METHOD OF THEORY OF DIMENSIONS IN EXPERIMENTAL RESEARCH OF SYSTEMS AND PROCESSES
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МЕТОД ТЕОРІЇ РОЗМІРНОСТЕЙ В ЕКСПЕРИМЕНТАЛЬНИХ ДОСЛІДЖЕННЯХ СИСТЕМ І ПРОЦЕСІВ

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DOI: https://doi.org/10.356.33/inmateh-65-24

Keywords: *technological process, dimension theory, experiment, Galileo criterion, Euler number, coefficient of friction, mathematical model.*

ABSTRACT
The method and parameters of experimental modelling of systems and processes in mechanical engineering are substantiated. The theory of similarity and dimensionality is used as an intermediate link between theory and experiment. The dimension of the factor space depends on the number of factors. The set of factors is grouped into dimensionless similarity criteria. The selected criteria are in certain dependence, such as the Galileo test, Euler and Reynolds numbers. Examples of application in experimental studies are given. The use of dimension theory in a factor-planned experiment allows reducing the number of factors, simplifies the mathematical interpretation of the response criterion and provides a graphical representation in the form of 3-D model.

РЕЗЮМЕ
Обґрунтовано методику та параметри експериментального моделювання систем і процесів в галузевому машинобудуванні. Використано теорію подібності і розмірності, як проміжну ланку між теорією і експериментом. Розмірність факторного простору залежить від числа факторів. Сукупність факторів згрупована в безрозмірні критерії подібності. Вибрані критерії знаходяться у визначеній залежності, наприклад критерій Галілея, числа Ейлера і Рейнольдса. Приведено приклади застосування в експериментальних дослідженнях. Використання теорії розмірності при факторному плановану експерименті дозволяє скоротити кількість факторів, спрощує математичну інтерпретацію характеру критерію відгуку і забезпечує графічне представлення у вигляді 3-D моделі.

INTRODUCTION
Nowadays, many technological processes and technical systems are evaluated by a large number of parameters that are mutually consistent and affect the indices of control or measurement. These indicators characterize the optimality of the system or process (*Dmytriv V.T. et al, 2020*). Modelling in the traditional form of a statistical model is one of the more scalable methods, which takes into account a large number of parameters, a significant amount of sample data and a significant range of variation. There are experimental research methods that reduce both the number of parameters and the amount of sample data. Such methods are as follows: analysis of basic parameters and its variants (*Kettaneha N. et al, 2005; Elgamal T. and Hefeeda M., 2015*), clustering methods (*Bouveyron C. and Brunet-Soumard C., 2014*), variational selection by checking the independence of factors (*Fan J. and Lv J., 2008; Fan J et al, 2011*) and the smallest angular regression (*Efron B. et al, 2004*).

For large data types, other methods have been developed, such as sequential updates for streaming data (*Schifano E.D. et al, 2016*) or matrix adjustments (*Liberty E., 2013*). These methods are aimed at simplifying data processing, reducing the characteristics of processes to linear functional dependences, but do not simplify the number of factors for multifactorial processes, where each parameter affects the optimization criterion.

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For a large number of variables in the experiment it is more appropriate to use the totality of the methods of the dimensional analysis and modern statistical modelling of experiments (*Islam M.F. and Lye L.M., 2009; Woods D. C. et al, 2017*). The dimensional method is widely used to analyse the various processes, for this the Buckingham Pi theorem is used (*Hu P. and Chang C.-kan, 2020*). On the example of a significant number of parameters, which are grouped by functional characteristics, the method of dimensional functions shows the possibility of implementing the model on the measured data in real time (*Wang Y. et al, 2019*).

Experimental and analytical modelling of processes at the level of physical phenomena and patterns, including technical, technological and at the level of operational control of processes, requires a significant number of factors to be taken into account. These factors characterize the relationship between the parameters of the technological process and the physical and mechanical characteristics of the components involved in the technological process. To generalize and determine the characteristics of the technological process, it is advisable to have structured dependencies that describe in the first approximation the physics of the process and will give possibility to determine the critical limits of factors.

