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MATHEMATICAL MODELING OF MAGNETOELASTIC VIBRATIONS OF A CONDUCTING SHELL IN A MAGNETIC FIELD

Abstract*: The coupled problem of magnetoelasticity for a flexible orthotropic shell is considered in the article, taking into account the orthotropy of conducting properties, and the effect of thickness on the stress-strain state of an orthotropic shell is investigated.*
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Introduction

The effect of the coupling of dynamic and mechanical displacements of electrically conductive bodies with an electromagnetic field is due to ponderomotive Lorentz forces.

Significant effects of ponderomotive interaction occur under high-frequency vibrations at large displacement amplitudes, pulsed magnetic fields, and current-carrying elements.

The construction of optimal designs of modern technology operating in magnetic fields is associated with the widespread use of structural elements, such as flexible thin-walled shells.

The impact of non-stationary fields on metal thin-walled elements leads to the appearance of volumetric electromagnetic forces that, under certain field parameters, can cause large deformations of structures.

Recently, the issue of determining the stress state of flexible orthotropic shells operating in an alternating magnetic field, taking into account the orthotropic electrical conductivity, has attracted considerable interest.

In recent decades, considerable attention in the literature has been paid to the study of the process of deformation of electrically conductive bodies placed in an external alternating magnetic field under the influence of non-stationary force, thermal and electromagnetic loads [1,2,3,4,5,6,7,8,9,10,11,12,13, 14,15,16,17,18,19,20,21,22].

Interest in research in this area is associated with the importance of quantitative studying and evaluating the observed effects of the relationship of non-stationary mechanical, thermal and electromagnetic processes and their practical application in various fields of modern technology in the development of new technologies, in the field of nanotechnology and microelectronics, and in modern measuring systems, etc.

I. NONLINEAR FORMULATION OF THE PROBLEM. Let us consider the nonlinear behavior of an orthotropic current-carrying conical shell from beryllium of variable thickness, changing in the meridional direction according to the following law $h = 5\cdot 10^{-4}(1-\alpha\, s/s_N)$ m. We assume that the shell is under the influence of mechanical force $P_\zeta = 5\cdot 10^3 \sin\omega t\ N/m^2$, external electric current $J_{\theta CT} = -5\cdot 10^5 \sin\omega t\ A/m^2$, and external magnetic field $B_{S0} = 0.1T$, and the shell has a finite orthotropic electrical conductivity $\sigma(\sigma_1, \sigma_2, \sigma_3)$.

We assume that the external electric current in the unperturbed state is uniformly distributed over the shell, i.e. the external current density does not depend on the coordinates. In this case, the shell is subjected to a combined loading of the ponderomotive Lorentz force and mechanical force. The contour of small radius $s = s_0$ is hinged, and the second contour $s = s_N$ is free in the meridional direction.

Let us assume that the geometric and mechanical characteristics of the body are such that an option of the geometrically nonlinear theory of thin shells in the quadratic approximation can be used to describe the deformation process. We also assume that the electromagnetic hypotheses are fulfilled with respect to the strength of electric field $\vec{E}$ and the strength of magnetic field $\vec{H}$ [1,2].

These assumptions are some electrodynamic analogs of the hypothesis of non-deformable normal lines and, together with the latter, constitute the hypotheses of magnetoelasticity of thin bodies. Acceptance of these hypotheses makes it possible to reduce the problem of the deformation of a three-dimensional body to the problem of the deformation of an arbitrarily chosen coordinate surface.

We refer the coordinate surface in the undeformed state to the curvilinear orthogonal coordinate system s and θ, where S – is the length of the generatrix arc (of the meridian), counted from some fixed point, θ – is the central angle in the parallel circle, counted from the selected plane. The coordinate lines $s = const$ and $\theta = const$ are the lines of the main curvatures of the coordinate surface. Choosing coordinate ζ along the normal to the coordinate surface of revolution, we refer the shells to the spatial coordinate system s, θ, ζ.

We assume that the vector of magnetic induction and the surface mechanical forces are known on the surface of the conical shell.

When obtaining a resolving system in Cauchy normal form, we choose $u, w, \theta_S, N_S, Q_S, M_S, B_\zeta, E_\theta$ as the main functions.

With these functions, we can select various combinations of cone fixing. We assume that all components of the excited electromagnetic field and the displacement field included in the equations of the magnetoelasticity problem do not depend on coordinate θ, and we also assume that the elastic and electromagnetomechanical characteristics of the shell material do not change along the parallel.

