EVALUATION OF THE PLACES FOR CREATION OF APIARIES AND OPTIMAL DISTRIBUTION OF THE BEE COLONIES
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ОЦЕНКА НА МЕСТАТА ЗА СЪЗДАВАНЕ НА ПЧЕЛИНИ И ОПТИМАЛНО РАЗПРЕДЕЛЕНИЕ НА ПЧЕЛНИТЕ КОЛОНИИ

Atanasov Atanas *[1)], Georgiev Ivan [2)]
[1)] University of Ruse, Agro-industrial Faculty / Bulgaria;
[2)] University of Ruse, Faculty of Natural Sciences and Education, Department of Applied Mathematics and Statistics / Bulgaria
Tel: +359 885 497 406; E-mail: aatanasov@uni-ruse.bg
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ABSTRACT
An approach for evaluation of the places for creation of apiaries and optimal distribution of bee colonies formed on the basis of the feeding capacities of the areas with flowering plants, the distances between these sites and the feeding areas is proposed. A multicriteria model with two main criteria is considered. The first maximizes the sum of the products of the weights for a given place multiplied by the number of colonies that will be positioned at that place. This criterion is divided into two sub-criteria, including the "subjective" and "objective" assessment of place preferences, respectively. The second criterion aims to minimize malnourished bee colonies. The model, with the proposed approach for "objective" assessment of potential distribution sites, can be applied both for cases without overpopulation of the area with bee colonies and for areas with overpopulation.

РЕЗЮМЕ
Предложен е подход за оценка на местата за създаване на пчелини и оптимално разпределение на формираните пчелни колонии на базата на хранителните капацитети на площите с цъфтящи растения, разстоянията между тези площи и местата за хранене. Разглежда се многокритериален модел с два основни критерия. Първият максимизира сумата от произведенията на теглата за дадено място, умножена по броя на колониите, които ще бъдат позиционирани на това място. Този критерий е разделен на два подкритерия, включващи съответно „субективна“ и „обективна“ оценка на предпочитанията за място. Вторият критерий цели минимизиране на недохранените пчелни семейства. Моделът, с предложения подход за „обективна” оценка на потенциалните места за разпространение, може да се приложи както за случаи без пренаселеност с пчелни колонии, така и за райони с пренаселеност.

INTRODUCTION
Providing sufficient quantity and quality of food resources to bee (*Apis mellifera macedonica*) colonies is essential for their optimal biological development. The selection of areas with appropriate bee pasture is a difficult task in terms of the variable location of areas with agricultural crops, the variable number of bee colonies in the area over the years, the regulations governing beekeeping, etc. All these factors can affect the sustainability of bee products in different years. It is very important for bee colonies to find the best location for setting up an apiary in a certain area in such a way that each colony has sufficient food resources and at the same time minimizes the proportion of colonies that will not be fed due to overpopulation. The food resources are very important for the honey bee population and its dynamics has been studied in a number of papers, see e.g.: *Atanasov & Georgiev, (2021), Georgiev & Vulkov, (2021),* including unhealthy dynamics (*Atanasov et al., 2021*).

The social behaviour of bees in a colony allows them to be considered as a very well-structured biological unit subordinated to a common goal. The study of social animals and social insects has led to the creation of mathematical models seeking an optimal solution to a problem (*Todorov & Dimov, 2020; Dimov et al., 2015*) satisfying one or more objective functions, under conditions of a set of constraints.

In order to avoid competition between bee colonies, many publications discuss optimal models for the distribution of bee colonies in predetermined locations (*Ramon et al., 2010; Rebysarah et al., 2011; Yuce et al., 2013; Maica et al., 2014*). These places have connections with fields of flowering plants to feed bee colonies. Different fields have different nectar secretion potentials (*Al-Ghamdi et al., 2016*). Some studies solve an optimization task aimed to optimize the beekeeper's preferences for individual places including accessibility and ease of maintenance, features related to the terrain, transport distances to settlements, security and surveillance, quality infrastructure and more (*Saritha & Vinod Chandra, 2017*). The optimization model in the article allows the possibility of overpopulation in the area, which leads to malnutrition of some bee colonies. In the presence of such a part, the optimization model minimizes it.