The purpose of this study is to develop methods for the theory of similarity and dimensionality, criterion values, as an intermediate component between theory and experiment. This will ensure a functional relation between whole sets of quantities that characterize the process at the level of the physical model and simplify the planned experiment for processes and systems characterized by a significant number of factors.

MATERIALS AND METHODS

Transformation of factor space by methods of dimension theory

The analysis of theoretical and experimental researches allows to conclude that in the theory of experiment planning the choice of process parameters is the most important. The accepted parameters should reflect all the main factors of the technological process, and their number should be minimal for the planned experiment. Reducing the number of factors reduces the number of experiments and increases the reliability of the response criterion.

The dimension of factor space depends on a number of factors, so to simplify the problem the method of dimension theory, namely the π-theorem, is used. The essence of this theorem is that the whole set of factors can be grouped into dimensionless similarity criteria of $\pi_1, \pi_2, \ldots, \pi_i$ (*Sonin A.A., 2001*; *Shen W., and Lin D. K. J., 2019*; *Albrecht M. C. et al, 2013*; *Jónsson D., 2014*).

The application of methods of the theory of similarity and dimensionality, criterion quantities, as an intermediate component between theory and experiment, provides a functional relationship between whole sets of quantities that characterize the process at the level of the physical model.

The application of the proposed technique is as follows. The whole set of factors is grouped into dimensionless similarity criteria. To find these quantities, the basic units of measurement are chosen and through them - the dimension of all other quantities. For example, the number of factors is *n* values, and *r* is the number of values that have independent dimensions, respectively, $n - r$ similarity criteria are obtained. The selected criteria are in a certain dependence.

For example, modelling of hydropneumodynamic processes is considered (*Dmytriv V. et al, 2018; Dmytriv V.T. et al, 2019; Dmytriv V. et al, 2019*). Galileo's criterion, Euler's and Reynolds numbers were an analogue of the momentum of kinetic energy, the specific indicator of the equivalent of energy consumption was the equivalent factor of profitability or productivity, the scale factor and others were used in study. Similarly for other modelling, for example, is considered the process of mixing a multicomponent mixture with the indices as follows. The analogue of the kinetic energy momentum of the bulk material particle is justified as a ratio of $p \cdot g/(\mu \cdot \omega)$, where: g – acceleration of gravity, m/s^2; μ – dynamic flow viscosity of the bulk component, kg/(m·s), ω - angular speed, s^{-1}, p – pressure in the mixing chamber of the components, kg/m^2.

All selected parameters must have a physical meaning and be controlled during the experiment, meet the requirements of compatibility and independence. The correct choice of these parameters is confirmed by a determinant consisting of the dimensions of these parameters, which should not be equal to zero.

Transformation of a multifactor space into a two-factor planned experiment

It is considered the transformation of multifactor space into a two-factor planned experiment on the example of air transportation in the vacuum pipeline of the milking machine. The process can be subjected to the method of proportionality and a combination of similarity numbers through the equation of relations:

$$\lambda = f(\Delta p, u, g, v, \rho, i_p, d) \tag{1}$$

where: λ - coefficient of friction; Δp – pressure drop at the section, Pa; v – air velocity, m/s; g – acceleration of gravity, m/s²; η – kinematic viscosity of the mixture, m²/s; ρ – air density at a given pressure, kg/m³; i_p – piezometric slope of the pipeline; d – diameter of the milk pipeline, m.

To find these quantities, the basic units of measurement are chosen and through them the dimension of all other quantities are expressed. In this case, n=7 quantities, and r = 3 is the number of quantities having independent dimensions, respectively, $n - r$ = 4 similarity criteria are obtained. The selected criteria are in a certain dependence. Basic units of dimension are: mass, length, time (Table 1). The main parameters of air movement are Δp, η, d.

