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APPLICATION OF THE SPECTRAL-GRID METHOD IN SOLVING THE STABILITY PROBLEM

Abstract: *The article discusses the spectral-grid method for solving the stability equation for two-phase flows taking into account the non-stationary effects of the force interaction of the phases.*

Key words: *gradient, polynomial, stability equations, wavenumber, growth rate.*

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Introduction

In the spectral grid method [1, 2, 3, 4, 5, 6] for a given number of elements N to achieve the required accuracy of calculations, it is necessary to correctly position the grid nodes and choose the number of polynomials p_j, the number of Chebyshev polynomials for approximating the solution by p_j j-th element. These questions are closely related, since by bringing the joining nodes closer together, one can reduce the number of polynomials on the elements and vice versa. In practice, it is apparently more convenient to choose a uniform mesh by setting different p_j on each element. Then the number of required polynomials depends on the relative magnitude of the gradients of the solution on a particular element. Solution gradients can often be estimated from asymptotic analysis.

In the problem of the stability of the boundary layer, it is well known [7] that near the wall - in the so-called critical layer - the behavior of the solution is determined by a rapid change in viscous solutions:

$$\psi \approx e^{-(k\,\mathrm{Re})^{\frac{1}{3}} y},\ k\,\mathrm{Re} >> 1.$$

Far from the wall, perturbations slowly decay according to the law:

$$\psi \approx e^{-ky},\ k >> 1.$$

It can be seen that the relative value of the gradients of the solution at the wall in $\sqrt[3]{\mathrm{Re}}$ times more than far from her. The refore, the number of nodes, and hence the number of polynomials, should be greater near the wall ~ in $\sqrt[3]{\mathrm{Re}}$ once. More accurate values p_j are selected in the process of calculations.

We now turn to the presentation of the algorithm of the spectral-grid method for the numerical solution of the equations of stability of two-phase flows

$$D^2\psi - ik\,Re\left(\mathrm{V} - \lambda - \frac{if}{k\tau}\right)D\psi + ik\,Re\,\frac{d^2\mathrm{V}}{dy^2}\psi + \frac{f}{S}D\varphi - i\lambda k\,\frac{f}{2S_1}\,(D\varphi - D\psi\,) + i\lambda k\frac{f}{S_1}D\psi -$$
$$-\frac{3}{2}\,Re\,f\sqrt{\frac{k}{S_1\tau}}\,(i - -1)\sqrt{\lambda}\,(D\psi - D\varphi\,) = 0, \qquad (1)$$

$$\text{-}D\psi - ik\tau\,\left(\mathrm{U} - \lambda - \frac{i}{k\tau}\right)D\varphi + ik\tau\,\frac{d^2\mathrm{U}}{dy^2}\,\varphi - i\lambda\frac{k}{2S_1}(\partial\psi - \partial\varphi\,) - i\lambda\frac{k}{S_1}D\varphi -$$
$$-\frac{3}{2}\sqrt{\frac{k\tau}{S_1}}\,(i - 1)\,\sqrt{\lambda}\,(D\varphi - D\psi) = 0\,, \qquad (2)$$

where $\lambda = \frac{w}{k}$, $\lambda = \lambda_r + i\lambda_i$ – unknown constant to be determined, λ_r – phase velocity, λ_i – slew rate, $f = \alpha_0 S_1$ – mass concentration of particles. If $\lambda_i > 0$, then the flow is unstable if $\lambda_i < 0$ – stable. If $\lambda_i = 0$, then the oscillations are neutral stable, the curve in which $\lambda_i = 0$ called the curve of neutral stability. Here $D = \frac{\partial^2}{\partial y^2} - k^2$, k – wave number.

Consider equations (1), (2) under the following boundary conditions:

$$\psi(\eta) = 0,\ \frac{d\psi}{d\eta} = 0,\ \varphi(\eta) = 0 \text{ at } \eta = \eta_0; \qquad (3)$$

$$\psi(\eta) = 0,\ \frac{d\psi}{d\eta} = 0,\ \varphi(\eta) = 0 \text{ at } \eta = \eta_i. \qquad (4)$$

For a specific type of flow $U(\eta)$, boundary conditions (3), (4) have a definite physical meaning. Integration interval $[\eta_0, \eta_1]$ split into a grid. $\eta_0 < \eta_1 < \ldots < \eta_N = \eta_l$ and thus we get n various elements:

$$[\eta_0, \eta_1],\ [\eta_1, \eta_2], \ldots, [\eta_j, \eta_{j+1}], \ldots, [\eta_{N-1}, \eta_N],$$
$$j = 0,1,2,\ldots,N-1.$$

