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SERIAL CONNECTION OF NTC THERMISTORS

***Abstract**: A working mathematical model of a technical system has been reached. The technical system provides for the serial connection of negative temperature coefficient thermistors. The built mathematical model possesses the properties of fullness, adequacy, productivity and economy to a sufficient degree. The use of such a model reduces the time and costs spent on research and makes efficient use of the mathematical modelling capabilities.*

***Key words**: NTC thermistor, working mathematical model, properties of mathematical models, principles of mathematical modeling.*
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Introduction

Extensive educational and scientific literature is devoted to the consideration of technical characteristics of negative temperature coefficient thermistors, the basic principles of their operation, and the methods of designing circuits with said thermistors. There are numerous examples of successful practical use of such equipment in various fields of human activity.

The purpose of this work is to build a working mathematical model of a technical system using a unified approach. This technical system provides for serial connection of negative temperature coefficient thermistors.

The dependence of the resistance R of such a thermistor on its temperature T is usually described in an expression (for an example, see [1; 2]), which takes the following form

$$R(T) = re^{\beta\left(T^{-1}-T_0^{-1}\right)},$$

where r is the thermistor's resistance at $T = T_0$; β is the coefficient constant for the given thermistor. However, within a relatively narrow temperature range, it can be assumed that

$$R(T) = \frac{r}{1+\beta\left(T-T_0\right)T_0^{-2}}.$$

A unified approach to building a working mathematical model that has necessary properties for a specific study is described in [3; 4]. Some properties of mathematical models are formulated, for instance, in [5; 6]. An example of building a mathematical model with the necessary properties for a study is presented in [7]; some of the results of this study were published in [8–10]. The particular features of using a unified approach to building mathematical models are described, for example, in [11; 12].

Problem statement

The serial connection of n thermistors is discussed below. The i-th thermistor shall be considered a body with high thermal conductivity, whose temperature T_i at the initial time point t_0 is equal to T_0, while $T_i \le T_1$, $i = 1, 2, \ldots, n$. Convective heat exchange occurs with the environment, the temperature of which is equal to T_0 on the surface of the thermistor area S_i, and the heat transfer coefficient is known and equal to α_i. For a relatively narrow temperature range from T_0 to T_1, it is considered that

$$R_i(T_i) = \frac{r_i}{1+\beta_i(T_i - T_0)T_0^{-2}},$$

$$C_i(T_i) = c_i\left[1+\gamma_i(T_i - T_0)\right],$$

where $R_i(T_i)$ and $C_i(T_i)$ are the resistance and total heat capacity of the *i*-th thermistor; r_i and c_i are the resistance and total heat capacity of the *i*-th thermistor at $T_i = T_0$; β_i and γ_i are positive constants. The electrical potential difference at the poles of the *i*-th element is

$$U_i = \frac{r_i I}{1+\beta_i(T_i - T_0)T_0^{-2}}, \qquad (1)$$

where I is the direct electric current intensity flowing through the thermistors.

The electrical potential difference

$$U = \sum_{i=1}^{n} U_i \qquad (2)$$

is of interest in the study. Let us design a working mathematical model of the object of study that has sufficient properties of fullness, adequacy, productivity and economy.

Problem solution

The results obtained in [13] are to be used in order to solve the problem. These results allow us to build a hierarchy of mathematical models of this object of study and determine the conditions under which it is possible to find the desired value U with a relative error of no more than the given value δ_0.

If the differences $T_i - T_0$ are small enough, then according to (1), the desired value is obtained using the formula

$$U_0 = \sum_{i=1}^{n} r_i I. \qquad (3)$$

Conditions under which the obtained formula is applicable are to be determined. A steady-state heat exchange process is considered for this reason. In this case, according to the calculations in [13], the steady-state value U_i is determined using the formula

$$U_i^* = \frac{2r_i I}{1+\sqrt{1+4\beta_i r_i I^2 \alpha_i^{-1} S_i^{-1} T_0^{-2}}},$$

and for the given temperature range

$$\frac{r_i I^2}{\alpha_i S_i(T_1 - T_0)} \le 1+\beta_i(T_1 - T_0)T_0^{-2}, \qquad (4)$$

then the steady-state value of the sought value is equal to

$$U_* = \sum_{i=1}^{n} U_i^*. \qquad (5)$$

The relative error of the value U_0 is

$$\delta(U_0) = \left|\frac{U - U_0}{U}\right| = \frac{U_0}{U} - 1 \le \frac{U_0}{U_*} - 1.$$

In the event of inequation

$$\frac{U_0}{U_*} - 1 \le \delta_0$$

formula (3) can be used with a relative error of no more than δ_0 to find the desired value. Consequently, in the event of inequation

$$U_0 \le (1+\delta_0)U_* \qquad (6)$$

the mathematical model (3) possesses the properties of fullness, adequacy, productivity and economy to a sufficient degree.