Table 1

Dimensionality of the parameters of the equation for milk-air mixture transporting through the milk pipeline of the milking machine

Operation factors	λ	Δp	v	g	η	ρ	i_p	d
M	0	1	0	0	0	1	0	0
L	0	-1	1	1	2	-3	0	1
T	0	-2	-1	-2	-1	0	0	0

The correct choice of parameters is confirmed by the determinant, which consists of the dimension of these parameters, which should not be equal to zero:

$$\Delta = \begin{array}{c} \\ M \\ L \\ T \end{array} \begin{vmatrix} \Delta p & \eta & d \\ 1 & 0 & 0 \\ -1 & 2 & 1 \\ -2 & -1 & 0 \end{vmatrix} = 1 \neq 0$$

Taking into consideration the choice of basic units, the equation (1) can be written as:

$$\lambda = F((\Delta p^{\alpha v} \cdot \eta^{\beta v} \cdot d^{\gamma v})/v;\ (\Delta p^{\alpha g} \cdot \eta^{\beta g} \cdot d^{\gamma g})/g;\ (\Delta p^{\alpha \rho} \cdot \eta^{\beta \rho} \cdot d^{\gamma \rho})/\rho; 1) \tag{2}$$

The coefficients of $\alpha v, \ldots, \beta g, \ldots \gamma \rho$ were determined on the condition that each complex is dimensionless quantity. The values of αU, βU, γU were determined in the way of:

$$\frac{\Delta P^{\alpha v} \cdot \eta^{\beta v} \cdot d^{\gamma v}}{v} = \frac{(M^1 \cdot L^{-1} \cdot T^{-2})^{\alpha U} \cdot (L^2 \cdot T^{-1})^{\beta U} \cdot (L^1)^{\gamma U}}{L^1 \cdot T^{-1}} = M^{\alpha v} \cdot L^{-\alpha v + 2\beta v + \gamma v - 1} \cdot T^{-2\alpha v - \beta v + 1} = 1 \tag{3}$$

From the equation (3) a system of equations of the power expressions is composed and solved by finding the values of powers at their parameters:

$$\begin{cases} \alpha v = 0 \\ -\alpha v + 2\beta v + \gamma v - 1 = 0 \\ -2\alpha v - \beta v + 1 = 0 \end{cases} \rightarrow \begin{cases} \alpha v = 0 \\ \gamma v = -1 \\ \beta v = 1 \end{cases} \tag{4}$$

Accordingly, the similarity criterion for velocity was determined by the ratio:

$$\Pi_v = \frac{\eta}{d \cdot v} \tag{5}$$

By analogy for other criteria, the values of indicators are found and the ratio for other similarity criteria is written:

$$\Pi_g = \frac{\eta^2}{d^3 \cdot g},\ \Pi_\rho = \frac{\Delta p \cdot d^2}{\eta^2 \cdot \rho} \tag{6}$$

Instead of the parameters in equation (1), the following criteria will be substituted:

$$\lambda = f\left(\frac{\eta}{d \cdot v}, \frac{\eta^2}{d^3 \cdot g}, \frac{\Delta p \cdot d^2}{\eta^2 \cdot \rho}, i_p\right) \tag{7}$$

In the equation of (7), the first component is the inverse of the Reynolds number (1/Re), the second component is the inverse of the Galilean criterion (1/*Ga*), and the third component is the Euler number (*Eu*), in which the η^2/d^2 ratio has an explicit physical content of the mixture velocity.

In equation (7), the kinematic viscosity is repeated in the first and second terms, so the inverse of the Galilean criterion is multiplied by the Euler number and the result is the second and third terms. After these transformations, the following equation is obtained:

$$\lambda = f\left(\frac{\eta}{d \cdot v}, \frac{\Delta p}{d \cdot \rho \cdot g}, i_p\right) \tag{8}$$

The third term of the (8) equation is a constant value during the experiment, so it is neglected.

The (8) criterion equation will take the form of:

$$\lambda = f\left(\mathrm{Re}, Eu\right);\ \frac{1}{\mathrm{Re}} = \frac{\eta}{d \cdot v};\ Eu = \frac{\Delta p}{d \cdot \rho \cdot g}. \tag{9}$$

For the planned experiment, where the response criterion is the coefficient of friction, there were two factors, the Reynolds number and the Euler number. These criteria take into account the main parameters on which the coefficient of friction is depended.

