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USING INEQUALITIES IN CHECKING THE CONVEXITY AND CONCAVITY OF THE FUNCTION

Abstract*: The use of inequalities in the verification of functions is one of the most pressing issues. This article gives examples of the use of inequalities in checking the convexity and concavity of a function.*

Key words*: function, interval, value, graph, convex, sunk.*
Language*: English*
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Introduction

It is important to check the convexity and concavity of the function when checking and plotting the function. We usually use the product to find the convex and concave ranges of a function. In this article, we provide information on how to check the convexity and concavity of a function using inequality.

For the function $y = f(x)$ an interval is optional x_1 and x_2 at different values

$$f\left(\frac{x_1 + x_2}{2}\right) > \frac{f(x_1) + f(x_2)}{2}$$

If the inequality is true, the function is called convex in this range.

For the function $y = f(x)$ an interval is optional x_1 and x_2 for values

$$f\left(\frac{x_1 + x_2}{2}\right) < \frac{f(x_1) + f(x_2)}{2}$$

If the inequality is true, the function is said to be submerged in this interval.

The geometric meaning of this is that the center of an arbitrary vat of a graph of a convex function lies at the bottom of the corresponding arc. (Figure 1)

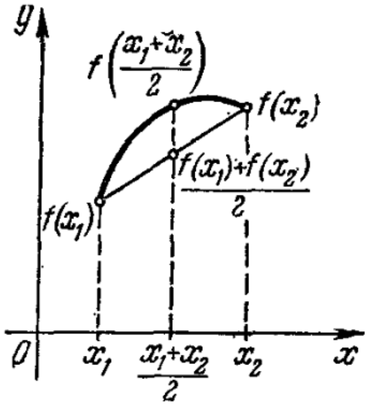

Figure 1.

The center of an arbitrary vat of a graph of a submerged function lies at the top of the corresponding arc. (Figure 2)

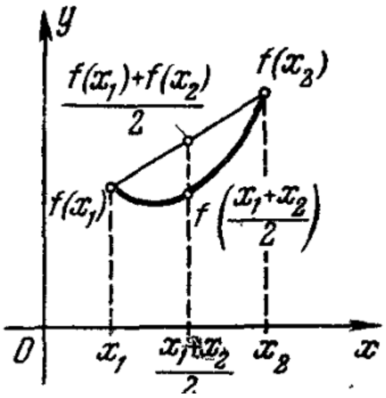

Figure 2.

Obviously, $y = f(x)$ the curve may be convex at one point and concave at another. The point that separates the convex part of a function from the concave part is called its inflection point. In Figure 3, point A is the inflection point.

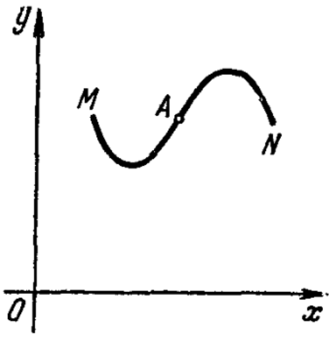

Figure 3.

We now give examples of how to determine the convexity and concavity of a function.

Example 1. $y = x^2$ determine the convexity and concavity of the function.

Solution. arbitrary values of the arguments of x_1 and x_2. Then:

$$y_1 = f(x_1) = x_1^2, \quad y_2 = f(x_2) = x_2^2,$$

$$f\left(\frac{x_1 + x_2}{2}\right) = \left(\frac{x_1 + x_2}{2}\right)^2$$

$y = x^2$ to check the function for convexity and concavity

$$\frac{f(x_1)+f(x_2)}{2}-f\left(\frac{x_1+x_2}{2}\right)$$

we determine the sign of the difference.

$$\frac{f(x_1)+f(x_2)}{2}-f\left(\frac{x_1+x_2}{2}\right)=\frac{x_1{}^2+x_2{}^2}{2}-\left(\frac{x_1+x_2}{2}\right)^2=$$

$$=\frac{2x_1^2+2x_2^2-x_1^2-2x_1x_2-x_2^2}{4}=$$

$$=\frac{x_1^2-2x_1x_2+x_2^2}{4}=\left(\frac{x_1-x_2}{2}\right)^2>0$$

(because of $x_1 \neq x_2$). because of the difference is positive $y=x^2$ the function graph is sunk. (Figure 4)

