# Non Algebraic Techniques for Digital Processing of Historic Painting Research Documentation

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**Abstract.** Conservation of historic painting requires comprehensive and correct information to be analyzed during the diagnostic and decision making process in a systematic and rational approach. This paper aims to contribute for the setting of an optimal working process tailored for digital image processing of historic painting research documentation, to be used also in other interdisciplinary areas. To the end it integrates the purely algebraic approach with expediencies derived from the specific scientific background of multidimensional and multimodal images of different types (MMI). Following key tasks have been addressed: 1)Presentation involving non algebraic methods whose main advantages are the functionality to the objectives of the scientific study, the relatively reduced computational complexity and the possibility of parallel processing of MMI; 2) Simulation of new digital expedients based on developed methods by using test databases containing different types of MMIs; 3) MMI analysis based on non-algebraic intelligent segmentation methods that expand the possibilities for proper detection, recognition and evaluation of the changes in the surveyed objects; 4) MMI processing based on adaptive interpolation, which is of small computational complexity and provides high quality interpolated areas (objects) in 2,5D, facilitating the decision-making process on the basis of the relevant information. The effectiveness of the proposed approaches in terms of scientific functionality, accuracy of the diagnosis, and low computational complexity is demonstrated with examples of applications to typical real data in the respective subject area.

**Keywords:** Digital Image Procession, Non Algebraic Digital Techniques, Historic Painting Research Documentation.

## 1 Introduction

Decision making process in conservation of historic painting necessitates the utilization of multimodal & multidimensional images (MMI) and attribute data reflecting its different aspects and coming from various sources, such as: multispectral cameras, IR

thermographs, sonic tomography, μ-chemical and μ-anatomical analyses, computer tomography obtained by nuclear magnetic resonance, positron emission and magnetic resonance tomography, etc . With the advent of non contact diagnostic methods that are almost completely supported by digital technologies, the use of MMIs for cultural heritage management has grown in various interdisciplinary areas bringing to the creation of many new types, subtypes and hybrids on base of timeline, application field and format, visualization method, technical systems and aspects and actually exist several types describing the spatial and temporal or spectral characteristics of the studied objects: besides in 3D, also in 2D; 2,5D; and 4D.

In the past, the expert deductions based on their complex analysis were schematically presented manually, in the form of simplified graphics: of single layers (stratigrammata) or integrated (fragmentarizations) layouts, indicating less or more precisely the interrelationships among the fragments. In the digital era, when all research documentation is produced in form of tensors (Pratt, 2007) (Woods, 2012) sampling the different signal classes that describe the spatial, temporal and spectral characteristics of examined objects, the need to can design, manipulate and share these technical visualizations via computer arises critically. To facilitate the process of interpreting information contained in MMIs, and to allow scientists to have a "full picture" of the studied artworks, a large number of methods and algorithms for their computer processing is used actually (Fieguth, 2011) (J. Toriwaki, 2009) (B. Cyganek, 2009) however there aren't yet binding standards or guidelines of the working process to follow by processing of scientific documentation in every specific cultural heritage sector. The growing number of applications and application fields and techniques causes several problems for the use of digitally processed MMI in terms of scholarly content, sustainability, evidence and long-term archiving of knowledge. The greatest drawback is that the algebraic approach – with the exception of few authors (Vavilov, 2013) - rests phenomenological and descriptive: it treats the digitized signals exclusively as morphological units of pixels/voxels with determined amplitude, but isolate from the scientific background relevant to the objectives of the study for which the image is processed.

This paper surveys selected cases of a wider research project (Stoyanova, Spectral Investigation of Serbian Baroque Icons for their Scientific Documentation., 2015), whose aim is to establish an optimal working process tailored for image processing of historic painting research documentation, producing accurate and concise technical visualizations easily comparable with related MMIs, more functional to the research objectives of the interdisciplinary assessment, to be used also in other interdisciplinary areas. Considering possibilities and limits of digital processing, it integrates existing algebraic approach with expediencies derived from the specific scientific background of analytic images of different types (MMI), merging the geometric representation with the artefact's diagnosis aimed to understand the technical & technological properties and indispensable for correct dating, attribution, conservation and restoration.

