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APPLIED QUASIPOTENTIAL METHOD FOR SOLVING THE COEFFICIENT PROBLEMS OF PARAMETER IDENTIFICATION OF ANISOTROPIC MEDIA

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Abstract. *A numerical method of quasiconformal mappings for solving the coefficient problems of finding eigenvalues of the conductivity tensor having information about its directions in an anisotropic medium using applied quasipotential tomographic data is generalized. The corresponding algorithm is based on the alternate solving of problems on quasiconformal mappings and parameter identification. The results of numerical experiments of imitative restoration of environment structure are presented.*

Keywords: applied quasipotential tomography, quasiconformal mapping, identification, anisotropy

ZASTOSOWANIE METODY QUASIPOTENCJALNEJ DO ROZWIĄZYWANIA ZADANIA IDENTYFIKACJI PARAMETRYCZNEJ OŚRODKÓW ANIZOTROPOWYCH

Streszczenie. *Opracowano uogólnioną numeryczną metodę mapowania quasi-formalnego w celu rozwiązania zadań znalezienia wartości własnych tensora przewodnictwa posiadając informacje o jego kierunkach w ośrodku anizotropowym z zastosowaniem quasipotencjalnych danych tomograficznych. Podstawą algorytmu jest alternatywne rozwiązanie problemów związanych z mapowaniem quasi-formalnym i identyfikacją parametrów. Przedstawiono wyniki numerycznych symulacji odtworzenia struktury ośrodka.*

Slowa kluczowe: tomografia quasipotencjałowa, mapowanie quasikonformalne, identyfikacja, anizotropia

Introduction

Today, solving the problem of image reconstruction of a conductivity tensor (CT) in anisotropic media finds its application in an increasing number of areas. In particular, in robotics, geology, medicine, etc. (see, e.g., [6–8]). Despite the low resolution of the resulting images, geometric flexibility, harmlessness to the environment contributes to the continuation of relevant studies in order to improve the accuracy and speed of the calculations [6, 8].

The aim of this work is to generalize the numerical quasiconformal mapping method [2 – 4] for solving the coefficient problems of finding eigenvalues of the CT having information about its directions in an anisotropic medium using applied quasipotential tomographic (AQT) data.

1. The parameter identification problem of anisotropic media using AQT data

We consider the quasiideal processes of particles movement (in particular, liquids, electric charges) in a single-connected curvilinear domain (anisotropic layer or plate, which is some tomographic cross-section) G_z (Fig. 1a), limited by a smooth closed curve $\partial G_z = \{(x,y): \ x=\tilde{x}(\tau), \ \ y=\tilde{y}(\tau), \ \ 0\le\tau\le 2\pi,$ $\tilde{x}(0)=\tilde{x}(2\pi)=\tilde{x}_0, \ \ \tilde{y}(0)=\tilde{y}(2\pi)=\tilde{y}_0$, where $\tilde{x}(\tau)$, $\tilde{y}(\tau)$ are defined continuosly differentiated functions, $O(\tilde{x}_0,\tilde{y}_0)$ is given refference point, under conditions of various situational states (injections), generated by the action of the differences in the applied potentials to the selected sections of the boundary [1, 2, 4, 6, 8–10]. Suppose that we known not only distributions of potentials, but also local velocities of matter at the same points [2, 4]. In this case, the problem of parameters identification of the quasiideal stream using AQT data is traditionally reduced to finding an infinite number of functions-quasipotentials $\varphi=\varphi^{(p)}(x,y)$ and a single anisotropy tensor $\sigma=\left(\sigma_{\alpha\beta}(x,y)\right)_{\alpha,\beta=1,2}$ in the domain G_z, for which the equations:

$$(\sigma_{11}\varphi_x'^{(p)}+\sigma_{12}\varphi_y'^{(p)})_x'+(\sigma_{21}\varphi_x'^{(p)}+\sigma_{22}\varphi_y'^{(p)})_y'=0 \quad (1)$$