A similar problem has been solved by *Aderinto et al., (2020),* where the distribution of bees in the apiary is optimized to improve honey production, which in turn maximizes profits and minimizes the cost of maintaining hives in terms of nutrition, labour and storage. Other studies have considered a similar problem for optimal distribution of bee colonies, optimizing several criteria simultaneously *(Atanasov & Georgiev, 2021*). When simultaneously optimizing several criteria, the Pareto optimal set of solutions is sought, after which one is chosen (*Triantaphyllou, 2013*). In the optimization models considered so far (*Maica et al., 2014; Atanasov & Georgiev, 2021*), the constraints are:

Constraint 1) $\sum_{\substack{j=1 \\ j\in C_i}}^{p} x_{ij} - X_i = 0 \; \forall i = \acute{1,m}$

Constraint 2) $\sum_{\substack{i=1 \\ i\in F_j}}^{m} x_{ij} - b_j \leq 0 \; \forall j = \acute{1,p}$ (1)

Constraint 3) $\sum_{k=1}^{n} z_{ki} - E_i - X_i = 0 \forall i = \acute{1,m}$

Constraint 4) $\sum_{i=1}^{m} z_{ki} \leq d_k \forall k = \acute{1,n}$

where:

m is the number of relocation sites,

p - the number of plants or plants clusters,

n - number of colonies,

X_i - number of colonies to be relocated at site $i, i = \acute{1,m}; X_i \in Z^+$,

x_{ij} - fraction of colonies of X_i (relocation of in site i, which feed on a plant cluster j, that can be accommodated by plant cluster $j, i = \acute{1,m}, \; j = \acute{1,p}; \; x_{ij} \in Z^+$

b_j - carrying capacity of plant cluster $j; b_j \in R^+$

w_i - priority weight given to site $i; w_i \in Z^+$

z_{ki} - number of hives from the k-th apiary, that can be relocated at site $i, \; k = \acute{1,n}, \; i = \acute{1,m}; \; z_{ij} \in Z^+$

d_k - number of hives from the k^{-th}apiary; $d_k \in Z^+$

E_i - number of hives distributed in the i^{-th} site that will not be fed; $E_i \in R^+$

$j \in C_i$ – plant cluster j is connected to site i

$i \in F_j$ if site i is connected to plant cluster j

Constraint 1 represents the distribution of the number of colonies relocated at a site connected to the connected plant clusters. *Constraint 2* represents the contribution of a site (in terms of number of colonies) to the carrying capacity of the connected plant cluster. *Constraint 3* shows how the number of colonies will be relocated to the sites with the assurance that the colony will not be subdivide into parts. *Constraint 4* allows, part or all of the hives to be relocated in different sites.

The objective functions are:

$$\mathrm{Max} f_1(X) = \sum_{i=1}^{m} w_i X_i \wedge \mathrm{Min} f_2(E) = \sum_{i=1}^{m} E_i \quad (2)$$

Important aspects in formulating the mathematical model are the distances of a potential location of apiaries to the fields with flowering vegetation (food source), the nectar secretion capacities of these fields, as well as the competition with the other locations of apiaries.

In the considered models the assessment, which puts the beekeeper in the separate places for apiaries location, is to a large extent subjective and insufficiently substantiated.

Our study aims to complement the proposed models by proposing a solution to more accurate assessment in the individual places.

MATERIALS AND METHODS

A model for estimating the potential site has been designed

A model has been designed to assess the potential location M_i, $i=1...m$, where hives can be placed, comparable to a coefficient characterizing the extent to which this place is desired. When constructing this coefficient, the following will be taken into account:

1) the distance c_{ij} from place M_i to the source of feeding S_j, $j=1...m;$

2) carrying capacity of plant cluster b_j, $j=1...m$ (measured in the number of bee colonies that can be fed);

3) competition with other places.

One of the important factors that determine the choice of bees to carry nectar and pollen from the source of feeding to the bee colony is the distance and the energy spent by the bees in flight. Since the energy consumed by the bee per flight is linearly increasing and in practice after 2500m, the bee consumes energy equivalent to the food supply it has collected, this linearity will be taken into account when determining the efficiency factor. Let us construct a linear function f, matching a number $c_{ij} \in [0,1]$ at any distance between a place M_i and given source of feeding S_j. If a potential place is located on the area of a source of nectar (rather the distance between them is negligibly small), then on, c_{ij} the number 1 is compared. If the distance between the potential site and the source of nectar is 2500 m or more, the number 0 is compared. Let this number c_{ij}, let's call it "useful distance", it takes the value 0 at no utility of the real distance (above 2500m) and 1 at maximum utility (when the location of the apiary and the location of the food source coincide). The function f has the following form:

$$f(x) = \begin{cases} \frac{-x}{2500} + 1, x \in [0; 2500] \\ 0, x \notin [0; 2500] \end{cases} \quad (3)$$

