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MATHEMATICAL MODELING OF MAGNETO-ELASTIC VIBRATIONS OF AN ANNULAR PLATE IN A MAGNETIC FIELD

Abstract: *The stress state of a flexible annular plate is analyzed in the article under the action of a time-variable mechanical force and a time-variable external electric current, taking into account mechanical and electromagnetic anisotropy.*

Key words: *plate, magnetic field, magneto elasticity.*
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Introduction

In recent decades, considerable attention in the literature has been paid to the study of the deformation process of electrically conductive bodies placed in an external constant magnetic field under the influence of power, thermal and electromagnetic loads [1,2,3,4,5,6, 7,8,9,10,11,12,13,14,15,16,17,18,19].

Interest in research in this area is associated with the importance of a quantitative study and assessment of the relationship effects observed between mechanical, thermal and electromagnetic processes and their practical application in various fields of modern technology in the development of new technologies, and in the field of microelectronics, modern measuring systems, etc. Modern technology places great demands on structural elements exposed to fields of various natures. This circumstance leads to the need to create new calculation methods that fully and adequately take into account the properties of real materials and the processes occurring in them.

That is why in recent years, nonstationary dynamic problems of electroelasticity and electromagneto-elasticity have attracted the attention of researchers. At the same time, if in electro-elasticity, a relatively large number of completed results are currently known (for both static and dynamic problems), then in electro-magneto-elasticity, the number of such studies is limited. At present, the least studied are the problems of nonstationary dynamics of elastic conducting bodies under the action of mechanical and electromagnetic fields.

I. STATEMENT OF THE PROBLEM. THE EQUATIONS OF MAGNETOELASTICITY.

Let us assume that an electrically conductive body is in a magnetic field formed both by an electric current in the body itself (internal magnetic field) and by a source located at a distance from the body (external magnetic field). The body has finite electrical conductivity and does not possess the property of spontaneous polarization and magnetization. We also assume that, in the general case, surface currents and external charges are absent.

Let us present the equations of magnetoelasticity for similar bodies in the Euler coordinates [2]:

equations of motion:

$$\frac{\partial t_{ki}}{\partial x_k}+\rho\left(F_i+F_i^{\wedge}\right)=\rho\frac{dV_i}{dt}, \qquad (1)$$

$$\rho F_i^{\wedge}=\varepsilon_{ilm}J_l B_m+\rho_e E_i, \qquad (2)$$

Maxwell's equations:

$$\varepsilon_{ijk}\frac{\partial H_k}{\partial x_j}=J_i+\frac{\partial D_i}{\partial t},\quad \frac{\partial B_i}{\partial x_i}=0, \qquad (3)$$

$$\varepsilon_{ijk}\frac{\partial E_k}{\partial x_j}=-\frac{\partial B_i}{\partial t},\frac{\partial D_i}{\partial x_i}=\rho_e. \qquad (4)$$

The following designations are introduced in relations (1)-(4): t_{ij} - components of the tensor of internal stresses; ρF_i - components of the vector of volumetric mechanical forces; $\rho F_i^{\wedge}$ - components of the vector of Lorentz volumetric forces; E_k, D_k, H_k, B_k, J_k - components of the vectors of the intensity and induction of the electric field, the intensity and induction of the magnetic field, respectively; $J_k=J_k^*+\rho_e V_k$ - components of the density vector of total current; J_k^* - conduction current density; $\rho_e V_k$ - convective current density; ρ_e - the density of electric charges; ρ - the density of the substance in its current state; V_k - components of the velocity vector; $\frac{d}{dt}=\frac{\partial}{\partial t}+V_k\frac{\partial}{\partial x_k}$ - total time derivative.

We assume that the geometric and mechanical characteristics of the body are such that a version of the geometrically nonlinear theory of thin plates in the quadratic approximation is applicable to describe the deformation process.

For the considered case of quadratic nonlinearity [1, 3, 4], we assume that deformations and shears are small in comparison with the angles of rotation of the element, and the angles themselves are substantially less than unity.

We also assume that electromagnetic hypotheses are fulfilled with respect to the electric field strength $\vec{E}$ and the magnetic field strength $\vec{H}$ [1].

The elastic properties of the material correspond to an anisotropic body, the main directions, the elasticity of which coincide with the directions of the corresponding coordinate lines. The electro-magnetic properties of the material are characterized by tensors of electrical conductivity σ_{ij}, magnetic permeability μ_{ij}, dielectric constant ε_{ij} $(i,j=1,2,3)$.