are fulfilled when for each p interconnected conditions of Dirichlet and Neumann are given [1, 2, 4, 6–10]. Here (1) is a consequence of the movement law by Ohm, Darcy type etc. $\bar{j}^{(p)}=\sigma grad\varphi^{(p)}$ and the continuity equation $div\bar{j}^{(p)}=0$ [1, 2, 4, 6, 8–10]; $p=1,2,...$ is the injection number (see, e.g., [1, 2, 4]); $\sigma_{\alpha\beta}=\sigma_{\alpha\beta}(x,y,...)$ are bounded continuously differentiated in the domain G_z functions that characterizing the conductivity and anisotropy of the medium [3].

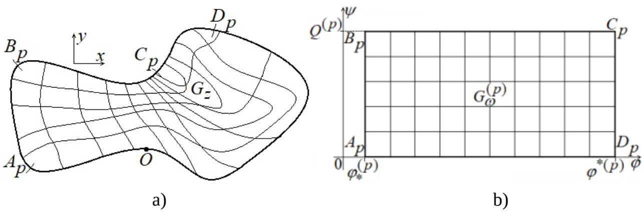


Fig. 1. Tomographic cross-section G_z (a) and the corresponding domains of complex quasipotential $G_\omega^{(p)}$ (b)

In practice, it is not possible to obtain an infinite number of data (measurements at the boundary), and therefore scientists apply a different kind of simplification (see, e.g., [1, 2, 4, 6–10]). First of all, they consider the finite number of injections of current through the tomographic section. It, similar to [2, 4], we simulate by sets of values $\{\tau_A^{(p)},\tau_B^{(p)},\tau_C^{(p)},\tau_D^{(p)}\}$, according to which

$$A_p=\left(\tilde{x}(\tau_A^{(p)}),\tilde{y}(\tau_A^{(p)})\right), \ \ B_p=\left(\tilde{x}(\tau_B^{(p)}),\tilde{y}(\tau_B^{(p)})\right),$$
$$C_p=\left(\tilde{x}(\tau_C^{(p)}),\tilde{y}(\tau_C^{(p)})\right), \ \ D_p=\left(\tilde{x}(\tau_D^{(p)}),\tilde{y}(\tau_D^{(p)})\right).$$

Corresponding for this injection boundary of the domain G_z with given four marked points is denoted by $\partial G_z^{(p)}$ $(z^{(p)}=x^{(p)}+iy^{(p)})$. We propose to set the local velocity distributions at sections of constant applied potentials, and at other lines, both the constant stream values and the distributions of potentials [2, 4]. This, in comparison with the known world analogues (see, e.g., [1, 6, 8, 10]), provides the possibility of both the physical providing of experiment and the application of our developed complex analysis methods.

In this case, the mathematical model of AQT [8], similar to [1, 2, 4], we write in the form (1) and conditions:

$$\varphi^{(p)}\Big|_{A_pB_p}=\varphi_*^{(p)},\ \varphi^{(p)}\Big|_{C_pD_p}=\varphi^{*(p)},$$
$$\left|\vec{j}^{(p)}\right|\Big|_{B_pC_p\cup A_pD_p}=0; \quad (2)$$

$$\varphi^{(p)}(M)\Big|_{A_pD_p}=\underline{\varphi}^{(p)}(M),\ \varphi^{(p)}(M)\Big|_{B_pC_p}=\bar{\varphi}^{(p)}(M),$$
$$\left|\vec{j}^{(p)}(M)\right|\Big|_{A_pB_p}=\Psi_*^{(p)}(M),$$
$$\left|\vec{j}^{(p)}(M)\right|\Big|_{C_pD_p}=\Psi^{*(p)}(M), \quad (3)$$