Boundary conditions (3), (4) written in dots η_0, η_N, and the requirements for the continuity of the solution of the equations (1), (2) and their derivatives up to (M - 1) - th order are of the form:

$$\psi_j^{(t)}(\eta_j) = \psi_{j+1}^{(t)}(\eta_j),\ t = 0,1,2,3;\ j = 1,2,\ldots,N-1; \quad (5)$$

$$\psi_j^{(p)}(\eta_j) = \psi_{j+1}^{p(t)}(\eta_j),\ p = 0,1;\ j = 1,2,\ldots,N-1; \qquad (6)$$

where t and p indicate the order of the derivative.

Solutions ψ_j, φ_j eqs. (1), (2) can be represented as a series in the Chebyshev polynomials of the first kind. To do this, each element $[\eta_j, \eta_{j+1}]$ map to interval [-1, +1] by using:

$$\eta = \frac{m_j}{2} + \frac{l_j}{2} y, \qquad (7)$$
$$m_j = \eta_j + \eta_{j+1}, \quad l_j = \eta_j + \eta_{j-1}.$$

across l_j indicated length j - th element.

Equations (1), (2) after applying transformation (7) take the form:

$$F_j \psi_j^i + M_j \varphi_j = 0; \qquad (8)$$

$$S_j \psi_j + k_j \varphi_j = 0, \qquad j = 1,2,\ldots,N, \qquad (9)$$

where

$$F_j = \frac{1}{ik_j \operatorname{Re}_j} D_j^2 - \left(U_j(y) - \lambda - \frac{if}{k\tau}\right) D_j +$$
$$+ \frac{d^2 U_j}{dy_j^2} + \frac{f}{S_1} \lambda D_j;$$

$$M_j = \frac{f}{ik\tau} D_j; \qquad S_j = \frac{f}{ik\tau} D_j - \frac{\lambda}{S_1} D_j;$$

$$k_j = -\left(U_j(y) - \lambda - \frac{i}{k\tau}\right) D_j + \frac{d^2 U_j}{dy_j^2};$$

$$D_j = \frac{d^2}{dy_j^2} - k^2; \quad k_j = \frac{l_j}{2} k; \quad \operatorname{Re}_j = \frac{l_j}{2} \operatorname{Re}.$$

In this case, the boundary conditions and continuity conditions for (8) are

$$\psi_1(-1) = 0,$$
$$\frac{d\psi_1}{dy}(-1) = 0,$$
$$l_j^{-t} \psi_j^{(t)}(+1) = l_{j+1}^{-t} \psi_{j+1}^{(t)}(-1),\ t = 0,1,2,3;\ j = 1,2,\ldots,N1,$$

where the conditions of continuity for pure gas (these conditions are set only at the inner nodes of the grid).

$$\psi_N(+1) = 0,$$
$$\frac{d\psi_N}{dy}(+1) = 0, \qquad (10)$$

similar conditions for (9) have the form

$$\varphi_1(-1) = 0,$$
$$l_j^{-p} \psi_j^{(p)}(+1) = l_{j+1}^{-p} \psi_{j+1}^{(p)}(-1), \qquad (11)$$
$$p = 0,1;\ j = 1,2,\ldots,N-1,$$

where the continuity conditions for particles.

$$\psi_N(+1) = 0.$$

We seek an approximate solution to problem (8), (9) at each of the elements in the form of the following series:

$$\psi_j(y) = \sum_{n=0}^{p_j} a_n^{(j)} T_n(y),$$
$$\varphi_j(y) = \sum_{n=0}^{p_j} d_n^{(j)} T_n(y), \qquad (12)$$
$$U_j\left(y_l^{(j)}\right) = \sum_{n=0}^{p_j} b_n^{(j)} T_n\left(y_l^{(j)}\right),$$

where $T_n(y)$ – Chebyshev polynomials of the first kind; $y_l^{(j)} = \cos\left(\frac{\pi l}{p_j}\right), (l = 0,1,2,\ldots,p_j;\ j = 1,2,\ldots,N)$ – Chebyshev polynomial nodes; p_j – number of polynomials per j- th element.

