# SIMULATION OF ROOTS VACUUM PUMP ROTOR GEOMETRY
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# *МОДЕЛЮВАННЯ ГЕОМЕТРІЇ РОТОРА ВАКУУМНОГО НАСОСА РУТС*

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## ABSTRACT
*Mathematical model for designing the surface geometry of the Roots pump rotor based on the Cassini oval principle was derived. The polar coordinate system was used, and the radius vector, the direction of which was set by the φ angle, characterizes the location of the point on the surface of the rotor. The distance of this point from the axis of rotor rotation was set by the calculated value of the $\rho_R$ polar radius vector. The $\gamma$ angle of rotors rotation characterizes their mutual orientation in the plane of rotation. Peculiarities of the choice of the a and b parameters that satisfy the shape of the rotor surface geometry are considered. An example of rotor geometry is given for rotor radius R = 50 mm, rotor rounding radius r = 20 mm, parameters a = 33.166 and b = 28. Rotor geometry depends on normalized parameters of a and b, which are constant for a given shape of the surface and constructive dimensions. A mathematical model of the usable cross-sectional area of the pump has been developed. The usable cross-sectional area of the pump was simulated by the geometry of the rotors. The area of the rotor was determined by the geometry of the surface, which was described by an elliptic integral of the 2nd kind. The usable cross-sectional area for the given parameters is modelled. The results of simulation in the form of graphical dependences are given. Parameters a and b must meet the condition of $\sqrt{2}/2 < b/a < 1$. Under such conditions, the geometry of the rotor surface will be a Cassini oval. The rotation of the two rotors against each other will be by rolling one surface over another.*

## ABSTRACT
*Наведено математичну модель проектування геометрії поверхні ротора насоса Рутс за принципом овалу Кассіні. Використано полярну систему координат, а радіус-вектор, напрям якого задається кутом $\varphi$, характеризує розміщення точки на поверхні ротора. Віддаль цієї точки від осі обертання ротора задається розрахованим значенням полярного радіус-вектора $\rho_R$. Кут повороту роторів $\gamma$ характеризує взаємну їх орієнтацію у площині обертання. Розглянуто особливість вибору параметрів a і b, які задовольняють форму геометрії поверхні ротора. Наведено приклад геометрії ротора для радіуса обертання роторів R = 50* [mm], *радіуса заокруглень ротора r =* 20 [mm], *параметрів a* = 33.166 *і b* = 28. *Геометрія ротора залежить від нормованих параметрів a і b, що є сталими для заданої геометрії ротора і конструктивних розмірів. Розроблено математичну модель корисної площі поперечного перерізу насоса. Корисна площа перерізу насоса моделюється геометрією роторів. Площу, яку займає ротор визначали геометрією поверхні, яка описується еліптичним інтегралом 2-роду. Проведено моделювання корисної площі поперечного перерізу для заданих параметрів. Наведено результати моделювання у вигляді графічних залежностей. Параметри a і b повинні відповідати умові: $\sqrt{2}/2 < b/a < 1$. За таких умов геометрія поверхні ротора буде овалом Кассіні. Обертання двох роторів один проти одного проходитиме методом обкачування однієї поверхні по іншій.*

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## INTRODUCTION

Two-rotor vacuum pumps are widely used in industry, medicine, mechanical engineering and agriculture. Roots pumps are two-rotor. Such pumps have two rotors located in the pump casing and rotate synchronously. Ideally, the two rotors are in contact with each other, rolling around each other. They have no contact with the pump casing. Therefore, it is important to calculate the optimal geometry (surface shape) of the rotors, which will ensure the process of rolling the rotors on their surfaces. Having regard to the scope of use, for example for milking systems, it is necessary to take into account the forced pulsations of the medium flow due to the peculiarities of the milking equipment. Pressure regulator (*Dmytriv V. et al., 2017*), features of the pulsator operation of the milking machine (*Dmytriv V. et al., 2019*) and the milking system (*Dmytriv V. et al., 2020; Medvedskyi O. et al., 2018*) create the forced pulsations and extensive pressure losses. Such conditions require the special operation of vacuum pumps.