We will determine the "value" of given field source of feeding, S_j, concerning a place M_i, taking into account the "useful distances", the capacity b_j and the competition between the other places. Because given source of feeding can be accessed by places other than M_i, the "value" of the place source of feeding S_j, about place M_i can be defined as part of the capacity b_j, proportional to the square of the "useful distance" and inversely proportional to the sum of all "useful distances" to the location of the source of feeding S_j. The latter is motivated by the fact that if a feeding place has a connection with only one apiary place, then the "value" of this feeding place for the given apiary place is expressed by the part of the food stocks at that feeding place, which does not have to be spent on flight- $b_j c_{ij}$. This part is weighted on average in terms of "useful distances" with other places. The "value" of a place of feeding S_j, for given place for apiary M_i, this is the part of the capacity b_j, which remains after taking into account the competition (potential sharing) with other places and the energy spent by the bees to fly to that place. In order to obtain a final coefficient ρ_i, preferably in a given place, the sum of the "values" of all feeding places in relation to the given apiary place is taken:

$$\rho_i = \sum_{j=1}^{n} b_j c_{ij} \frac{c_{ij}}{\sum_{k=1}^{m} c_{ij}} = \sum_{j=1}^{n} \frac{b_j c_{ij}{}^2}{\sum_{k=1}^{m} c_{ij}} \quad (4)$$

In determining the coefficient ρ_i, the preferences of the beekeeper were not taken into account when assessing the potential locations for bee colonies. For the purpose of the numerical experiment, when choosing a place in accordance with the wishes of the beekeeper, priority weights w_i^1, are given, taking into account the proximity of the apiary to the main road, the proximity to the settlement, the possibility of guarding and monitoring the apiary. With these requirements in mind, the following "subjective" priority weights have been identified:

$w_1^1 = 4$; $w_2^1 = 3$; $w_3^1 = 5$; $w_4^1 = 6$; $w_5^1 = 1$; $w_6^1 = 7$; $w_7^1 = 7$;

We propose a practical technological solution for evaluation of the places for creation of 7 apiaries with different numbers of bee hives as follows: $d_1 = 100$; $d_2 = 30$; $d_3 = 22$; $d_4 = 96$; $d_5 = 34$; $d_6 = 10$; $d_7 = 25$, with 317 total number, spaced at a certain distance from fields with flowering vegetation's with caring capacity of plant cluster b_1 = 64; b_2 = 14; b_3 = 66; b_4 = 10; b_5 = 7; b_6 = 20 and b_7 = 218 shown in (Table 1.).

Using formula (3) and (4), the "objective" priority weights are also calculated:

w_1^2 =0.4347; w_2^2 =0.3745; w_3^2 =0.1185; w_4^2 =0.0101; w_5^2 =0.0466; w_6^2 =0.0085; w_7^2 =0.0071.

The total priority weight w_i is designed as a convex linear combination of the "subjective" w_i^1 and "objective" w_i^2 priority weights:

$$w_i = \mu w_i^1 + (1-\mu) w_i^2 \tag{5}$$

where $\mu \in [0,1]$. At $\mu = 0$, full priority is given to the "objective" priority weights, at $\mu = 1$, full priority is given to the "subjective" priority weights. When setting intermediate values to μ: 0< $\mu < 1$, the "subjective" and "objective" priorities with different weight are taken into account. As a result, different objective functions $f_1(X)$, are obtained, which would lead to different solutions of problem (1)-(2). Criteria $f_1(X)$ a set of two sub-criteria in which the subjective assessment of the beekeeper for on-site preferences and the objective assessment for on-site preferences are calculated by formulas (3)-(4). These two criteria are combined into one - $f_1(X)$, by selecting the parameter μ.

The study was conducted in 2019, based on the assessment existing bee forage resources as Acacia (*Robinia pseudoacacia*), Sunflower (*Helianthus annuus*) and the number of bee colonies kept in the Northeast part of Bulgaria in village Batishnica. Geographical location of experimental apiaries in Batishnica are $m_1, m_2, m_3, m_4, m_5, m_6, m_7$ – 43°32'21.53"N, 25°50'33.58"E; 43°33'17.02"N, 25°49'27.51"E; 43°33'22.91"N, 25°48'57.99"E; 43°32'13.69"N, 25°52'26.25"E; 43°31'46.43"N, 25°52'17.40"E; 43°31'35.65"N, 25°51'59.15"E; 43°31'34.93"N, 25°52'10.88"E and at an altitudinal range of 288 m. In the region major honey source plants are *Robinia pseudoacacia* with total land area 6.3 ha and *Helianthus annuus* with total land area 317.7 ha.