where A_pB_p and C_pD_p are selected equipotential lines; B_pC_p and A_pD_p are impermeable boundary stream lines; $\vec{n}$ is unit vector of outer normal; M is a running point of the corresponding curve. Functions $\Psi_*^{(p)}(M)=\Psi_*^{(p)}(\tau,...)$ $(\tau_B^{(p)}\le\tau\le\tau_A^{(p)})$, $\bar{\varphi}^{(p)}(M)=\bar{\varphi}^{(p)}(\tau,...)$ $(\tau_C^{(p)}\le\tau\le\tau_B^{(p)})$, $\underline{\varphi}^{(p)}(M)=\underline{\varphi}^{(p)}(\tau,...)$ $(\tau_A^{(p)}\le\tau\le\tau_D^{(p)})$, $\Psi^{*(p)}(M)=\Psi^{*(p)}(\tau,...)$ $(\tau_D^{(p)}\le\tau\le\tau_C^{(p)})$, as in [2], can be constructed by interpolating experimentally obtained their values $\bar{\varphi}^{(p)}_{\bar{i}^{(p)}}$, $\underline{\varphi}^{(p)}_{\underline{i}^{(p)}}$, $\Psi^{(p)}_{*j_*^{(p)}}$, $\Psi^{*(p)}_{j^{*(p)}}$ having some $\tau=\bar{\tau}^{(p)}_{\bar{i}^{(p)}}$, $\tau=\underline{\tau}^{(p)}_{\underline{i}^{(p)}}$, $\tau=\tau^{(p)}_{*j_*^{(p)}}$, $\tau=\tau^{*(p)}_{j^{*(p)}}$ at the sections B_pC_p, A_pD_p, A_pB_p, C_pD_p, respectively $(0\le\bar{i}^{(p)}\le\bar{m}^{*(p)}+1,$ $0\le\underline{i}^{(p)}\le\underline{m}_*^{(p)}+1,$ $\Psi^{(p)}_{*j_*^{(p)}},$ $\Psi^{*(p)}_{j^{*(p)}}>0,$ $\varphi_*^{(p)}\le\underline{\varphi}^{(p)}_{\underline{i}^{(p)}}\le\varphi^{*(p)},$ $\varphi_*^{(p)}\le\bar{\varphi}^{(p)}_{\bar{i}^{(p)}}\le\varphi^{*(p)},$ $0\le j_*^{(p)}\le n_*^{(p)}+1,$ $0\le j^{*(p)}\le n^{*(p)}+1)$. CT components with equal elements of the additional diagonal [10] are defined as follows:

$$\sigma_{11}=(\lambda_1-\lambda_2)cos^2\theta+\lambda_2,\ \sigma_{22}=(-\lambda_1+\lambda_2)cos^2\theta+\lambda_1,$$
$$\sigma_{12}=(\lambda_1-\lambda_2)sin\theta cos\theta,\ \sigma_{21}=\sigma_{12}, \quad (4)$$

eigenvalues λ_1, λ_2 of corresponding to (1) matrix we search in the form:

$$\lambda_1=\lambda_1(x,y,a_{s_a,0},...,a_{0,s_a})=\sum_{k_a,r_a=0}^{s_a,k_a}a_{k_a-r_a,r_a}x^{k_a-r_a}y^{r_a},$$

$$\lambda_2=\lambda_2(x,y,b_{s_b,0},...,b_{0,s_b})=\sum_{k_b,r_b=0}^{s_b,k_b}b_{k_b-r_b,r_b}x^{k_b-r_b}y^{r_b}, \quad (5)$$

and we consider the angles distribution function of extreme value directions of the conductivity coefficient $\theta=\theta(x,y)$, similar to [1, 8, 9], a priori known. Here $a_{k_a-r_a,r_a}$, $b_{k_b-r_b,r_b}$ $(k_a=0,...,s_a,$ $r_a=0,...,k_a,$ $k_b=0,...,s_b,$ $r_b=0,...,k_b)$ are the parameters that are defined during the problem solving process.

The problem lies in image reconstruction of the CT. Here, the related is the calculation of the corresponding dynamic meshes and velocity fields.