The authors considered the design features of the chamber geometry of the two-rotor pump at the condition of the gradual change of the gap between the rotors during the meshing of the rotors (*Li Y.-B. et al., 2018*). The flow in this geometry is numerically studied, and its effect on the pressure pulsation and radial exciting force is analysed based on the dynamic mesh and local mesh reconstruction. Numerical methods, in particular the computational fluid dynamics (CFD) method are widely used to simulate the operation of such pumps (*Peng Q.L. et al., 2018*). The authors analysed the effect of reflux on pump capacity and made sure that the result of CFD simulation is consistent with the result of the experiment.

A new type of rotor profile with a variable coefficient of trochoid shape was proposed and the method of achieving both the tightness and the higher volumetric efficiency was investigated (*Hwang Y.W. and Hsieh C.F., 2006*). The authors studied a new curve of the chamber (*Hsieh C.F., 2015*). The curve is formed around the longer axis of the rolling arc end point of the roller ellipse with the using of an elliptical roller track. Based on the curve model, the authors modelled a rotary pump. The results showed improved flow characteristics. Other authors have developed a new type of rotor profile, which has obviously improved pump capacity (*Kang Y.H. and Vu H.H., 2014*). The authors studied the capacity of the four rotor profiles by calculating the volume and analysed the flow field based on the number of pump leaves. The results showed that the shape of the rotors significantly affected the capacity of the pump. The authors also studied the effect of the number of leaves of the rotor and its geometry on the capacity of the pump (*Xing L. et al., 2020; Xing L. et al., 2021*).

To simulate the capacity of rotary pumps, the authors numerically studied the internal flow field in an involute-vane rotary pump using the *k-e* model of RNG turbulence and dynamic mesh (*Li Y.B. et al., 2013*). Also the influence of the pressure angle on the transient characteristics of the flow field inside the rotary pump was analysed. Other authors studied the effect of turbulence simulation on CFD prediction of local velocity fields of twin-screw compressors (*Kethidi M. et al., 2011; Kovacevic, Rane, Manolis, and Stosic, 2015*). The authors also investigated the local pressure loss in the suction chamber of the screw compressor and predicted the pressure loss with the CFD (*Arjeneh M. et al., 2015*). Using the results of the study, the authors (*Kovacevic A. et al., 2007*) described the mechanism of combining CFD and other software for design, in which the interactive control of the entire process of designing the screw compressor was obtained using a holistic control system. Also, the authors proposed the shape of the rotor in the form of an elliptical roulette curve and theoretically modelled the profile and flow of the medium (*Hsieh C-F., 2015*).

Widespread use of the Roots pumps has been proven by studies of its parameters and characteristics depending on thermodynamic parameters, as evidenced by theoretical dependencies (*Zhao F. et al., 2020*).

Analysis of research shows that the development of a mathematical model of the Roots pump rotors geometry, which would take into account the technical, thermodynamic characteristics of the medium pumping process, will increase the efficiency of such pumps. Therefore, the development of a mathematical model of the rotor geometry of the Roots pump and the study of its characteristics is an urgent research-and-development task.

## MATERIALS AND METHODS

The aim of this work was to develop a mathematical model of rotor geometry and study the characteristics of the Roots pump with rotors of simulated geometry.

### Root pump rotor geometry

Consider the rotor in the form of a Cassini oval. Scheme of the rotor is shown in fig. 1, a. For the Cassini oval, the following equation is true (*Karatas M., 2013*):

$$r_1^2 \cdot r_2^2 = a^2 \tag{1}$$

where $a$ – an arbitrary number that satisfies the expression of (1).

The rotor must have a curved plane with the $R_r$, radius, so that the opposite rotation of the two rotors is rolling without jamming. The distance between the centres of the rotor rounding is denoted as $O_1O_2 = 2 \cdot b$. The radius of rounding is denoted as *r*.