Then let us define the conditions under which mathematical model (5) can be applied. The unsteady-state heat exchange process is considered for this reason. In this case, according to the results from [13], we obtain a Cauchy problem

$$\frac{c_i r_i I T_0^2}{\beta_i U_i^2}\frac{dU_i}{dt} = \frac{\alpha_i S_i r_i I T_0^2 - \alpha_i S_i U_i T_0^2 - \beta_i I U_i^2}{\gamma_i r_i I T_0^2 - \gamma_i U_i T_0^2 + \beta_i U_i},$$

$$U_i(t_0) = r_i I, \qquad (7)$$

where $i = 1, 2, \ldots, n,$ and we can find the time point

$$t_i = t_0 + \frac{c_i}{\alpha_i S_i}\left[\frac{\gamma_i T_0^2}{\beta_i}\left(\frac{U_i^*}{r_i I} - 1 + \delta_0\right)\frac{r_i I}{U_i^*} + \left(\frac{r_i I}{2r_i I - U_i^*} + \frac{\gamma_i T_0^2}{\beta_i}\frac{r_i I - U_i^*}{2r_i I - U_i^*}\frac{r_i I}{U_i^*} - 1\right)\times \ln\left(2 - \frac{U_i^*}{r_i I} - \delta_0\right) - \left(\frac{r_i I}{2r_i I - U_i^*} + \frac{\gamma_i T_0^2}{\beta_i}\frac{r_i I - U_i^*}{2r_i I - U_i^*}\frac{r_i I}{U_i^*}\right)\ln\left(\frac{r_i I}{r_i I - U_i^*}\delta_0\right)\right],$$

for which

$$U_i(t_i) = \frac{U_i^*}{1-\delta_0}.$$

It is obvious that at $t \ge t_i$

$$\delta(U_i^*) = \left|\frac{U_i - U_i^*}{U_i}\right| = 1 - \frac{U_i^*}{U_i} \le \delta_0,$$

and the value U_i^* can be considered equal to $U_i(t)$ with a relative error of no more than δ_0. If $t_* = \max_{1\le i\le n} t_i$, then it is easy to show that at $t \ge t_*$

$$\delta(U_*) = \left|\frac{U - U_*}{U}\right| = \frac{\sum_{i=1}^{n}(U_i - U_i^*)}{\sum_{i=1}^{n} U_i} \le \delta_0.$$

Consequently, formula (5) can be used to find the desired value with a relative error of no more than δ_0.

If condition (6) is not met, then mathematical model (5) at $t \ge t_*$ possesses the properties of fullness, adequacy, productivity and economy to a sufficient degree.

The development of a new mathematical model in the event of the formation of a hierarchy of mathematical models of the object of study might lead to the clarification of the previously determined conditions for the applicability of the built mathematical models. Indeed, it is possible to clarify the condition of applicability of formula (3) using the mathematical model (2), (7). For this we need to calculate the time point

$$t_i = t_0 + \frac{c_i}{\alpha_i S_i}\left[\left(\frac{\gamma_i T_0^2}{\beta_i}\frac{r_i I - U_i^*}{2r_i I - U_i^*}\frac{r_i I}{U_i^*} + \right.\right.$$
$$\left. + \frac{r_i I}{2r_i I - U_i^*} - 1\right)\ln\left(1 + \frac{U_i^*}{r_i I}\delta_0\right) -$$
$$-\frac{\gamma_i T_0^2}{\beta_i}\delta_0 - \left(\frac{\gamma_i T_0^2}{\beta_i}\frac{r_i I - U_i^*}{2r_i I - U_i^*}\frac{r_i I}{U_i^*} + \right.$$
$$\left.\left. + \frac{r_i I}{2r_i I - U_i^*}\right)\ln\left(1 - \frac{U_i^*}{r_i I - U_i^*}\delta_0\right)\right],$$

for which

$$U_i\left(t_i\right) = \frac{r_i I}{1 + \delta_0}.$$

It is obvious that at $t \le t_i$

$$\delta\left(r_i I\right) = \left|\frac{U_i - r_i I}{U_i}\right| = \frac{r_i I}{U_i} - 1 \le \delta_0,$$

and the value $r_i I$ can be considered equal to $U_i\left(t\right)$ with a relative error of no more than δ_0. If $t^* = \min_{1 \le i \le n} t_i$, then it is easy to show that at $t \le t^*$

$$\delta\left(U_0\right) = \left|\frac{U - U_0}{U}\right| = \frac{\sum_{i=1}^{n}\left(r_i I - U_i\right)}{\sum_{i=1}^{n} U_i} \le \delta_0.$$

Consequently, formula (3) can be used to find the desired value with a relative error of no more than δ_0.

If condition (6) is met or $t \le t^*$, then the mathematical model (3) possesses the properties of fullness, adequacy, productivity and economy to a sufficient degree.

Results

The following statements, which make it possible to identify a working mathematical model of the object of study, are valid in case of inequation (4).

Statement 1. If condition (6) is met or $t \le t^*$, then the mathematical model (3) is considered as working.

Statement 2. If condition (6) is not met, then the mathematical model (5) at $t \ge t_*$ is selected as working.

Statement 3. If the inequation (6) does not hold, while the time interval from t^* to t_* is of interest, then the mathematical model (2), (7) is considered as working.

Conclusion

Thus, a unified approach was used to formulate the statements that allow us to define a mathematical model of a technical system. They allow a working mathematical model of a technical system to be established that provides for serial connection of negative temperature coefficient thermistor thermistors. The built mathematical model possesses the properties of fullness, adequacy, productivity and economy to a sufficient degree.

The use of such a mathematical model not only reduces the time and costs spent on conducting research, but also facilitates the rational use of mathematical modeling capabilities.

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