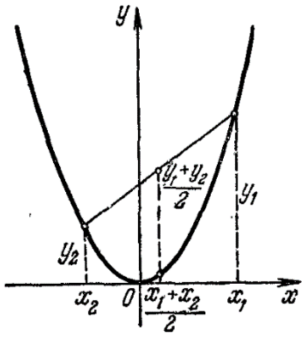

Figure 4.

Example 2. $y=\frac{1}{x}$ check that the function is convex and concave.

Solution. x_1 and x_2 the same sign values of the argument that are different from the arbitrary zero. Separation:

$$\frac{f(x_1)+f(x_2)}{2}-f\left(\frac{x_1+x_2}{2}\right)=\frac{\frac{1}{x_1}+\frac{1}{x_2}}{2}-\frac{2}{x_1+x_2}=$$

$$=\frac{x_1+x_2}{2x_1x_2}-\frac{2}{x_1+x_2}=\frac{(x_1+x_2)^2-4x_1x_2}{2x_1x_2(x_1+x_2)}=$$

$$=\frac{(x_1-x_2)^2}{2x_1x_2(x_1+x_2)}$$

If $x_1>0$ and $x_2>0$, then $x_1x_2>0$ and $x_1+x_2>0$, Separation will be plus. If $x_1<0$ and $x_2<0$, then $x_1x_2>0$ and $x_1+x_2<0$, Separation will be minus.

So, $x>0$ graphic sink, $x<0$ will be convex (Figure 5).

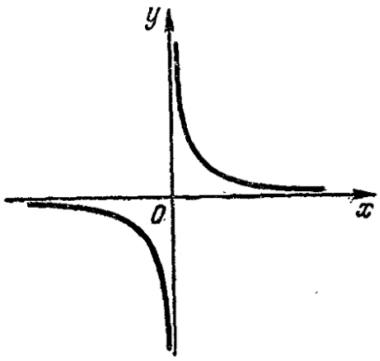

Figure 5.

Example 3. $y=a^x$ $(a>0, a\neq 1)$ check that the function is convex and concave.

Solution. x_1 and x_2 arbitrary values of the argument. Let’s look at the difference below.

$$\frac{f(x_1)+f(x_2)}{2}-f\left(\frac{x_1+x_2}{2}\right)=\frac{a^{x_1}+a^{x_2}}{2}-a^{\frac{x_1+x_2}{2}}=$$

$$=\frac{a^{x_1}+a^{x_2}-2a^{\frac{x_1+x_2}{2}}}{2}=\frac{\left(a^{\frac{x_1}{2}}-a^{\frac{x_2}{2}}\right)^2}{2}>0$$

As a result, the graph of the function is sunken (Figure 6).

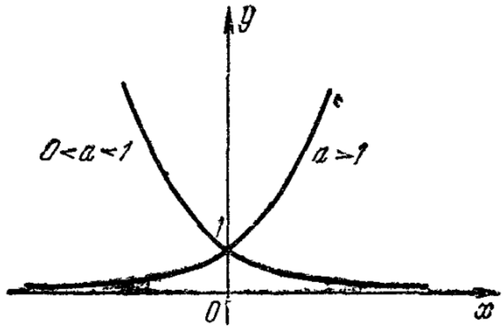

Figure 6.

Example 4 . $y=\log_a x$ check that the function is convex and concave.

Solution. x_1 and x_2 arbitrary positive values of the argument, Then

$$\log_a x_1+\log_a x_2=\log_a x_1x_2$$

or

$$\frac{\log_a x_1+\log_a x_2}{2}=\log_a\sqrt{x_1x_2} \qquad (1)$$

We know that, $\sqrt{x_1x_2}<\frac{x_1+x_2}{2}$ if $x_1\neq x_2$ and $x_1>0, x_2>0$, then

$y=\log_a x$ function growing in $a>1$, decreasing in $0<a<1$.