For Chebyshev polynomials of the first kind, the following recursive formula is valid

$$T_{k+1}(x) = 2xT_k(x) - T_{k-1}(x),$$
$$T_0(x) = 1, \qquad T_1(x) = x.$$

The derivatives of these polynomials are determined by the following recurrent formulas:

$$T'_{k+1}(x)=2T_k(x)+2xT'_k(x)-T'_{k-1}(x),\quad k\ge 1,$$

$$T'_0(x)=0,\; T'_1(x)=1;$$

$$T''_{k+1}(x)=4T'_k(x)+2xT''_k(x)-T''_{k-1}(x),\quad k\ge 1,$$

$$T''_0(x)=0,\; T''_1(x)=0;$$

$$T'''_{k+1}(x)=6T''_k(x)+2xT'''_k(x)-T'''_{k-1}(x),\quad k\ge 2,$$

$$T'''_0(x)=0,\; T'''_1(x)=0,\; T'''_2(x)=0.$$

Substituting series (12) into (8), (9) according to the Galerkin method, we require that the left side of equation (8) on each of the elements be orthogonal to the first $(p_j-4)-$ м and, similarly, the left-hand side of equation (9) to the first $(p_j-2)-$ m to Chebyshev polynomials, i.e.

$$(F_j\psi_j+M_j\varphi_j,T_n)=0,\quad n=0,1,\dots,p_j-4, \tag{13}$$

$$(S_j\psi_j+k_j\varphi_j,T_n)=0,\; n=0,1,\dots,p_j-2;\quad j=1,2,\dots,N \tag{14}$$

where *(f, g)* - dot product on a segment [-1.+1], i.e.

$$(f,g)=\int_{-1}^{+1}f(y)g(y)(1-x^2)^{-\frac{1}{2}}dy.$$

Conditions (10), (11) with the use of (12) are written in the form:

$$\sum_{n=0}^{p_1}(-1)^n a_n^{(1)}=0,\quad \sum_{n=0}^{p_1}(-1)^{n-1}n^2a_n^{(1)}=0,\quad \sum_{n=0}^{p_j}a_n^j=\sum_{n=0}^{p_{j+1}}(-1)^n a_n^{(j+1)}=0,$$

$$\frac{1}{l_j}\sum_{n=0}^{p_j}a_n^{(j)}n^2=\frac{1}{l_{j+1}}\sum_{n=0}^{p_{j+1}}(-1)^{n-1}n^2a_n^{(1)},$$

$$\frac{1}{l_j^2}\sum_{n=0}^{p_j}a_n^{(j)}T''_n(+1)=\frac{1}{l_{j+1}^2}\sum_{n=0}^{p_{j+1}}a_n^{(j+1)}T''_n(-1),$$

$$\frac{1}{l_j^3}\sum_{n=0}^{p_j}a_n^{(j)}T'''_n(+1)=\frac{1}{l_{j+1}^3}\sum_{n=0}^{p_{j+1}}a_n^{(j+1)}T'''_n(-1),\quad j=1,2,\dots,N-1,$$

$$\sum_{n=0}^{p_N}a_n^{(N)}=0,\quad \sum_{n=0}^{p_N}n^2a_n^{(N)}=0, \tag{15}$$

$$\sum_{n=0}^{p_1}(-1)^n d_n^{(1)}=0,\quad \sum_{n=0}^{p_j}d_n^j=\sum_{n=0}^{p_{j+1}}(-1)^n d_n^{(j+1)}=0,$$

$$\frac{1}{l_j}\sum_{n=0}^{p_j}d_n^{(j)}n^2=\frac{1}{l_{j+1}}\sum_{n=0}^{p_{j+1}}(-1)^{n-1}n^2d_n^{(j+1)},\quad j=1,2,\dots,N-1,$$

$$\sum_{n=0}^{p_N}d_n^{(N)}=0. \tag{16}$$

Thus, to determine $2\sum_{j=1}^{N}(p_j+1)$ unknown $a_n^{(j)},d_n^{(j)}(n=0,1,\dots,p_j,\ j=1,2,\dots,N)$ we have $2\sum_{j=1}^{N}(p_j+1)$ equations.

These equations will be: $\sum_{j=1}^{N}(p_j-3)+\sum_{j=1}^{N}(p_j-1)$ orthogonality equations (13), (14), $4N$ - conditions (15) and $2N$ - conditions like (16).

A system of linear algebraic equations (13), (15), (14), (16) write in matrix form

$$(A-\lambda B)x=0 \tag{17}$$

here *A* and *B* are complex matrices.

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