These two major crops determined the main honeybee pasture in the study area. Other vegetation types are vineyards (*Vitis*), oak (*Quercus*), maize (*Zea mays*), wheat (*Triticum*), barley (*Hordeum vulgare*), soybeans (*Glycine max*). If all the factors remain constant, the productivity of the bee colonies is in correlation with nectar secretion potential of bee forage species and the existing honeybee colony density. It is very important for the proper development of bee colonies to find the best location of a bee hive in the certain study area.

The distance between an apiaries site and a plant cluster is consistent with the maximum flight distance D, of the bee species *Apis mellifera macedonica*, which has a productive distance of up to 2500 m. This distribution was made to investigate the flight of bees to the extent that they were productive in collecting nectar and pollen and transporting it to the colony.

Table 1

The distance between an apiaries site and a plant cluster
(d_k –number of hives from the k^{-th}apiary, b_j – carrying capacity of plant cluster j, kg/h; D - flight distance, m)

	b_1	b_2	b_3	b_4	b_5	b_6	b_7
d_1	1737	2103	876	1533	1408	1214	0
d_2	1490	654	1880	D >2500	D >2500	D >2500	0
d_3	1600	1285 m	D >2500	D >2500	D >2500	D >2500	1214
d_4	D >2500	D >2500	D >2500	1417	2280	1370	2500
d_5	D >2500	D >2500	1982	1080	1602	730	1985
d_6	D >2500	D >2500	D >2500	1882	1840	1385	D >2500
d_7	D >2500	D >2500	D >2500	1998	1884	1468	D >2500

In the particular case, specific numbers of hives were placed on the places and the solution of the problem redistributed to the same places. The matrix of connections is represented by the following graph (Fig.1.).

The rectangles show the number of relocation sites m, the circles show the number of plants clusters p with Acacia and Sunflower. The apiaries m_1 and m_2 are located in a circle of plant cluster p_7, which does not take into account the influence of the distances between the locations of the apiaries m_1 and m_2 and plant cluster p_7.

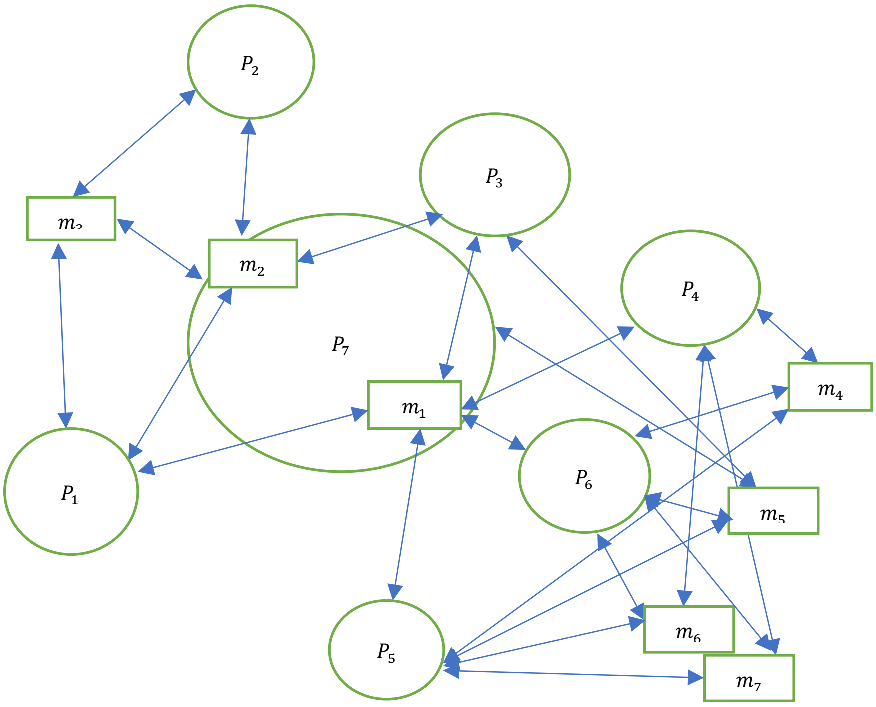

Fig. 1 - Matrix of connections

m - number of relocation sites; P - number of plants or plants clusters

A graph is composed of nodes and edges. Two kinds of nodes will be considered – the possible relocation of sites of beehives and clusters of plants. The distance between a relocation site and a plant cluster is consistent with the maximum flight distance of the bee species *Apis mellifera macedonica*.