After some transformations [3,9,21,22], we obtain a complete system of nonlinear differential equations of magnetoelasticity in the Cauchy form, which describes the stress-strain state of a current-carrying orthotropic conical shell under non-stationary action of mechanical and magnetic fields:

$$\frac{\partial u}{\partial s} = \frac{1-\nu_s\nu_\theta}{e_s h} N_S - \frac{\nu_\theta \cos\varphi}{r} u -$$
$$-\frac{\nu_\theta \sin\varphi}{r} w - \frac{1}{2}\theta_S^2\ ; \quad \frac{\partial w}{\partial s} = -\theta_S\ ;$$
$$\frac{\partial \theta_S}{\partial s} = \frac{12(1-\nu_s\nu_\theta)}{e_s h^3} M_S - \frac{\nu_\theta \cos\varphi}{r}\theta_S\ ;$$
$$\frac{\partial N_S}{\partial s} = \frac{\cos\varphi}{r}\left[\left(\nu_S \frac{e_\theta}{e_S} - 1\right) N_S + e_\theta h\left(\frac{\cos\varphi}{r} u + \frac{\sin\varphi}{r} w\right)\right] -$$
$$- P_S + h J_{\theta CT} B_\zeta -$$
$$-\sigma_1 h\left[E_\theta B_\zeta + 0.5\frac{\partial w}{\partial t} B_\zeta\left(B_S^+ + B_S^-\right) - \frac{\partial u}{\partial t} B_\zeta^2\right] + \rho h \frac{\partial^2 u}{\partial t^2};$$
$$\frac{\partial Q_S}{\partial s} = -\frac{\cos\varphi}{r} Q_S + \nu_S \frac{e_\theta}{e_S}\frac{\sin\varphi}{r} N_S +$$
$$+ e_\theta h \frac{\sin\varphi}{r}\left(\frac{\cos\varphi}{r} u + \frac{\sin\varphi}{r} w\right) - P_\zeta - 0.5 h J_{\theta CT}\left(B_S^+ + B_S^-\right) -$$
$$-\sigma_3 h\left[-0.5\, E_\theta\left(B_S^+ + B_S^-\right) - 0.25\, \frac{\partial w}{\partial t}\left(B_S^+ + B_S^-\right)^2 - \right. \qquad (1)$$

$$-\frac{1}{12}\frac{\partial w}{\partial t}\left(B_S^+ - B_S^-\right)^2 + 0.5\frac{\partial u}{\partial t}B_\zeta\left(B_S^+ + B_S^-\right) +$$

$$+\frac{h}{12}\frac{\partial \theta_S}{\partial t}B_\zeta\left(B_S^+ + B_S^-\right)] + \rho h\frac{\partial^2 w}{\partial t^2};$$

$$\frac{\partial M_S}{\partial s} = \frac{\cos\varphi}{r}\left[\left(\nu_S\frac{e_\theta}{e_S} - 1\right)M_S + \frac{e_\theta h^3}{12}\frac{\cos\varphi}{r}\theta_S\right] +$$

$$+Q_S + N_S\theta_S -$$

$$-\frac{\sin\varphi}{r}\left(\nu_S\frac{e_\theta}{e_S}M_S + \frac{e_\theta h^3}{12}\frac{\cos\varphi}{r}\theta_S\right)\theta_S + \frac{h^3}{12}\frac{\partial^2\theta_S}{\partial t^2};$$

$$\frac{\partial B_\zeta}{\partial s} = -\sigma_2\mu\left[E_\theta + 0.5\frac{\partial w}{\partial t}\left(B_S^+ + B_S^-\right) - \frac{\partial u}{\partial t}B_\zeta\right] + \frac{B_S^+ - B_S^-}{h};$$

$$\frac{\partial E_\theta}{\partial s} = -\frac{\partial B_\zeta}{\partial t} - \frac{\cos\varphi}{r}E_\theta.$$

Here $N_s, N_\theta -$ are the meridional and circumferential forces; $S -$ is the shear force; $Q_s, -$ is the cutting force; $M_s, M_\theta -$ are the bending moments; u, $w -$ are the displacement and deflection, $\theta_S -$ is the normal line rotation angle; $P_S, P_\zeta -$ are the mechanical load components; E_θ - is the circumferential component of the electric field strength; B_ζ - is the normal component of magnetic induction; B_s^+, B_s^- - are the known components of magnetic induction from the surface of the shell; $J_{\theta cm}$ - is the component of the electric current density from an external source; e_s, e_θ - are the moduli of elasticity in s, θ directions, respectively; ν_S, ν_θ - are the Poisson ratios characterizing the transverse compression under tension in the direction of the coordinate axes; $\mu -$ is the magnetic permeability; $\omega -$ is the circular frequency; $\sigma_1, \sigma_2, \sigma_3$ are the principal components of the electrical conductivity tensor.