The paper is organized as follows: Sect.2 gives some background on the possibilities and limits of algebraic approach in view of the typical scientific goals pursued in diagnostic of historic painting; Sect.3 briefly surveys selected non algebraic expedients for

digital processing, while in Sect. 4 we address the interpolation of the related visualizations. To demonstrate the utility of the proposed expediencies, examples of applications to typical real data in the respective subject area are included.

## 2 The Limits of the Purely Algebraic Approach

The algebraic methods proposed in the literature are based mainly on frequency transformations, statistical analysis and interpolation in the high-order time domain, which complicates the processing and leads to significant delays in their execution. Their major disadvantage is the impossibility of adapting to the nature of the images in specific local areas, especially in images obtained from different sources, often resulting in poor reproduction and inability to repeatedly interpolate with relatively high quality (Woods, 2012) (B. Cyganek, 2009) (K. O. Egiazarian, 2015). Consequently, finding neighbouring pixels from one object in the sequence of images represents a serious problem. Linear interpolation variations are most often used, resulting in a more inaccurate reproduction of the volume structure of individual objects in the images. However, the main disadvantage of the purely algebraic approach is that it only describes the quantitative parameters of the measured signals and formulates the techniques for their manipulation, but rests isolated from the scientific background and objectives of the research.

For representation of images in computer vision systems now are widely used the statistical decomposition of SVD (Singular Value Decomposition), KLT (Karhunen-Loeve Transform) and PCA (Principal Component Analysis): (Ventzas, 2012), (M. Moonen, 1995), (Gerbrands, 1981), (Orfanidis, 2007), (E. Drinea, 2001), (Reed, 2005). The decomposed images they produce seem to be optimal because most of their energy is concentrated in a minimum number of components, but the accuracy of the digital transforms is validated only in respect to the visible. However, the algebraic values of the perceived signals not always and not completely correspond to the integral physic-chemical characteristics.

The limited use of these decompositions is related to their great computational complexity, which increases considerably with increasing the size of the image matrix. Another limitation imposes the distortion of the original scientific data they cause. In the most cases, the signals describing images are photometric values matching the energy characteristics of visible, ultraviolet, infrared or XR radiation at its emission, propagation and absorption, but photometry examines also the properties of radiation sources and receivers, including the human eye: factors that are rarely considered by algebraic manipulation of images. Moreover, the properties of light cannot be described only by intensity. Radiation is characterized by phase, wavelength, amplitude, and polarization; by one or other spectral composition. Different light streams have different diffraction coherence, they can interfere. Low intensity of an element on radiography doesn't mean "area of any interest", but indicate materials with low XR absorption: by algebraic compression they should not be eliminated, but rather enhanced. Exactly as in radiography, CT aims to individuate different materials on base of minimal density differences. The representation of the measured data is based usually on a grey scale, which provides also possibility for qualitative and quantitative evaluation: measured (in Hounsfield

units, HU) is the linear attenuation, internal to an element of the layer, which depends on the specific density, ρ (Rho), and on the composition of the material. By wood analysis, i.e., considering the wood species, it allows to determine the age. The specific density (Rho) however is often confused with spectral density which is linked to radiation energy (M=dΦ/ds, the light flux emitted by a determined surface), and to the energy of illumination (E=dΦ/ds ) which allows to quantify the light flux falling on that surface:

$$M(v) = \frac{\lambda^2}{c} M(\lambda); \ \ E(v) = \frac{\lambda^2}{c} E(\lambda).$$

Radiant flux emitted, reflected, transmitted or received by a *surface*, per unit solid angle per unit projected area (radiance, $L_{e,\Omega}$), is sometimes confusingly called "intensity", while radiance of a *surface* per unit frequency or wavelength (spectral radiance, $L_{e,\Omega,v}$ *or* $L_{e,\Omega,\lambda}$) is confused with "spectral intensity". Great confusion is made also between irradiance flux density ($E_e$ ) and intensity; radiosity ($J_e$) and intensity; spectral radiosity ($J_{e,v}$ *or* $J_{e,\lambda}$) and spectral intensity; radiant exitance, the emitted component of radiosity ($M_e$) and intensity; spectral exitance (or spectral emittance, $M_{e,v}$ *or* $M_{e,\lambda}$) and spectral intensity, etc (IUPAC, Compendium of Chemical Terminology, 1997).