The Cassini oval equation is as follows (*Karatas M., 2013*):

$$[(x-b)^2 + y^2][(x+b)^2 + y^2] = a^4 \tag{2}$$

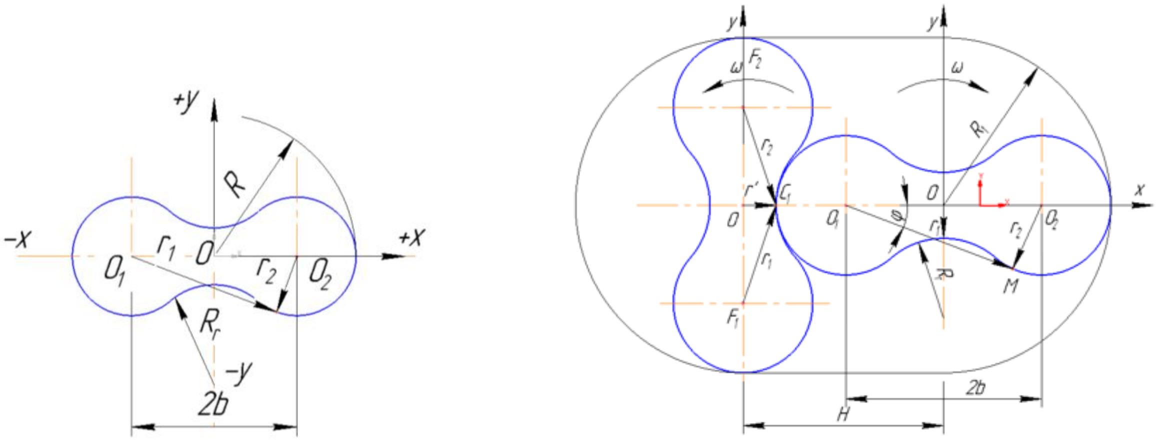
Where:

$(x-b)^2 + y^2 = r_1^2$ – the radius vector of the left part of the rotor relative to the *y*-axis (fig. 1);

$(x+b)^2 + y^2 = r_2^2$ – the radius vector of the right part of the rotor relative to the *y*-axis (fig.1);

$a$ and $b$ – the set of real positive numbers;

$x$ and $y$ – running coordinates of the Cartesian coordinate system.



a b

**Fig. 1 – Rotor diagrams (a) and rotor mutual placement (b)**

For the polar coordinate system: $x = \rho_R \cdot \cos\varphi\,;\ \ y = \rho_R \cdot \sin\varphi$. Taking into account that $x^2 + y^2 = \rho_2^2$, the equation (1) will be transformed:

$$(x^4 + 2x^2y^2 + y^4) = (x^2 + y^2)^2 = \rho_R^4$$

Then

$$\rho_R^4 - 2b^2(\rho_R^2 \cos^2\varphi - \rho_R^2 \sin^2\varphi) + b^4 - a^4 = 0$$

$$\rho_R^4 - 2b^2 \cdot \rho_R^2 \cdot \cos(2\varphi) + b^4 - a^4 = 0 \tag{3}$$

Equation (3) is solved as a quadratic equation and, accordingly, the following is obtained:

$$\rho_R = \sqrt{b^2\cos(2\varphi) \pm \sqrt{b^4 \cdot \cos^2(2\varphi) - b^4 + a^4}} \tag{4}$$

To model the geometry of the rotors rotation, the dimensions are set according to the scheme (Fig. 1, b). In the initial position, the rotors will be mutually located according to Fig. 1.b. Taking into account the dependence of (4), the rotation of the two rotors was modeled according to the following dependences:

- for vertically placed rotor:

$$\rho_R = \sqrt{b^2\cos(2\varphi + \gamma) \pm \sqrt{b^4 \cdot \cos^2(2\varphi + \gamma) - b^4 + a^4}} \tag{5}$$

- for a horizontally placed rotor the $\varphi$ angle is shifted by 90°

$$\rho_R = \sqrt{b^2\cos(2\varphi + 90 - \gamma) \pm \sqrt{b^4 \cdot \cos^2(2\varphi + 90 - \gamma) - b^4 + a^4}} \tag{6}$$

where:

$\gamma$ – the angle of rotors rotation, taking into account the rotation of the rotors in the opposite direction to each other (for a vertically located rotor, the $\gamma$ angle of rotation is positive, and for a horizontally located rotor, the $\gamma$ angle is negative).