The solution to boundary value problems of magnetoelasticity is associated with certain difficulties.

This is explained by the fact that the resolving system (1) is a system of eighth-order differential equations of the hyperbolic-parabolic type with variable coefficients. The components of the ponderomotive Lorentz force include nonlinear terms, caused by the consideration of shell displacements during its deformation.

II. METHODS FOR SOLVING A COUPLED PROBLEM. The methods developed for the numerical solution to a new class of coupled problems of magnetoelasticity of the theory of orthotropic conical shells of revolution with orthotropic electrical conductivity are based on the subsequent application of the Newmark finite difference scheme, linearization method, and discrete orthogonalization [2,3,4,5,6,7,8,9,10,18,19,20,21, 22]. To effectively use the methods proposed, we assume that when an external magnetic field appears, there are no sharp skin (surface) effects along the shell thickness and the electromagnetic process along the ζ coordinate quickly reaches a mode close to the steady-state one.

III. NUMERICAL EXAMPLE. ANALYSIS OF RESULTS. Let us study the behavior of an orthotropic shell depending on the change in the shell thickness. The problem for an orthotropic beryllium cone of variable thickness $h = 5\cdot 10^{-4}\left(1 - \alpha\ s/s_N\right)$m is calculated for different values of parameter $\alpha = \{0.2; 0.3; 0.4; 0.5\}$ characterizing the thickness variability in the meridional direction.

In this case, we write the boundary conditions in the following form

$$s = s_0 = 0: u = 0,\ w = 0,\ M_s = 0,\ B_z = 0.3\sin\omega t;$$
$$s = s_N = 0.05m: Q_s = 0,\ N_s = 0,\ B_\zeta = 0. \quad (2)$$

Initial conditions have the following form

$$\vec{N}(s,t)\,|_{t=0} = 0,\ \dot{u}(s,t)\,|_{t=0} = 0,\ \dot{w}(s,t)\,|_{t=0} = 0. \quad (3)$$

When solving the problem, the parameters take the following values:

$$s_0 = 0,\ m;\ s_N = 0.05m;\ h = 5\cdot 10^{-4}(1-\alpha\ s/s_N)\,m;$$
$$\alpha = \{0.2;\ 0.3;\ 0.4;\ 0.5\},\ r = r_0 + s\cos\varphi;\ \ r_0 = 0.05m,$$
$$\sigma_1 = 0.279\cdot 10^8 (\Omega\times m)^{-1},\ \sigma_2 = 0.321\cdot 10^8 (\Omega\times m)^{-1},$$
$$\sigma_3 = 1.136\cdot 10^8 (\Omega\times m)^{-1},\ \nu_S = 0.03;\ \nu_\theta = 0.09;$$
$$e_S = 28.8\cdot 10^{10}\ N/m^2;\ e_\theta = 33.53\cdot 10^{10}\ N/m^2;$$
$$\omega = 314.16\sec^{-1};P_\zeta = 5\cdot 10^3 \sin\omega t\ H/м^2;\ \ P_r = 0;$$
$$\tau = 1\cdot 10^{-2}\sec;\ \ \mu = 1.256\cdot 10^{-6}\ \ H/м;\ \rho = 2600\,kg/m^3;$$
$$J_{\theta CT} = -5\cdot 10^5 \sin\omega t\ \ A/m^2;\ B_S^{\pm} = 0.5Тл;$$
$$B_{S0} = 0.5\sin\omega t;\ \Delta t = 1\cdot 10^{-3}\sec; 0\le t\le 1\cdot 10^{-2}\sec.$$

The solution to the problem was found on the time interval $\tau = 1\cdot 10^{-2}\sec$; the time integration step was chosen as $\Delta t = 1\cdot 10^{-3}\sec$.

In the case under consideration, the anisotropy of the electrical resistivity of beryllium is $\eta_3/\eta_1 = 4.07$. In the figures below, the graphs (1, 2, 3, 4) correspond to the following values of parameter $\alpha = \{0.2; 0.3; 0.4; 0.5\}$.

Figure1 shows the distribution of deflection w along the meridian of the shell at time point $t = 5\cdot 10^{-3}\sec$ for various values of parameter α. It was established that the maximum values of deflections along the shell occur approximately in the vicinity of value $s = 0.4m$.

This is explained by the fact that, according to the boundary conditions, the left end is hinged, and the right end of the shell is free in the meridional direction. Besides, the thickness of the shell, from the left end to the right end, decreases up to 2 times at α=0.5.