Another example illustrating the dihotomy between algebraic and integrated scientific approach gives the fact that a strictly plane-parallel flow of light (dω=0), from a photometric point of view has any energy (dΦ=0), instead in optics/physics we often speak properly about plane electromagnetic waves. Relation between photometric and electrodinamic values is simple only by plane waves, but by rule it is very complex and ambiguous (B.I.Stepanov, 1989). As result, mathematically "accurate" digital transforms may not be functional to the scientific goals of the image processing for inacceptable data distortions.

As regards the receivers, their matrix consists of light-sensitive elements (pixels) capable of generating an electrical signal proportional to the intensity of the light flux reaching them, independently of its chromatic components. For to receive information about the colour, each pixel is covered by a red, blue or green filter and, ideally, is able to pass only "its own" colour. Each firm uses to this purpose different transistors that have different chromatic sensibility.

In vibrothermography (also known as ultrasonic lock-in thermography or sonic IR imaging) the images are derived from acoustic signals: the mechanic vibrations produce voluminous heating concentrated mainly in the critical points and their most opportune registration is in 3D tomography: a 2D visualization should strongly distort their spatial nature. The most opportune visualization for IR thermograms, instead, is in 2D, as optical IR heating is flat, it expands over the surface and produces noise that hinders the correct reading already of the 2D thermograms, let a 3D visualization apart.

The reported examples show that digital algebraic approach is not omnipotent, but rather very limited and that the physical properties of the signals also should be borne in mind when selecting the computational methods; the limits of their applicability should be very rigorously defined, their concepts used correctly for not to get beyond

the permissible limits. As for preservation of artworks the maximal fidelity to the original is fundamental, and, as well known, every processing is a less or more invasive distortion, every manipulation of artworks' research data must be motivated by well defined scientific reasons and objectives.

## 3    Techniques

To facilitate the process of interpreting visual information contained in MMI, a large number of algorithms for their computer processing have been designed primarily for representation, improvement of their visual quality and description; analysis such as matching, segmentation, coding, diagnosis, control; compression; filtration of specific noises; pseudo-coloring; detection and segmentation of selected objects; restoration, (3D) visual reconstruction; extraction of signs for automatic classification of objects in them (Rosenfeld & Kak, 1982) (Yaroslavsky, 1985) (B. Cyganek, 2009) (Fieguth, 2011) (J. Toriwaki, 2009);  for storage in large databases (Big Data) (P. Bühlmann, 2016) etc. In this paper we focus only on the non algebraic expediencies for processing of historic painting research data.

### 3.1    Presentation

**Format Selection and Recording.** Format can vary from 80 x 80 pixels to 1024 x 1024, the most popular format actually is 240 x 320 elements. Increasing their number increases the possibility to distinguish details. The amplitude of every pixel/voxel can be expressed arithmetically or in bytes (in the last case it can be 8, 12, 14 or 16). Greater the bytes of the signal, greater the dynamic of the diapason it can reproduce. Format and bytes determine the volume of the file. As known, every major digital cameras manufacturer develops and uses its own variant: Kodak - DCR, Canon - CRW, CR2, etc. These different formats distinguish primarily for the processing algorithms required for to obtain the "normal" images. A particularly wide creative freedom (and at the same time opportunity for most accurate setting and calibration of the image) provides a file in RAW format. The RAW file contains just a grayscale image and data to restore its color using a sophisticated interpolation algorithm. Thus, at a much smaller volume (compared to uncompressed TIFF format) the RAW file contains much more image information. The obtained optical images provide conservators and researchers with previously unavailable material.

Before recording images to external media, it is desirable to use only three operations on files: conversion from RAW to TIFF format, image rotation by the required number of degrees, image framing. Using other operations on images is appropriate at the stage of their target application. In fact only the crop and rotate do not distort the image received after shooting. The conversion from RAW format to TIFF also causes certain distortion, but it is necessary to ensure the availability of image editing tools: TIFF format is universal, RAW format (used by manufacturers of digital cameras) is available only in Adobe Photoshop CS2 version 9 with a module CAMERA RAW.