The geometry of the rotor is characterized by the $\rho_R$ polar radius, as the distance from the center of rotor rotation to the point on its surface in the function of $\varphi$ angle (fig. 2). Angle $\varphi$ is within $0 \le \varphi \le 2\pi$.

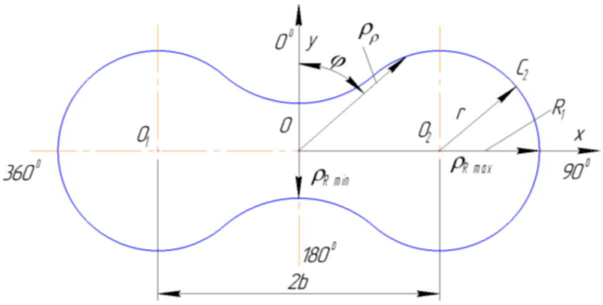



**Fig. 2 – Scheme to explain the $\rho_R$ polar radius as a parameter of rotor geometry**

It should be noted that the parameters of *a* (formula 1) and *b* (Fig. 1 and Fig. 2) are constants, while $r_1$ and $r_2$ are rotor rotation functions and characterize the point of contact of two rotors, which will move on the surface between two rotors depending on their mutual position.

**Mathematical model of the cross-sectional area of the working volume of the Roots two-rotor pump**

A mathematical model of the cross-sectional area of the working volume is developed for one rotor. The change in cross-sectional area for the second rotor will be shifted by an angle of $\frac{\pi}{2}$.

The cross-sectional area, limited by the $R_1$ stator radius, is calculated by the formula:

$$S_{\frac{1}{2}H} = S_{R_1} - S_{ROT}, \tag{7}$$

Where:

$S_{R_1}$ – half of the stator cross-sectional area, limited by the $R_1$ stator radius (Fig. 1, 2);

$S_{ROT}$ – half of the cross-sectional area of the rotor

$$S_{R_1} = \frac{1}{2}\int_{\gamma}^{\pi} R_1^2(\varphi)d\varphi = \frac{1}{2}R_1^2(\pi-\gamma) \tag{8}$$

$$S_{ROT} = \frac{1}{2}\int_{\gamma}^{\pi} \rho_R^2(\varphi)d\varphi = \frac{1}{2}\int_{\gamma}^{\pi} b^2\ (2\varphi-\gamma) + \frac{1}{2}\int_{\gamma}^{\pi}\sqrt{b^4\cdot\cos^2(2\varphi-\gamma)-b^4+a^4}\cdot d\varphi \tag{9}$$

Integrals are defined separately for two parts of equation (9). For the first part of the equation (9):

$$\frac{1}{2}\int_{\gamma}^{\pi} b^2\cos(2\varphi-\gamma)\,d\varphi = \frac{b^2}{2}\cdot\frac{\sin(2\varphi-\gamma)}{2}\int_{\gamma}^{\pi} = \frac{b^2}{4}[\sin(2\pi-\gamma)-\sin\gamma]. \tag{10}$$

For the second part of equation (9) is obtained:

$$\frac{1}{2}\int_{\gamma}^{\pi}\sqrt{b^4\cdot\cos^2(2\varphi-\gamma)-b^4+a^4}\cdot d\varphi = \frac{1}{2}\int_{\gamma}^{\pi} a^2\sqrt{1-\frac{b^4\sin^2(2\varphi-\gamma)}{a^4}}\cdot d\varphi. \tag{11}$$

