations. An entire paragraph is devoted to solving quadratic equations using the second formula (with an even coefficient).

Quite a lot of attention is paid to solving problems on the application of the formula of the roots of a quadratic equation. Then incomplete quadratic equations and Vieta's theorem are considered.

We believe that the topics in this textbook are presented specifically and accurately, there is an impressive amount of tasks for practical use in algebra lessons.

The study of the topic *"Quadratic Equations"* ends with the presentation of the section *"Factorization of a quadratic trinomial"*. The authors of the textbook in the section *"For those who are interested"* offer a rather interesting technique for the formulaless solution of a quadratic equation, which, in our opinion, is a kind of alternative to the standard methods of other compilers.

Consider standard examples of solving two quadratic equations from this author's textbook of the form $x^2 - 12x + 36 = 0$ and $7x^2 - 25x + 23 = 0$.

a) $x^2 - 12x + 36 = 0$.

Coefficients $a = 1$, $b = -12$, $c = 36$.

We calculate the discriminant D according to the well-known formula:

$D = b^2 - 4ac = (-12)^2 - 4136 = 144 - 144 = 0$.

According to the analysis of the discriminant, we see that $D = 0$.

We apply the formula of the roots of the quadratic equation and calculate the only root:

$$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a},$$

$$x_1 = \frac{12 + \sqrt{0}}{2} = \frac{12}{2} = 6,$$

$$x_2 = \frac{12 - \sqrt{0}}{2} = \frac{12}{2} = 6,\ x_1 = x_2 = 6.$$

Let us show that the roots of the equation are found correctly. In the equation $x^2 - 12x + 36 = 0$, the coefficient $p = -12$, and the constant term $q = 36$. The sum of the found numbers $(x_1 + x_2) = 6 + 6 = 12$, and their product $(x_1 \cdot x_2) = 6 \cdot 6 = 36$. Hence, according to the theorem converse to the Vieta theorem, these numbers are the roots of the equation $x^2 - 12x + 36 = 0$.

Answer: $x = 6$.

b) $7x^2 - 25x + 23 = 0$.

Coefficients $a = 1$, $b = -12$, $c = 36$.

We also calculate the discriminant D according to the well-known formula:

$D = b^2 - 4a = (-25)^2 - 4723 = 625 - 644 = -19$.

According to the analysis of the discriminant, we see that $D < 0$.

Answer: no roots.

Thus, when comparing all three author's textbooks available in the school library of Askino Secondary School №1 Republic of Bashkortostan, we can make the current conclusion that the material on quadratic equations in all the considered educational and methodological sets is presented in an accessible and clear.

Much attention is paid to both the theoretical and practical parts, because it is important to be able to apply the knowledge gained in solving equations, problems, where fractional rational equations act as mathematical models.

The tasks for all textbook authors are very diverse and are aimed at studying equations, determining the number of solutions, and also finding roots.

4. Computer experiment to find the roots of the reduced quadratic equation.

In order to confirm the correctness of the solution of all the above quadratic equations from different author's algebra textbooks, we developed a program in the school programming language PascalABC.NET [19] to find the roots of the above quadratic equation based on the analysis of the discriminant.

This idea arose on the basis that in the computer science course of the 8th grade of the author's textbook [20], provided for by the Federal State Educational Standard, when programming branching algorithms in the chapter "Fundamentals of Algorithmization", a compound operator is affected, which allows you to write a program for analysis quadratic equation.

Below is a listing of the program in the specified programming language, taking into account the use of an incomplete conditional branching construct [21], a comment for each line, indicating all the keywords of the language and the "tail" of the fractional part.

```
program Equation; {the name of the program}
uses crt; {connecting the library standard screen cleaning module}
var a, b, c : real; {description of variables – coefficients of the quadratic equation}
D : real; {variable description – discriminant}
x, x1, x2 : real; {description of variables – roots of a quadratic equation}
Begin ClrScr; {program start, clear screen}
writeln ('Solving a quadratic equation'); {comment}
write ('Enter coefficient a = '); {query coefficient a}
readln (a); {reading coefficient a}
write ('Enter coefficient b = '); {query coefficient b}
readln (b); {reading coefficient b}
write ('Enter coefficient c = '); {query coefficient c}
readln (c); {reading coefficient c}
writeln ('Compute the discriminant D:'); {comment}
D := (b * b) – (4 * a * c); {discriminant calculation}
writeln ('Discriminant D = ', D); {output value}
If (D < 0) Then writeln ('No roots'); {check condition for D < 0}
If (D = 0) Then {check condition for D = 0}
begin
x := (–b) / (2 * a); {calculation of the root of the equation}
writeln ('Root of equation x = ', x:0:3); {output value}
end;
If (D > 0) Then {check condition for D > 0}
begin
x1 := (–b + Sqrt (D)) / (2 * a); {calculation of the first root of the equation}
x2 := (–b – Sqrt (D)) / (2 * a); {calculation of the second root of the equation}
writeln ('First root x1 = ', x1:0:3); {output first value}
writeln ('Second root x2 = ', x2:0:3); {output second value}
end;
End. {end of program}
```

Let's run the program for execution to find the roots of the given quadratic equation, indicated above, by entering the coefficients and receiving the work protocols.

```
Solving a quadratic equation

Enter coefficient a = 8
Enter coefficient b = 2
Enter coefficient c = -1

Compute the discriminant D:
Discriminant D = 36

First root x1 = 0.250
Second root x2 = -0.500
```

Figure 1 – Protocol №1

```
Solving a quadratic equation

Enter coefficient a = 1
Enter coefficient b = 16
Enter coefficient c = 63

Compute the discriminant D:
Discriminant D = 4

First root x1 = -7.000
Second root x2 = -9.000
```

Figure 2 – Protocol №2

```
Solving a quadratic equation

Enter coefficient a = 1
Enter coefficient b = -12
Enter coefficient c = 36

Compute the discriminant D:
Discriminant D = 0

Root of equation x = 6.000
```

Figure 3 – Protocol №3

```
Solving a quadratic equation

Enter coefficient a = 7
Enter coefficient b = -25
Enter coefficient c = 23

Compute the discriminant D:
Discriminant D = -19

No roots
```

Figure 4 – Protocol №4

Thus, based on the developed program template, we can analyze any quadratic equation, calculate the discriminant by the specified coefficients and, accordingly, its roots.

It should be added that the analytical and computer approaches to solving quadratic equations completely complement each other.

This means that a well-composed analytical (mathematical) model of any of the above author's

textbooks allows you to develop an algorithm for the correct solution of a quadratic equation.

In turn, the mathematical model smoothly turns into writing a program code in one of the educational programming languages and allows you to conduct a computer experiment even within the framework of a rural general education school, which is very important for students from the periphery.

It is no secret that affordable and high-quality computer equipment is still not available in many rural and village schools.

Therefore, many students are still sitting on their phones, where they work in an installed application for one of the programming languages.

5. Conclusion.

Thus, in the course of the analysis of the available 8th grade algebra school textbooks, we identified and settled on the fact that the most optimal for the perception of students in rural schools and the periphery is the author's textbook edited by S.A. Telyakovsky [12].

We are sure that it is important for 8th grade students to be able to apply both analytical and computer approaches to solving the given quadratic equations, which is the basis for creating a basis in preparation for the main state exam.

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