**Optimization of the Images.** After polynomial approximation algorithms have been carried out, further processing can be applied as on the denoised image, as well as on the images of the polynomial coefficients: these can be identified less or more with the components of spectral frequencies. That's why it is to expect that detection of determined anomalies can be optimized analyzing the images of determined spectral wavelengths (1). In such case, the result will depend not only on the selected wavelength ("spectral coefficient"), but also on the entire sum of coefficients, used for the approximation. This operation depends on the purposes and as usually this is the individualization of determined textures on a noisy background, their quality is evaluated on base of criteria that contain areas of gradual and gitter mutation of the signal.



**Fig.1** A whitened script in VIS light (left) has been reconstructed in UV at 375 nm (right). (M. N. Zhizhin, 2018)

The surface of the paint layer - when visual recording has been performed directly on the original - can never be completely smooth, so perfect sharpness over the entire area of the frame by the hundredfold increase has not been achieved so far. The use of complex algorithms in this case has the goal to enhance the signal/noise relation and to optimize the possibility to reveal existing defects/anomalies. In many cases this can be achieved processing single portions of the images independently one from other. The results (2) demonstrate that such macro imaging can reveal hidden details of the original even when recorded in normal mode. This operation is often accompanied by

**Modifications of the histogram**. Whenever depth of digitization reaches 14 byte, the pixel amplitudes of the single images are collocated in a relatively narrow diapason. It is necessary to stretch the histogram between the minimum and maximum values.
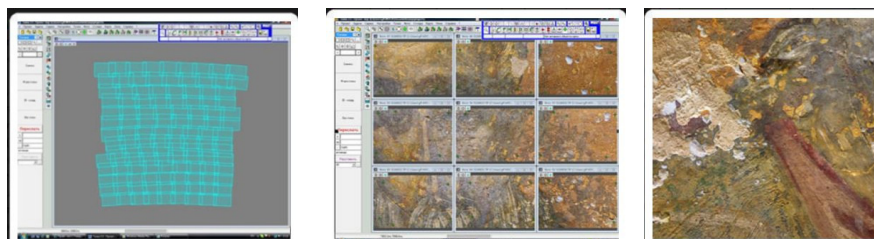


**Fig.2** A high-resolution image can be obtained with normal digital camera presenting the painting as a mosaic of details selected in order to minimize distortion of form and color, and recording it in computer RAW format, with a sufficient resolution of 8-10 megapixels (N.G. Bregman, V.V. Chistyakov, 2008).

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Images usually contain abnormally high and low signal areas that are not necessary to the analysis and for the matching of the low contrast details they can be visualized in a determined amplitude (an amplitude window) in the frame of the histogram.

**Selection of the Chromatic Palet.** In many cases MMI have pseudo colors, i.e. relation between pallet and histogram is determined by the operator, selecting among a variety of existing types. It should be remembered that, since the human eye and the photographic emulsion react differently to different color relationships, photographic effect may differ from the visually observed, and the contrast can also be different. Chromatic transformation of halftone images allows to render better visible details with similar characteristics presenting them in contrasting colors. In the rule, for low contrasting areas are recommended contrasting chromatic pallets and vice versa.

**Colour Inversion** is a useful expedient based on the psychophysical fact that the visibility of interference and distortions is lower near abrupt changes in brightness (edges, objects) than in places where the brightness change smoothly, i.e., where the intensity of the "contour" signal is low. From the standpoint of object localization in pictures by means of the optimal linear estimator, "contours" are what is obtained by picture "whiting" (3). The stronger this "contour" part of the signal described the object is (specifically, the sharper the picture of the object subjectively is), the more reliable is localization. Hence this procedure is strongly recommended by processing of radiographies and, in general, of dark images.
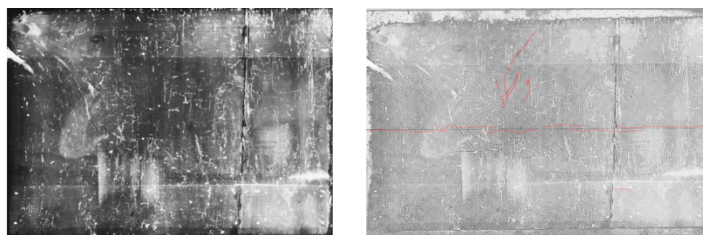


**Fig. 3** The digital XR in RAW (left) after color inversion and binary transformation revealed a subtle preparatory drawing lying under the levcas and paint layer, otherwise invisible (right, in red) (Stoyanova & G. Maximova, An Integrated Technical-Technological, 2017).