Let's make a substitution in the equation (11) $U = 2\varphi-\gamma \rightarrow \frac{dU}{d\varphi} = 2,\ \ d\varphi = \frac{1}{2}dU.$

Given the change, the dependence (11) will look like:

$$\frac{1}{2}\int_{\gamma}^{\pi} a^2\sqrt{1-\frac{b^4\sin^2(2\varphi-\gamma)}{a^4}}\cdot d\varphi \rightarrow \frac{a^2}{4}\int_{\gamma}^{\pi}\sqrt{1-\frac{b^4\sin^2(U)}{a^4}}\cdot dU \tag{12}$$

For the integral of

$$\int\sqrt{1-\frac{b^4\sin^2(U)}{a^4}}\cdot dU \tag{13}$$

there is a special function (normal elliptic integral of the 2nd kind) $E\left(U\left|\frac{b^4}{a^4}\right.\right)$.

Then, after integration, the integral of the dependence (12) will have the form:

$$\frac{a^2}{4}\int_{\gamma}^{\pi}\sqrt{1-\frac{b^4\sin^2(U)}{a^4}}\cdot dU = \frac{a^2\cdot E\left(U\left|\frac{b^4}{a^4}\right.\right)}{4} \tag{14}$$

The reverse will be replaced: $U = 2\varphi-\gamma$.

Then:

$$\frac{a^2\cdot E\left(U\middle|\frac{b^4}{a^4}\right)}{4} \to \frac{a^2\cdot E\left(2\varphi-\gamma\middle|\frac{b^4}{a^4}\right)}{4} \tag{15}$$

After integrating of the equation (11) we obtain the equation (16):

$$\frac{1}{2}\int_{\gamma}^{\pi}\sqrt{b^4\sin^2(2\varphi-\gamma)-b^4+a^4}\cdot d\varphi = \frac{a^2\cdot E\left(2\varphi-\gamma\middle|\frac{b^4}{a^4}\right)}{4}\Bigg|_{\gamma}^{\pi} = \frac{a^2\cdot\left[E\left(2\pi-\gamma\middle|\frac{b^4}{a^4}\right)-E\left(\gamma\middle|\frac{b^4}{a^4}\right)\right]}{4}. \tag{16}$$

Taking into consideration the dependences of (8), (10) and (16) the useful cross-sectional area for one rotor of the Roots pump is described by equation:

$$S_{\frac{1}{2}H} = \frac{R_1^2}{2}(\pi-\gamma) - \frac{1}{4}\cdot\left(b^2[\sin(2\pi-\gamma)-\sin\gamma] + a^2\left[E\left(2\pi-\gamma\middle|\frac{b^4}{a^4}\right) - E\left(\gamma\middle|\frac{b^4}{a^4}\right)\right]\right) \tag{17}$$

where $\mathrm{E}$ – normal elliptic integral of the 2nd kind,

$$E(\gamma, K) = \int_0^{\gamma}\sqrt{1-K^2\sin^2\theta}\cdot d\theta,$$

where $K^2 = \frac{b^4}{d^4}$; $\gamma = 0; 15; 30 \ldots 180^{\circ}$ for half of the rotor.

Accordingly, the normal elliptic integral of the 2nd kind will be:

$$E\left(2\pi-\gamma\middle|\frac{b^4}{a^4}\right) = \int_0^{2\pi-\gamma}\sqrt{1-\frac{b^4}{a^4}\cdot\sin^2(\theta)}\cdot d\theta; \tag{18}$$

$$E\left(\gamma\middle|\frac{b^4}{a^4}\right) = \int_0^{\gamma}\sqrt{1-\frac{b^4}{a^4}\cdot\sin^2(\theta)}\cdot d\theta. \tag{19}$$

The parameters are set and the rotor surface shape and cross-sectional area of the useful volume of the Roots pump are simulated.