The system of equations of magnetoelasticity must be closed by relations connecting the vectors of intensity and induction of the electromagnetic field and by Ohm's law, which determines the density of current conductivity in a moving medium. If an anisotropic body is linear with respect to the magnetic and electrical properties, then the governing equations for the electromagnetic characteristics of the field and the kinematic equation for electrical conductivity, as well as expressions for the Lorentz forces, taking into account the external current in the Lagrange variables, is written, respectively, in the following form [2, 11]:

$$\vec{B}=\mu_{ij}\vec{H},\ \vec{D}=\varepsilon_{ij}\vec{E}, \qquad (3)$$

$$\vec{J}=\sigma_{ij}\,\Gamma F^T F^{-1}\left[\vec{J}_{cm}+\vec{E}+\vec{\upsilon}\times\vec{B}\right] \qquad (4)$$

$$\rho\vec{f}^{\wedge}=\Gamma^{-1}F^{-1}\left[\vec{J}_{cm}\times\vec{B}+\sigma_{ij}\left(\vec{E}+\vec{\upsilon}\times\vec{B}\right)\times\vec{B}\right] \qquad (5)$$

Thus, (1), (2) with relations (3) - (5) constitutes a closed system of nonlinear equations of magneto-elasticity of current-carrying orthotropic bodies with orthotropy of conducting properties.

When investigating the deformation of a circular orthotropic plate in a magnetic field, we refer it to a cylindrical coordinate system r,θ,z, so that the middle plane of the plate is connected with the polar coordinate system and the center of the plate is at the origin.

Consider an annular plate in a one-dimensional statement along the spatial coordinate r; suppose that $\partial/\partial\theta=0,\ v=0,\ E_r=0,\ B_\theta=0,\ S=0,\ H_\theta=0,$ $F_\theta=0,\ F_\theta^{\wedge}=0,\ h=h(r),$ where S– is the shear force, v – is the circular displacement.

An account for the diagonal form of the electrical conductivity tensors, the complete system of equations, which makes it possible to describe the geometrically nonlinear model of the magnetoelasticity of orthotropic annular plates, consists of [1, 6, 7]:

equations of magnetoelasticity

$$\frac{\partial(rN_r)}{\partial r}-N_\theta+r(F_r+\rho F_r^{\wedge})=r\rho h\frac{\partial^2 u}{\partial t^2};$$
$$\frac{\partial(rQ_r)}{\partial r}+r(F_z+\rho F_z^{\wedge})=r\rho h\frac{\partial^2 w}{\partial t^2}; \qquad (6)$$
$$\frac{\partial(rM_r)}{\partial r}-M_\theta-rQ_r-rN_r\vartheta_r=0;$$
$$-\frac{\partial B_z}{\partial t}=\frac{1}{r}\frac{\partial(rE_r)}{\partial r};$$
$$\sigma_2\left[E_\theta+0{,}5\frac{\partial w}{\partial t}(B_r^++B_r^-)-\frac{\partial u}{\partial t}B_z\right]=$$
$$=-\frac{\partial H_z}{\partial t}+\frac{H_r^+-H_r^-}{h};$$

expressions for deformations

$$\varepsilon_r=\frac{\partial u}{\partial r}+\frac{1}{2}\vartheta_r^2;\ \varepsilon_\theta=\frac{u}{r};\ \chi_r=\frac{\partial\vartheta_r}{\partial r};$$
$$\chi_\theta=\frac{1}{r}\vartheta_\theta, \qquad (7)$$

where $\vartheta_r=\frac{\partial w}{\partial r}$ is the angle of rotation of the normal;

elasticity relations

$$N_r=\frac{e_r h}{1-\nu_r\nu_\theta}(\varepsilon_r+\nu_\theta\varepsilon_\theta);$$
$$N_\theta=\frac{e_\theta h}{1-\nu_r\nu_\theta}(\varepsilon_\theta+\nu_r\varepsilon_r); \qquad (8)$$
$$M_r=\frac{e_r h^3}{12(1-\nu_r\nu_\theta)}(\chi_r+\nu_\theta\chi_\theta);$$
$$M_\theta=\frac{e_\theta h^3}{12(1-\nu_r\nu_\theta)}(\chi_\theta+\nu_r\chi_r).$$

In (6) - (8), the following notations are taken: $\nu_r = \nu_{\theta r}$, $\nu_\theta = \nu_{r\theta}$, $e_r\nu_\theta = e_\theta\nu_r$; ν_r, ν_θ – is Poisson's ratios; e_r, e_θ – are Young's moduli; u, w – are the displacements; N_r, N_θ – are the tangential forces; M_r, M_θ – are the bending moments; Q_r – is the generalized shear force; χ_r, χ_θ – are the main curvatures of the middle surface of the plate; N_r, $B_r^{\pm}$ – are the known values of the tangential components of the magnetic induction on the surfaces of the plate.

The components of the Lorentz force are as follows:

$$\rho F_r^{\wedge} = \sigma_1 h\left[E_\theta B_z - \frac{\partial u}{\partial t}B_z^2 + 0{,}5\frac{\partial w}{\partial t}(B_r^+ + B_r^-)B_z\right];$$

$$\rho F_z^{\wedge} = -\sigma_2 h\left[0{,}5E_\theta(B_r^+ + B_r^-) - 0{,}25\frac{\partial w}{\partial t}(B_r^+ + B_r^-)^2 + \frac{1}{12}\frac{\partial w}{\partial t}(B_r^+ - B_r^-)^2 - 0{,}5\frac{\partial u}{\partial t}(B_r^+ + B_r^-)B_z\right]. \quad (9)$$

III. NUMERICAL EXAMPLE. ANALYSIS OF ELECTROMAGNETIC EFFECTS.

Consider a nonlinear magnetoelasticity problem on the stress-strain state of an annular plate of variable stiffness under the action of unsteady magnetic field and arbitrary mechanical load. The plate is elastic orthotropic, made of material with finite electrical conductivity, located in an external magnetic field with strength vector $\vec{H}_0$. The plate is a conductor of a uniformly distributed external electric current of $\vec{J}_{rCT}$ density. Let the magneto-statics problem for the unperturbed state be considered as solved, that is, the vectors of the magnetic induction of the initial state for the outer and inner regions are known.