## 3.2    Simulation of New Expedients Based on Tested Methods

**Monochromatic filtering of VIS light (alias: "study in false color").** Screening in monochromatic light is based on the rules for color separation and helps to identify restoration retouches, to define their extension, make better visible artist's signatures or other details. The human eye hardly distinguishes between near hues of the spectral scale and often perceives as the same colors of different composition. For example, faded blue and green are almost indistinguishable from each other, but illuminating them with light at whose source is set a blue color filter, the blue object will appear much brighter than the green as it reflects more blue light than green. For monitoring in monochromatic light usually is used a standard filter or combination of such (for to

render the acquisitions comparable), selected after the principle of complementary colors: when the details are, for example, in greenish-blue coloration, a red filter should be used, for blue – an yellow, and so on. With the same success the different parts of the painting can be distinguished by placing a digital transparent chromatic filter over the studied image.

**Chromatic transformation of halftone images (see &3.1)**. In some cases of XR analysis, this expediency can be used also for material characterization based on parallelism between chromatic scale and elemental indexes (Stoyanova & G. Maximova, 2017), (Stoyanova & L.Pavlova, Deep Belief Networks for Multimodal, Images-Based Non Contact Material Characterization, 2017).

**Morphologic filtration** is a logical operation applied to binary images that changes the form of the field of interest. The modification of the pixels is accomplished in the frame of the moving over the image structure building element which can have different forms and measures, i.e. a mask 3 x 3 . Morphological processing is based on the interaction of two fundamental operations: erosion and dilation, which cause strong distortion, that's why we recommend to avoid it – when possible – replacing the image in VIS with spectral coefficients (as in & 3.1) and in (M. Stoyanova, 2015).

**Highlighting the surface relief.** To explore painting techniques, it is necessary to make photos in incident and raking light changing the source of light and its direction.

The compact design of diode lamps lighting strips allows to bring the light source close to the original. Thanks to this homogeneously directed lighting, running almost parallel to the plane of the photographed fragment, one can capture many of the nuances of painting texture, revealing more clearly the topography of the paint layer, and highlight the relief of brushstrokes. Therefore, if we want to have a complete understanding of the style of the artist's manner, it is important to photograph one and the same fragment in the same scale in different lighting modes or, when this is not possible, apply to the « lighting » functions of the image processing programs. Apart from scientific interest, this view has a direct practical purpose.

## 3.3 Intelligent Segmentation

**Definition of borders** is a typical segmentation task and is solved using gradient filters. The Sobel filter calculates the measures of the bidimensional spatial gradient in the original halftone image and produces a black field with maximal visible area borders, where the gradient of the signal is the highest. An effective gradient filter is the Laplassian, but is very sensible to the noise fluctuations of the signal and more frequently is used the combination of Laplassian and Gaussian (LoG) filters. A simple method for enhancing the different areas of the image could be their binary segmentation (M. Stoyanova, 2015) using Fourier Transform (FT) or working with the spectral coefficients. The difference between these approaches is that FT decomposes a function of time (a *signal*) into the frequencies that make it up, while the spectral images filter the radiation at determined wavelengths.

**Binary image segmentation of the spectral coefficients.** Segmentation of objects and structures, such as delineating the boundaries and area of objects, etc., is almost always

a classification. (Boykov, 2006) This means that data is only part of the required information. An a priori knowledge of the studied objects is fundamental. Developed methods for segmentation of structures find application in the separation of signs, different types of measurements in a given subject area and 3D visualization of the separated objects. Generally, they can be graded based on: Gray level distribution, Texture, and hybrid methods. The first have found the greatest application. They can be divided into Region Growing, Threshold Based, Active Contours, Deformable Models, Graph Cuts, Base Clustering based and Level set. In (M. Stoyanova, 2015) instead for digital segmentation we propose a technique derived from well tested methods for spectral analysis.