## RESULTS

The geometry of the rotor surface is modeled in the following sequence. The *R* radius of rotation of the rotor is taken. The distance *b* based on the condition of *R* > *b* is set. The *a* parameter is determined by the formula of $a = \sqrt{R\cdot r}$, where *r* = *R* – *b* – rounding radius of the rotor (Fig. 1). Parameters *a* and *b* must meet the condition: $\sqrt{2}/2 < b/a < 1$.

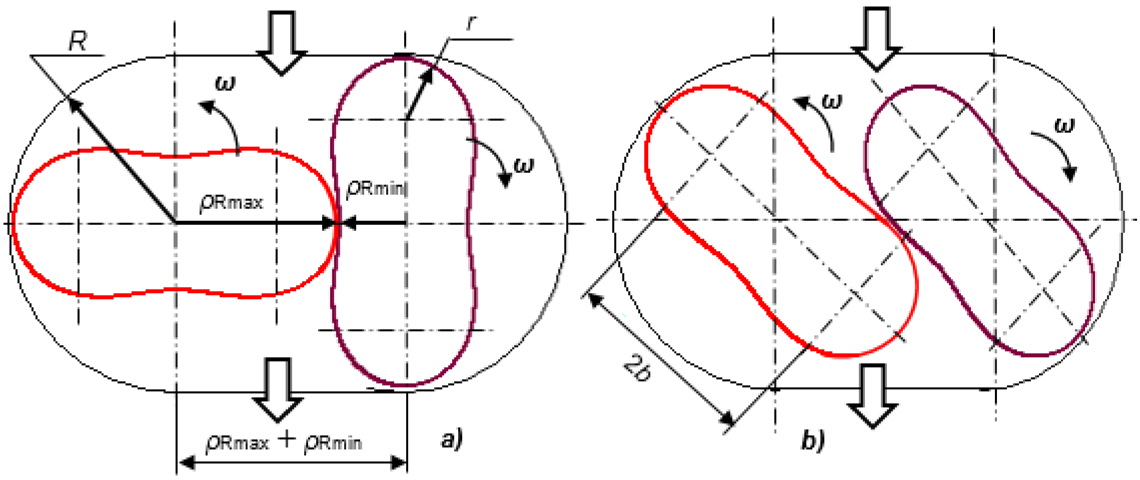



**Fig. 3 – The results of simulation of the geometry of the Roots pump rotors**

*a – rotors in antiphase; b – rotation of rotors on 135 degrees; parameters for simulation the geometry of the rotors:*
*R* = 50 [mm]; *r* = 20 [mm]; *a* = 33.166; *b* = 28

To model the effective cross-sectional area of the pump, the normal elliptic integrals of the 2nd kind of (18) and (19) are determined by the numerical method. The results of the numerical solution of normal elliptic integrals of the 2nd kind are given in Table 1.

**Table 1**

**Results of numerical integration of normal elliptic integrals of the 2nd kind**

| Angle of rotor rotation γ, radian | $\mathbf{E}\left(2\cdot\pi-\gamma\left\|\frac{b^4}{a^4}\right.\right)$, from the equation (18) | $\mathbf{E}\left(\gamma\left\|\frac{b^4}{a^4}\right.\right)$, from the equation (19) |
|---|---|---|
| 0 | 5.383236147 | 0.00000000 |
| 15 | 5.123096019 | 0.26016592 |
| 30 | 4.871864747 | 0.51161234 |
| 45 | 4.636451935 | 0.74721944 |
| 60 | 4.420778189 | 0.96313815 |
| 75 | 4.223737983 | 1.16037375 |
| 90 | 4.038140391 | 1.3460468 |
| 105 | 3.852407175 | 1.5316819 |
| 120 | 3.655095251 | 1.72884326 |
| 135 | 3.439054015 | 1.94461587 |
| 150 | 3.203303729 | 2.1801105 |
| 165 | 2.951811138 | 2.43147962 |
| 180 | 2.691618072 | 2.69161807 |

To simulate the change in the effective cross-sectional area of a two-rotor pump, it is sufficient to simulate the rotation of the rotors from 0 to 180 degrees.