This distribution was made to investigate the flight of bees to the extent that they were productive in collecting nectar and pollen and transporting it to the colony. The maximum flight distance for bees to search for food is taken into account in the assessment of each apiary location. This reduces the stress in the colony of the lack of supply of nectar and pollen into the hive. Food sources are various flowering plant species around the apiaries. The productivity of the various forage species depends on the nectar secretion potential of each plant.

RESULTS

The ability of flowering plant fields to feed bee colonies is defined by the name "carrying capacity of a plant cluster". The carrying capacity of a plant cluster depends on the nectar secretion potential of each plant and the area occupied by flowering plants. The preference of the beekeeper and feeding of bee colonies over the location sites can also be considered by giving each site a priority weight. In constructing the proposed mathematical model, we estimated the places for creation of apiaries based on main pastures of *Robinia pseudoacacia* and *Helianthus annuus.* The role in feeding bees from other flowering plants in the study area is too small and was not taken into account in the site assessment in our model.

The problem is solved via Matlab software operations research capabilities. The aim in our experiment is to find the Pareto optimal solutions (*Blunter et. al., 2004*) of problem (1) - (2).

For this purpose, a generalized criterion is constructed:

$$F(X) = \lambda f_1(X) + (1-\lambda) f_2(X), \tag{6}$$

where $\lambda \in [0,1]$.

From the fact that the constraints (1) are linear, it follows that the permissible range is convex. Criteria (2) are also linear and to find the Pareto optimal solutions can be obtained by setting a set of different values of the parameters $\lambda \wedge \mu$. Different numerical experiments were performed (at different values of $\lambda \wedge \mu$). Different solutions have been obtained. The beekeeper can choose a solution depending on the specific situation and the characteristics of the beekeeping area. Some of the results are shown in Table 2, 3, 4, 5.

Table 2

Number of hives from the k-th apiary that can be relocated at site i, at $\lambda = 0.5, \mu = 0.5$
(d_k – number of hives from the k^{-th} apiary, m_k – number of relocation sites)

z_{ki}	m_1	m_2	m_3	m_4	m_5	m_6	m_7
d_1	100	0	0	0	0	0	0
d_2	30	0	0	0	0	0	0
d_3	22	0	0	0	0	0	0
d_4	96	0	0	0	0	0	0
d_5	34	0	0	0	0	0	0
d_6	10	0	0	0	0	0	0
d_7	25	0	0	0	0	0	0

Table 3

Number of hives from the k-th apiary that can be relocated at site i, at $\lambda = 0.5, \mu = 0.1$
(d_k – number of hives from the k^{-th} apiary, m_k – number of relocation sites)

z_{ki}	m_1	m_2	m_3	m_4	m_5	m_6	m_7
d_1	100	0	0	0	0	0	0
d_2	30	0	0	0	0	0	0
d_3	19	0	0	0	0	3	0
d_4	96	0	0	0	0	0	0
d_5	0	0	0	0	0	34	0
d_6	10	0	0	0	0	0	0
d_7	25	0	0	0	0	0	0

Table 4

Number of hives from the k-th apiary that can be relocated at site i at $\lambda = 0.1, \mu = 0$
(d_k – number of hives from the k^{-th} apiary, m_k – number of relocation sites)

z_{ki}	m_1	m_2	m_3	m_4	m_5	m_6	m_7
d_1	0	0	93	0	0	7	0
d_2	0	0	0	0	0	0	30
d_3	0	0	22	0	0	0	0
d_4	0	0	96	0	0	0	0
d_5	0	0	34	0	0	0	0
d_6	0	0	10	0	0	0	0
d_7	0	0	25	0	0	0	0

Table 5

Number of hives from the k-th apiary that can be relocated at site i at $\lambda = 1, \mu = 0.9$
(d_k – number of hives from the k^{-th} apiary, m_k – number of relocation sites)

z_{ki}	m_1	m_2	m_3	m_4	m_5	m_6	m_7
d_1	0	100	0	0	0	0	0
d_2	0	3	0	20	0	0	7
d_3	0	0	22	0	0	0	0
d_4	0	84	12	0	0	0	0
d_5	0	0	34	0	0	0	0
d_6	0	0	10	0	0	0	0
d_7	0	0	0	0	25	0	0

In all the obtained solutions there is a complete feeding of the bee colonies, which is a prerequisite for obtaining honey. Regardless of which solution the beekeeper chooses, each of them is applicable.