Transformation of a multifactor space into a three-factor planned experiment

The technological process of bulk material mixing in a low vacuum environment is considered. The dependence of homogeneity and energy efficiency on the determining parameters can be represented by the connection equations:

$$v_m = f(\omega, g, R_\rho, R_D, \alpha, m_b, \rho_{10}, \mu_{10}, Q, p); \tag{10}$$

$$N_m = f(\omega, g, R_\rho, R_D, \alpha, m_b, \rho_{10}, \mu_{10}, Q, p), \tag{11}$$

where ω – angular speed of rotation of the dosing and mixing disk, [s^{-1}]; g – acceleration of gravity, [m/s^2]; R_ρ – radius of the curvature of agitator, [m]; R_D – disk radius, [m]; α – the angle of inclination of the generating line of disk, [grad]; m_b – number of blades or agitators on the disk, pcs.; ρ_{10} – density of bulk material, [kg/m^3]; μ_{10} – the dynamic viscosity of bulk material flow, [kg/(m·s)]; Q – productivity of the mixing component, [kg/s]; p – vacuum in the chamber of the components mixing, [kg/m^2].

In this case, $n = 10$ quantities, and $r = 3$ is the number of quantities having independent dimensions, respectively, $n - r = 7$ similarity criteria are obtained. The selected criteria are in certain dependence. Length, mass and time were accepted as basic units. The dimensions of the required and defining parameters of the work process are given in Table 2.

Table 2

The dimension of the parameters of the mixer of the feed mixture micro-components

Operation factors	$v_{зм}$	$N_{зм}$	ω	g	R_ρ	R_D	α	m_b	ρ_{10}	μ_{10}	Q	p
M	0	1	0	0	0	0	0	0	1	1	1	1
L	0	0	0	1	1	1	0	0	-3	-1	0	-2
T	0	0	-1	-2	0	0	0	0	-	-1	1	0

From Table 2 the basic units of measurement for the considered technological process are chosen. As the main parameters ω, ρ_{10}, p were considered. The correct choice of these parameters is confirmed by the determinant, which consists of the dimension of these parameters, which should not be equal to zero. Once such a determinant is solved, the following is obtained:

$$\Delta = \begin{array}{c} \\ M \\ L \\ T \end{array}\begin{vmatrix} \omega & \rho_{10} & p \\ 0 & 1 & 1 \\ 0 & -3 & -2 \\ -1 & 0 & 0 \end{vmatrix} = -1 \neq 0$$

Taking into consideration the choice of basic units the equation (10) can be written as:

$$v_m = \Phi(1; g/(\omega^{\beta g}\rho^{\gamma g}p^{\lambda g}); R_\rho/(\omega^{\beta h}\rho^{\gamma h}p^{\lambda h}); R_D/(\omega^{\beta d}\rho^{\gamma d}p^{\lambda d}); 1; \mu_{10}/(\omega^{\beta\mu}\rho^{\gamma\mu}p^{\lambda\mu}); 1). \tag{12}$$

The coefficients of $\beta g, \gamma h, \ldots \lambda\mu$ were determined on the condition that each complex is dimensionless quantity. The values of $\beta g,\ \gamma g,\ \mu g$ were determined in the way of:

$$g\Big/(\omega^{\beta g}\rho^{\gamma g}p^{\lambda g}) = L^1\cdot T^{-2}\Big/\left[\left|T^{-1}\right|^{\beta g}\left|M^1\cdot L^{-3}\right|^{\gamma g}\left|M^1\cdot L^{-2}\right|^{\lambda g}\right] =$$

$$= L^1\cdot T^{-2}\big/(T^{-\beta g}\cdot M^{\gamma g+\lambda g}\cdot L^{-3\gamma g-2\lambda g}) = T^{-2+\beta g}\cdot M^{-\gamma g-\lambda g}\cdot L^{1+3\gamma g+2\lambda g} = 1$$

$-2+\beta g = 0$, whence βg = 2

$-\gamma g-\lambda g = 0$, whence γg = -1

$1+3\gamma g+2\lambda g = 0$, whence λg = 1

Accordingly, this similarity criterion was determined by the ratio:

$$\Pi_g = \frac{\omega^2 p}{g\rho_{10}} \tag{13}$$

In a similar manner the other criteria were determined:

$$\Pi_{R_\rho} = \frac{R_\rho p}{\rho_{10}};\ \Pi_D = \frac{R_D p}{\rho_{10}};\ \Pi_\mu = \frac{\mu_{10}p^2\omega}{\rho_{10}} \tag{14}$$