## 4    Adaptive Interpolation

How can be interpolated the information generated during the study of an artwork or the resulting stratigrammata into a global layout? For the conservators this means that they have to decide to which purposes such adaptive interpolations may serve, therefore - on which principle they will integrate them. By rule they are interested in visualizing several related MMS in such a way that the inherent differences can be easy compared and explored by trained specialists in order to understand the relationships among strata in a determined and in similar artifacts. Further, changes to the status of objects need also to be easily communicated, and to this end a 'change-tracking' device should be put in place to flag up changes and **fiducial points should be introduced to simplify comparisons among different types of images.**
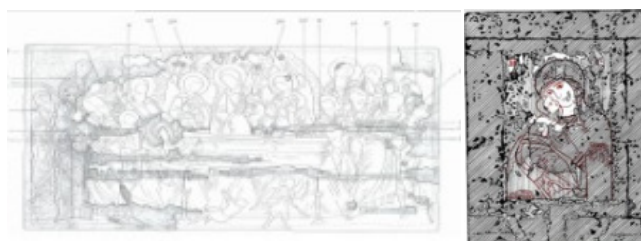


**Fig.4** Stratigram of the gesso layer of a Dormition icon indicating damages and later interventions (left); fragmentarization of a Vladimirskaja icon (right) that indicates with appropriate geometric coding the areas belonging to different epochs (M.Stoyanova, 2014).

The structural characteristics of historic painting make them particularly amenable to layered graph drawing methods (M.Stoyanova, 2014). Praxis has shown (4) that the most adapted approach is to visualize sets of related layers (stratigrammata) in 2,5D, i.e. to produce interdependent, two-dimensional, layered layouts with spatial elements, adapted to the research objectives, and then interpolate them in a global schematic layout (fragmentarization),

Stratigramma and fragmentarizations give a graphic schematic presentation of the data obtained during the microscopic and spectral/radiological analyses of multilayer

painting. They contribute to understand the made-up of the work, its status of conservation, to reconstruct virtually its vicissitudes and to produce referential standards for optical recognition useful as archaeometric/authenticity certification tools.

In stratigrammata objects may be modeled and analyzed according to theme or view in a larger model. Some of the main problems by their technical visualizations are the interpolation and the modeling, i.e. how to subset the sampled research images into individual objects or layers based upon theme or convenience; each of which can then be decomposed into a set of abstract geometric primitives - schematic figures, points, lines, faces and volumes; and a set of relationships describing how the object may be reconstructed from these primitives and how the individual spatial objects interact. To allow users to compare the single strata, we want to draw them in different parallel planes, but with interdependent layouts, called 2,5D because the third dimension is used in a way fundamentally different from 2d and 3D.

The adaptive interpolations of more or of all stratigrammata (fragmentarizations), from the point of view of scientific restoration serve the preservation of the original painting, weakened by time and often representing a conglomerate of paint layers going back to different epochs. It is no coincidence that for scientific publications of unique monuments are used different forms of mapping, 2,5D designs, schematizations and reconstructions of paintings that, unfortunately, is not a norm for documenting iconography. There is not an established methodology and pre-defined processes in relation to the working processes terms or documentation which should be adopted and followed. The development of innovative visualization techniques including integration of multi-sensor data creates continuously new premises for enhanced exploitation of information pertaining to art diagnostic and for on-line resources, overcoming the concept of the digital library towards a kind of information system, better corresponding to the specific demands of a broader interdisciplinary auditorium.

# 5    Conclusion

The rapid development of new modern computing and communication systems, scientific equipment and diagnostic and control technologies imply the continuous development and improvement of methods and algorithms for processing large volumes of multi-dimensional and multimodal information about objects and scenes in specific applications. In this connection, the requirements for compression of the information, the time for its processing, the accuracy of the detection and the automatic segmentation of the investigated objects have increased considerably. From the other side, as every processing causes the distortion of the images in a certain degree, and documentation of cultural heritage particularly requires max truth to the original, it is important to select only these operations that serve the objectives of the study. The interdisciplinary approach by digital presentation and preservation of cultural heritage we illustrated here opens new horizons that impose the development of new fields of cooperation among mathematicians and hard scientists. This is an on-going challenge, which will need to be improved and stream-lined as the trend evolves.

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