We investigate the stress-strain state of a metallic boron/aluminum annular plate of constant thickness h, inner radius r_0, outer radius r_1, under the influence of the normal component of mechanical load P_Z and an external magnetic field with a given vector of magnetic induction $\vec{B}^{(e)}$.

The boundary conditions are

$$s = r_0 = 0: u = 0,\ w = 0,\ \vartheta_r = 0,\ B_z = 0.5\sin\omega t;$$

$$s = r_N = 0.0009m:\ N_r = 0,\ Q_r = -100,\ M_r = 0,\ E_\theta = 0.$$

The initial conditions are

$$\vec{N}(s,t)\big|_{t=0} = 0,\quad \dot{u}(s,t)\big|_{t=0} = 0,\quad \dot{w}(s,t)\big|_{t=0} = 0$$

The parameters of the plate and the material are:

$r_0 = 0.005m$; $r_1 = 0.009m$; $h = 5\cdot10^{-4}(1-\gamma r^2/r_0)m$; $\gamma = 0.7$,
$\sigma_1 = 0.454\cdot10^8(\Omega\times m)^{-1}$, $\sigma_2 = 0.200\cdot10^8(\Omega\times m)^{-1}$, $\nu_r = 0.262$;
$\nu_\theta = 0.320$; $e_r = 22.9\cdot10^{10}\ N/m^2$; $e_\theta = 10.7\cdot10^{10}\ N/m^2$;
$\omega = 314.16\ \sec^{-1}$; $P_z = 5\cdot10^3\sin\omega t\ H/м^2$; $P_r = 0$;
$\tau = 1\cdot10^{-2}\sec$; $\mu = 1.256\cdot10^{-6}\ H/м$; $\rho = 2600\ kg/m^3$;
$J_{\theta CT} = 3\cdot10^7\sin\omega t\ A/m^2$; $B_r^{\pm} = 0.5Tл$;
$B_{r0} = 0.5\sin\omega t$; $\Delta t = 1\cdot10^{-3}\sec$; $0 \le t \le 1\cdot10^{-2}\sec$.

The solution to the problem is determined over time interval $\tau = 10^{-2}\sec$, the integration step over time is taken as $\Delta t = 1\cdot10^{-3}\sec$ at one hundred points of integration over the length of the shell. The maximum values are obtained at time step $t = 5\cdot10^{-3}\sec$. Fig. 1 shows the graphs of deflection change $w(r)$ depending on the radial coordinate of the plate at time point $t = 5\cdot10^{-3}\sec$ for three values of magnetic induction. Graphs $1\div3$ correspond to magnetic induction 1. $B_{z0} = 0.1$; 2. $B_{z0} = 0.2$; 3. $B_{z0} = 0.5$, respectively.

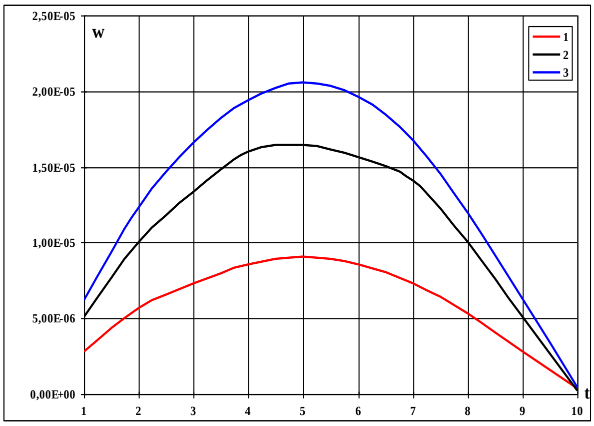

Figure 1. Graphs of deflection changes in time $w(t)$ for the values of the normal component of external magnetic induction

Analyzing the numerical results obtained, it can be seen that with an increase in the values of normal component of external magnetic induction, the deflection of the plate increases.

IV. CONCLUSION.

The article deals with the coupled problem of magnetoelasticity for a flexible orthotropic conductive annular plate taking into account the anisotropy of the conductive properties. A solution was obtained for the nonlinear problem of magnetoelasticity of an annular plate taking into account anisotropic electrical conductivity.

The analysis of the results obtained allows us to evaluate the influence of the normal components of magnetic induction on the stress state of a flexible orthotropic annular plate. Based on the results presented, the magnetoelastic nonlinear problem for a conductive annular plate must be considered in a coupled form.

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