Therefore, the maximum deflections occur near the right end of the shell. When considering the effect of thickness, the stress of the conical shell is taken as the sum of mechanical stresses and Maxwell stresses, i.e. the general stress state is taken into account.

Figure 2 shows the distribution of maximum stress values $\sigma_{22}^- + T_{22}^-$ along the meridian of the shell at time $t = 5\cdot 10^{-3}\sec$ of the inner surface of the shell for various values of parameter α.

The figures show the complex behavior of the shell depending on the boundary conditions under the action of mechanical and magnetic fields. It should be noted that with increasing parameter α the maximum values of deflections and stresses increase.

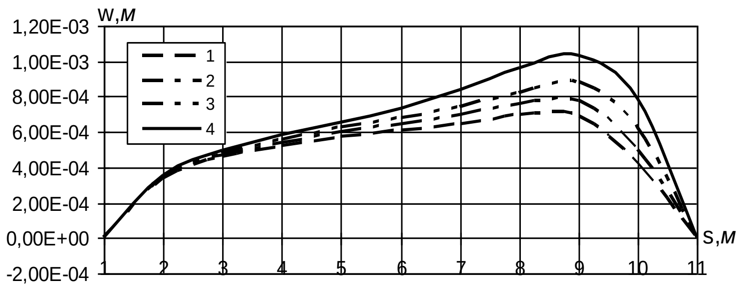

Figure 1. Distribution of w over s at time $t = 5\cdot 10^{-3}\sec$ for different values of parameter α .

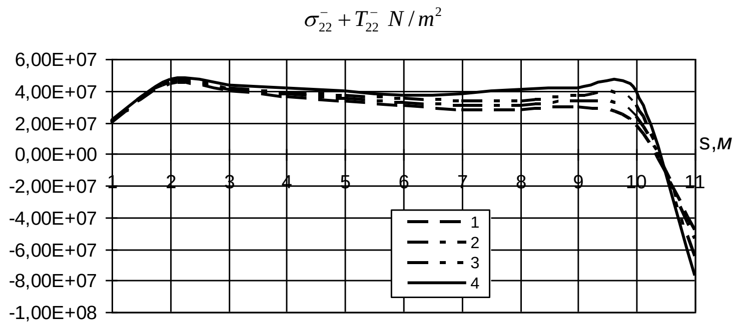

Figure 2. Distribution of $\sigma_{22}^- + T_{22}^-$ over s at time $t = 5\cdot 10^{-3}\sec$ for different values of parameter α .

IV. CONCLUSION.

The analysis of the stress state of a flexible shell under the action of a time-varying mechanical force and a time-varying external electric current was conducted, taking into account mechanical and electromagnetic orthotropy. The results obtained indicate the effect of thickness on the deformation of the shell and the necessity to consider this factor in the calculation schemes. As seen, the thickness variability has a significant effect on changes in the stress-strain state of the shell, and an account for the geometric nonlinearity makes it possible to significantly refine the deformation pattern.

References:

1. Ambartsumyan, A., Bagdasaryan, G.E., & Belubekyan, M.V. (1977). *Magnetoelasticity of Thin Shells and Plates* [in Russian], Nauka, Moscow.
2. Grigorenko, Y. M., & Mol'chenko, L. V. (2010). *Fundamentals of the Theory of Plates and Shells with Elements of Magnetoelasticity (Textbook)* (IPTs, 2010). Google Scholar
3. Mol'chenko, L. V., Loos, I. I., & Indiaminov, R. S. (2008). "Determining the stress state of flexible orthotropic shells of revolution in magnetic field," *Int. Appl. Mech.*, 44, 882–891. https://doi.org/10.1007/s10778-008-0102-6, Google ScholarCrossref
4. Indiaminov, R. (2008). "On the absence of the tangential projection of the lorenz force on the axsymmetrical stressed state of current-carrying conic shells," *Int. Jour. Comp. Techn.* 13, 65–77. Google Scholar
5. Mol'chenko, L. V., Loos, I. I., & Indiaminov, R. S. (2009). "Stress–strain state of flexible ring plates of variable stiffness in a magnetic field," *Int. Appl. Mech.*, 45, 1236–1242. https://doi.org/10.1007/s10778-010-0264-x, Google ScholarCrossref
6. Mol'chenko, L. V., & Loos, I. I. (2013). "The stress state of a flexible orthotropic spherical shell subject to external current and mechanical force in a magnetic field," *Int. Appl. Mech*. 49, 528–533. https://doi.org/10.1007/s10778-013-0587-5, Google ScholarCrossref
7. Mol'chenko, L. V., Loos, I. I., & Fedorchenko, L. M. (2016). "Deformation of a flexible orthotropic spherical shell of variable stiffness in a magnetic field," *Int. Appl. Mech*, 52, 56–61. https://doi.org/10.1007/s10778-016-0732-z, Google ScholarCrossref
8. Bian, Y. H., & Zhao, H. T. (2016). "Analysis of thermal-magnetic-elastic stresses and strains in a thin current-carrying cylindrical shell," *Int. Appl. Mech.*, 52, No. 4, 437–448. MathSciNet Article Google Scholar
9. Indiaminov, R. S. (2015). "Magnetoelastic deformation of a current-carrying orthotropic conical shell with an orthotropy of conductive properties," *Bulletin of the University of Kiev* 5, 81–86. Google Scholar
10. Indiaminov, R., Akbaev, U., & Dustmuradov, A. (2016). "Nonlinear deformation of flexible orthotropic shells of variable thickness in the non stationary magnetic field," *International Journal of Engineering Innovation & Research* 5, 234–238. Google Scholar
11. Indiaminov, R., Butaev, R., & Mavlanov, S. (2018). "Research of deformation of the current carrying orthotropic shells in nonlinear statement," *ISJ Theoretical*, *Applied Science* 09(65), 25–30, https://doi.org/10.15863/TAS.2018.09.65.5
12. Indiaminov, R. S., & Butaev, R. (2020). "Stress-strain state of current-carrying shells in a magnetic field," *ISJ Theoretical & Applied Science*, 05(85), 489–492, https://doi.org/10.15863/TAS.2020.05.85.90
13. Indiaminov, R., Hotamov, A., & Akhmedov, F. (2017). "Magnetoelastic deformation of the current carrying shells with the orthotropy of conductive properties," *International Journal of Engineering Innovation & Research* 5, 344–350. Google Scholar
14. Indiaminov, R., & Rustamov, S. (2019). "Axisymmetric magnetoelastic shells deformation with account for anisotropy of conductive properties," *ISJ Theoretical & Applied Science*, 11(79), 554–559. https://doi.org/10.15863/TAS.2019.11.79.115, Google ScholarCrossref
15. Indiaminov, R. S., Narkulov, A. S., & Zarpullaev, U. K. (2020). "Mathematical modeling of magnetoelastic vibrations of a rod in a magnetic field," *ISJ Theoretical & Applied Science*, 03(83), 327–332, https://doi.org/10.15863/TAS.2020.03.83.60, Google ScholarCrossref

16. Indiaminov, R., Kholjigitov, S., & Narkulov, A. S. (2020). "Nonlinear vibrations of a currentcarrying anisotropic cylindrical shell in a magnetic field," *ISJ Theoretical & Applied Science* 01(81), 205–211. https://doi.org/10.15863/TAS.2020.01.81.38, Google ScholarCrossref
17. Indiaminov, R. S., & Butaev, R. (2020). "Nonlinear integro-differential equations of bending of physically nonlinear viscoelastic plates," *IOP Publishing. Conf. Series: Materials Science and Engineering*, 7, https://doi.org/10.1088/1757-899X/869/5/052048. Google ScholarCrossref
18. Indiaminov, R., Narkulov, A., & Butaev, R. (2021). "Magnetoelastic strain of flexible shells in nonlinear statement", AIP Conference Proceedings, 2021, 2365, 02 0002. https://doi.org/10.1063/5.0056840
19. Indiaminov, R., Butaev, R., & Narkulov, A. (2021). "Nonlinear deformation of a current shell in a magnetic field", *AIP Conference Proceedings*, 2021, 2365, 02 0001. https://doi.org/10.1063/ 5.0056840
20. Indiaminov, R., & Yusupov, N. (2021). *"Mathematical Modeling of Magnetoelastic Vibrations of Current Conductive Shells in the Non Stationary Magnetic Field,"* International Conference on Information Science and Communications Technologies (ICISCT), pp. 1-4, doi: 10.1109/ICISCT52966.2021.9670 308.
21. Indiaminov, R.Sh., & Abdullaev, A. (2022). Simulation of magnetoelastic deformation of a plate in an alternating magnetic field. *ISJ Theoretical & Applied Science*, 01(105), 751-755. Doi: https://dx.doi.org/10.15863/TAS.2022.01.105.55
22. Indiaminov, R. Sh. (2021). Mathematical modeling of magnetoelastic vibrations of an annular plate in a magnetic field. *ISJ Theoretical & Applied Science*, 12 (104), 1275-1279. Doi: https://dx.doi.org/10.15863/TAS.2021.12.104.141