Taking into consideration the criteria of (13) and (14), equation (10) is written as follows:

$$\nu_m = \Phi(\Pi_g, \Pi_h, \Pi_d, \Pi_\mu, \alpha, m_л) \tag{15}$$

The product of criteria (13) and (14) or their division gives the new criteria, respectively:

$$\Pi_h : \Pi_d = \frac{R_\rho}{R_D},\ Q : \Pi_\mu\cdot\Pi_g = \frac{Q\cdot\omega}{\mu_{10}\cdot p\cdot g}$$

After the transformations, a new criterion equation is obtained:

$$\nu_m, N_m = \Phi\left(\frac{Q\cdot\omega}{\mu_{10}\cdot p\cdot g}; \frac{R_\rho}{R_D}; \alpha; m_b\right) \tag{16}$$

where: $\dfrac{Q\cdot\omega}{\mu_{10}\cdot p\cdot g}$ – dimensionless factor of feeding of a blending agent on mixing;

R_ρ/R_D – the scale factor; α, m_b – dimensionless factors.

Since the number of blades on the disk is constant, the m_b factor is not taken into account during the experiment.

When the vacuum changes from 0 to 6 kPa, the dynamic flow viscosity is constant.

At atmospheric pressure the vacuum is 0, so the division by 0 in the dimensionless factor gives infinity. Instead of a vacuum, a coefficient that is calculated as a ratio is introduced:

$$K_p = \frac{p_{atm}}{p_{abs}}, \tag{17}$$

where: p_{atm} – atmospheric pressure, kPa; p_{abs} – absolute pressure in the mixing chamber, kPa;

$p_{abs} = p_{atm} - p_V$.

Accordingly, at atmospheric pressure K_p = 1.0 at p_V = 3 kPa; K_p = 1.031 at p_V = 6 kPa; K_p = 1.064.

Thus, the derived similarity criteria in their physical content are as follows: $Q\cdot\omega/(K_p\cdot g)$ – analogue of the momentum of the kinetic energy of a particle adding into the mixture of bulk material; R_ρ/R_D – scale factor; α – the angle of the generating line of the disk, the dimensionless factor.

RESULTS

The results of experimental studies of the coefficient of friction

The nature of the change in the coefficient of friction of air depending on the Reynolds and Euler numbers according to (9) criterion equation is considered. Reynolds and Euler numbers are calculated from experimental data of parameters: the volume air flow of vacuum pipeline V = 0.0015-0.0060 m^3/s; the loss of vacuum gage pressure of the vacuum pipeline Δp = 0.6-2.2 [kPa]; the inner diameter of the vacuum pipeline d = 0.022-0.038 m.

The regression equation in natural values has the form of:

$$\lambda = 4.4218 - 1.3104\cdot10^5\cdot\frac{1}{\mathrm{Re}} + 0.0029\cdot Eu + 9.265\cdot10^7\cdot\left(\frac{1}{\mathrm{Re}}\right)^2 + 24.032\cdot\frac{1}{\mathrm{Re}}\cdot Eu - 3.9655\cdot10^{-7}\cdot Eu^2 \tag{18}$$

The calculated value of the Cochran test is G_{cal} = 0.2601, which is less than the tabular of G_T = 0.2624. Accordingly, the experiment is reproduced.

The calculated value of the F-criterion is F_{cal} = 0.2111, which is much lower than the tabular F_T = 2.3. Accordingly, the model is adequate.

The graphical representation of the regression equation is presented in the form of a three-dimensional plane (Fig. 1).