The model (1)-(2) is also applicable when the number of colonies exceeds the capacity of the food stocks of the areas with flowering plants. Then there will certainly be malnourished bee colonies. If the capacity of plant cluster decreases: $b_1 = 32; b_2 = 7; b_3 = 33; b_4 = 5; b_5 = 4; b_6 = 10; b_7 = 109$, and the remaining parameters are retained, the following results are obtained and shown in the Table 6, 7, 8, 9.

Table 6

Number of hives from the k-th apiary that can be relocated at site i at $\lambda = 0.5, \mu = 0.5$
(d_k –number of hives from the k^{-th}apiary, $m_k -$ number of relocation sites)

z_{ki}	m_1	m_2	m_3	m_4	m_5	m_6	m_7
d_1	100	0	0	0	0	0	0
d_2	30	0	0	0	0	0	0
d_3	0	0	0	0	0	0	22
d_4	35	61	0	0	0	0	0
d_5	0	0	0	0	0	0	34
d_6	10	0	0	0	0	0	0
d_7	25	0	0	0	0	0	0

Table 7

Number of hives from the k-th apiary that can be relocated at site i at $\lambda = 0.5, \mu = 0.1$
(d_k –number of hives from the k^{-th}apiary, $m_k -$ number of relocation sites)

z_{ki}	m_1	m_2	m_3	m_4	m_5	m_6	m_7
d_1	100	0	0	0	0	0	0
d_2	30	0	0	0	0	0	0
d_3	0	0	0	0	0	0	22
d_4	16	80	0	0	0	0	0
d_5	0	0	0	0	0	19	15
d_6	10	0	0	0	0	0	0
d_7	25	0	0	0	0	0	0

Table 8

Number of hives from the k-th apiary that can be relocated at site i at $\lambda = 0.1, \mu = 0$
(d_k –number of hives from the k^{-th}apiary, $m_k -$ number of relocation sites)

z_{ki}	m_1	m_2	m_3	m_4	m_5	m_6	m_7
d_1	0	100	0	0	0	0	0
d_2	0	0	0	0	0	0	30
d_3	0	0	22	0	0	0	0
d_4	0	0	96	0	0	0	0
d_5	0	0	28	0	0	6	0
d_6	8	0	2	0	0	0	0
d_7	25	0	0	0	0	0	0

Table 9

Number of hives from the k-th apiary that can be relocated at site i at $\lambda = 1, \mu = 0.9$
(d_k –number of hives from the k^{-th}apiary, $m_k -$ number of relocation sites)

z_{ki}	m_1	m_2	m_3	m_4	m_5	m_6	m_7
d_1	0	100	0	0	0	0	0
d_2	0	0	0	0	0	0	30
d_3	0	0	22	0	0	0	0
d_4	0	79	17	0	0	0	0
d_5	0	0	0	0	0	0	34
d_6	0	10	0	0	0	0	0
d_7	0	0	0	0	25	0	0

In all considered decisions the number of the malnourished colonies is 117, regardless of the distribution in the individual places. The distribution of colonies that can be fed is optimal according to the relevant criteria with selected parameters $\lambda \wedge \mu$, and for those that cannot be fed it is necessary to find another solution. Possible solutions are: relocation of the malnourished colonies to places other than the designed ones, increasing the area of the flowering plants, the type of the sown crops, improving the nectar-releasing potential of the crops, by applying good agrotechnical practices, etc.

CONCLUSIONS

An approach for “objective” assessment of the potential distribution sites of bee colonies formed on the basis of the feeding capacities of the areas with flowering plants, the distances between these sites and the feeding areas is proposed.

A multicriteria model with two main criteria is considered. The first maximizes the sum of the products of the weights for a given place multiplied by the number of colonies that will be positioned at that place. This criterion is divided into two sub-criteria, including the “subjective” and “objective” assessment of place preferences, respectively. The second criterion aims to minimize malnourished bee colonies.

The given model, with the proposed approach for "objective" assessment of potential distribution sites, can be applied both for cases without overpopulation of the area with bee colonies and for areas with overpopulation.

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