The effective cross-sectional area of the Roots twin-rotor pump is variable. It varies from 483 mm$^2$ to 1449 mm$^2$ depending on the angle of rotation of the rotors. The results of modeling the cross-sectional area are shown in Fig. 4.

The rotation of the rotors by 180 degrees was simulated. Rotating the rotor by 180 degrees covers the entire effective cross section of the pump for one rotor. This cross section characterizes the pumping area of the medium.

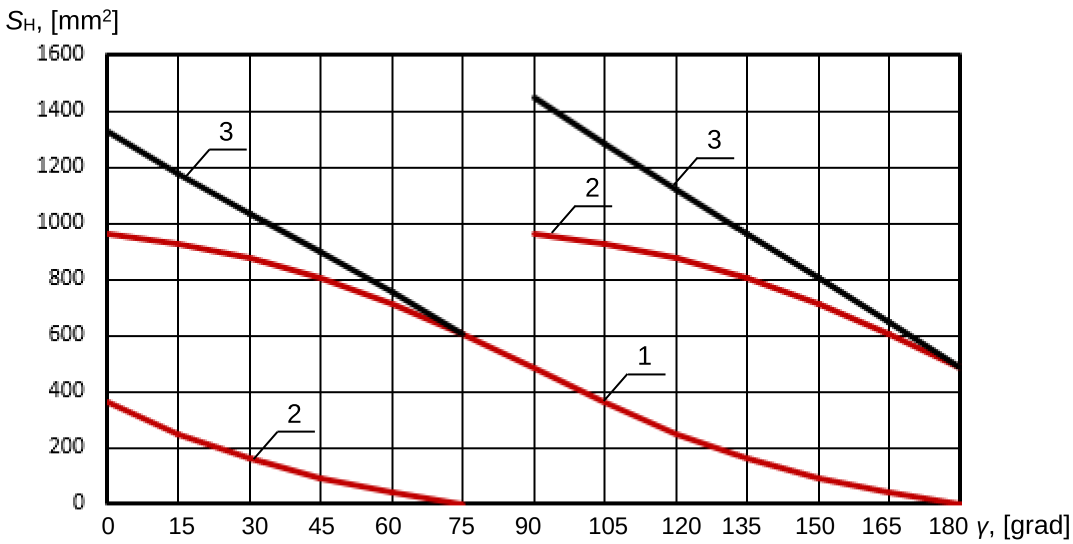



**Fig. 4 - Dependence of the effective cross-sectional area of the Roots two-rotor pump on the angle of the rotors rotation**

1 – *the rotor of vertical initial position;* 2 – *the rotor of horizontal initial position;* 3 – *the total cross-sectional area of the two-rotor pump*

## CONCLUSIONS

Mathematical model was developed as the equations (5) and (6). With the help of these equations the surface of the body of rotation which is different from the ideal circle is modeled. The polar coordinate system is used, and the radius vector, the direction of which is set by the $\varphi$ angle, characterizes the location of the point on the surface of the rotor.

The distance of this point from the axis of rotor rotation is set by the calculated value of the $\rho_R$ polar radius vector. The $\gamma$ angle of rotors rotation characterizes their mutual orientation in the plane of rotation (Fig. 3).

The useful cross-sectional area of the pump is modelled by the geometry of the rotors. The area of the rotor was determined by the geometry of the surface, which was described by an elliptic integral of the 2nd kind, equation (17). The geometry of the rotor depends on the normalized parameters *a* and *b*, which are constant for a given geometry of the rotor and constructive dimensions.

Parameters *a* and *b* must meet the condition of $\sqrt{2}/2 < b/a < 1$. Under such conditions, the geometry of the rotor surface will be a Cassini oval. The rotation of the two rotors against each other will be by rolling one surface over another.

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