So:

$$\log_a\frac{x_1+x_2}{2}>\log_a\sqrt{x_1x_2}\quad (a>1\ da)$$

$$\log_a\frac{x_1+x_2}{2}<\log_a\sqrt{x_1x_2}\quad (0<a<1\ da)$$

(1) the right-hand side of the inequality $\log_a\frac{x_1+x_2}{2}$ swapping

$$\frac{\log_a x_1+\log_a x_2}{2}<\log_a\frac{x_1+x_2}{2}\qquad (a<1)$$

and

$$\frac{\log_a x_1+\log_a x_2}{2}>\log_a\frac{x_1+x_2}{2}\qquad (0<a<1)$$

we create inequalities.

So, $a>1$ The graph is convex, $0<a<1$ will sink in (Figure 7)

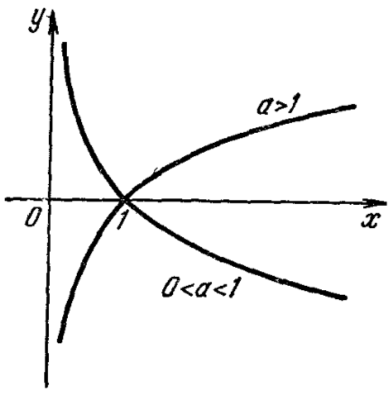

Figure 7.

Example 5. $y=\sin x\,(0<x<2\pi)$ check that the function is convex and concave.

Solution. x_1 and x_2 arbitrary values of the argument. In this case, both (0; π) will belong to the interval or $(\pi;2\pi)$ interval. Let's look at the difference below.

$$\frac{f(x_1)+f(x_2)}{2}-f\left(\frac{x_1+x_2}{2}\right)=\frac{\sin x_1+\sin x_2}{2}-\sin\frac{x_1+x_2}{2}=$$

$$=\sin\frac{x_1+x_2}{2}\cos\frac{x_1-x_2}{2}-\sin\frac{x_1+x_2}{2}=\sin\frac{x_1+x_2}{2}\left(\cos\frac{x_1-x_2}{2}-1\right)$$

because of being $|\cos a|\le 1, \cos\frac{x_1-x_2}{2}-1$ the expression is negative or equal to zero.

because of being $0<x<2\pi$,

$$\cos\frac{x_1-x_2}{2}-1<0$$

If $0<x_1<\pi,\ 0<x_2<\pi$, then will be $0<\frac{x_1+x_2}{2}<\pi$. As a result

$$\sin\frac{x_1+x_2}{2}>0$$

So, if x_1 and x_2 belongs to an interval $(0;\pi)$,

$$\sin\frac{x_1+x_2}{2}\left(\cos\frac{x_1-x_2}{2}-1\right)<0$$

and the difference under consideration is negative. Thus $(0;\pi)$ the graph of the function in the interval is convex (Fig. 8).

If $\pi<x_1<2\pi,\ \pi<x_2<2\pi$, then $\pi<\frac{x_1+x_2}{2}<2\pi$.

As a result

$$\sin\frac{x_1+x_2}{2}<0 \text{ and}$$

$$\sin\frac{x_1+x_2}{2}\left(\cos\frac{x_1-x_2}{2}-1\right)>0 .$$

So the difference is positive. In the interval $(\pi;2\pi)$ the function graph is sunk. (Figure 8)

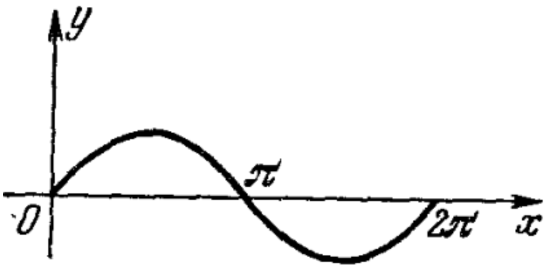

Figure 8.

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