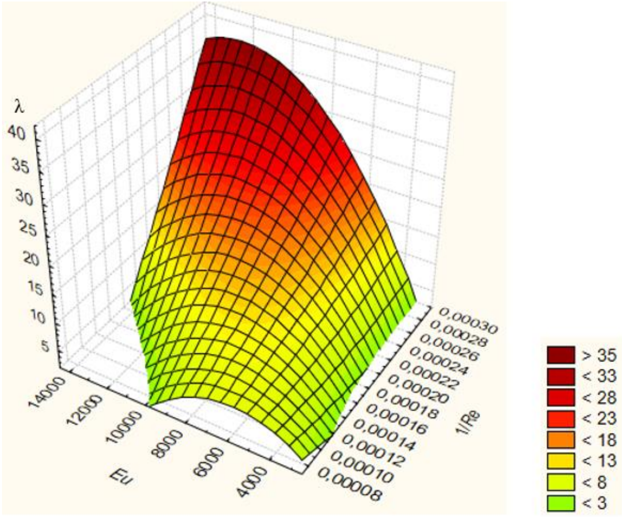

Fig. 1 – Dependence of the coefficient of friction λ of air in vacuum pipeline on the Euler and Reynolds numbers

The results of experimental studies of the homogeneity of bulk materials mixing

The results of a planned experiment are considered to study the homogeneity of mixing by the meter-mixer using the following factors: analogue of the kinetic energy impulse (x_1) – $Q \cdot \omega / (K_p \cdot g)$, the angle of the generating line of the metering disk α (x_2), scale factor of (x_3) – R_ρ / R_D. The response criterion is the homogeneity of the mixing of the meter-mixer.

The regression equation, which characterizes the dependence of the mixing homogeneity of the meter-mixer on the analogue of the kinetic energy impulse, the angle of the generating line of the metering disk and the scale factor in natural values, is as follows:

$$y = 0.0883921 \cdot \frac{Q \cdot \omega}{K_p \cdot g} + 0.088567 \cdot \alpha - 0.0990177 \cdot \frac{R_\rho}{R_D}. \tag{19}$$

The calculated value of the Cochran-criterion is G_{cal} = 0.09497 which is less than the tabular of G_T = 0.1377, accordingly, the experiment is reproduced.

The calculated value of the F-criterion is F_{cal} = 1.5882, which is less than the tabular F_T = 1.6, which confirms the adequacy of the model. The graphical representation of the regression equation is presented in the form of four-dimensional plane (Fig. 2).

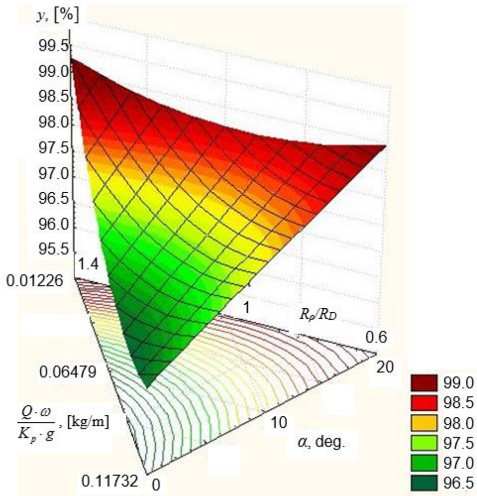

Fig. 2 - The dependence of the *y* mixing homogeneity of the meter-mixer on the analogue of the kinetic energy impulse of $Q \cdot \omega / (K_p \cdot g)$, the R_ρ / R_D scale factor and the α angle of the generating line of the metering disk

CONCLUSIONS

Analysis of the results of experimental studies of the friction coefficient where the factors are Reynolds and Euler numbers, showed the identity to the theoretical studies. The coefficient of friction is within the same limits. With vacuum and excess pressures, with decreasing of Reynolds number and increasing of Euler number, the gas friction coefficient increased. As the pressure loss and the diameter of the pipeline are increased the friction coefficient increased as well. An increase in the Reynolds number characterizes an increase as follows: the dynamic parameters of gas flow, velocity and flow modes - from laminar to turbulent, which consequently leads to an increase in Mach number.

The use of dimension theory in a factor-planned experiment allows reducing the number of factors, simplifies the mathematical interpretation of the response criterion and provides a graphical representation in the form of 3-D model. Fundamental similarity numbers confirm the validity of the model and expand the number of factors that directly through the similarity numbers characterize the physical essence of the processes. The use of the research results allows optimal tasks control according to specified criteria.

Using the method of the theory of similarity and dimensionality, criterion values, as an intermediate component between theory and experiment ensures a functional relationship between whole sets of quantities that characterize the process at the level of the physical model.

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