We can reduce (1) – (5) to the series of more general boundary value problems on quasiconformal mapping $\omega^{(p)}(z)=\varphi^{(p)}(x,y)+i\psi^{(p)}(x,y)$ of the physical domains $G_z^{(p)}$ (Fig. 1a) onto the corresponding domains of the complex quasipotential $G_\omega^{(p)}$ (Fig. 1b) by the way, similarly to [2–4], of introducing the stream functions $\psi^{(p)}=\psi^{(p)}(x,y)$, which are complex conjugated to $\varphi^{(p)}=\varphi^{(p)}(x,y)$ $(p=\overline{1,\tilde{p}})$, under (4) and (5) conditions:

$$\begin{cases}\sigma_{11}\varphi_x'^{(p)}+\sigma_{12}\varphi_y'^{(p)}=\psi_y'^{(p)},\\ \sigma_{21}\varphi_x'^{(p)}+\sigma_{22}\varphi_y'^{(p)}=-\psi_x'^{(p)};\end{cases} \quad (6)$$

$$\varphi^{(p)}\Big|_{A_pB_p}=\varphi_*^{(p)},\ \varphi^{(p)}\Big|_{C_pD_p}=\varphi^{*(p)},$$
$$\psi^{(p)}\Big|_{A_pD_p}=0,\ \psi^{(p)}\Big|_{B_pC_p}=Q^{(p)}; \quad (7)$$
$$\int_{MN}\left|\vec{j}^{(p)}\right|dl=Q^{(p)},\ M\in B_pC_p,\ N\in A_pD_p;$$

$$\psi^{(p)}(M)\Big|_{A_pB_p}=\psi_*^{(p)}(M),\ \psi^{(p)}(M)\Big|_{C_pD_p}=\psi^{*(p)}(M),$$
$$\varphi^{(p)}(M)\Big|_{A_pD_p}=\underline{\varphi}^{(p)}(M),\ \varphi^{(p)}(M)\Big|_{B_pC_p}=\bar{\varphi}^{(p)}(M), \quad (8)$$

where $G_\omega^{(p)}=\{(\varphi,\psi):\ \ \varphi_*^{(p)}\le\varphi\le\varphi^{*(p)},\ \ 0\le\psi\le Q^{(p)}\}$; $\psi_*^{(p)}(M)=\int_{A_pM}\Psi_*^{(p)}(M)dl,$ $\psi^{*(p)}(M)=\int_{D_pM}\Psi^{*(p)}(M)dl;$ $Q^{(p)}$ are discharges of the vector fields (current) through the contact sections (A_pB_p and C_pD_p); dl is arc element of corresponding curve.

2. Synthesis of the numerical quasiconformal mapping method and ideas of alternating block parametrization

In [2, 4], algorithms for numerical solving of inverse nonlinear boundary value problems on quasiconformal mappings in curvilinear quadrilateral domains bounded by stream and equipotential lines are proposed, and in [3] such approaches are generalized to the case of anisotropy. Accordingly, solving the problem will be carried out applying these methods (using the corresponding notations; the algorithms obtained in the above-mentioned works will be fragments of wider structures, in particular, injectivity must be taken into account).

We reconstruct the CT, like in [2–4], provided minimize the residual sum of squares of expressions, obtained from Cauchy-Riemann-type conditions, with applying the ideas of regularization and having positive eigenvalues condition.

$$\Phi(x^{(1)},y^{(1)},...,x^{(\tilde{p})},y^{(\tilde{p})},a_{s_a,0},b_{s_b,0},...,a_{0,s_a},b_{0,s_b})=\frac{df}{}$$
$$=\sum_{p=1}^{\tilde{p}}\left(2\eta\left(\sum_{k_a,r_a=0}^{s_a,k_a}\frac{a^2_{k_a-r_a,r_a}}{100^{k_a}}+\sum_{k_b,r_b=0}^{s_b,k_b}\frac{b^2_{k_b-r_b,r_b}}{100^{k_b}}\right)+\Big((\sigma_{11}+\right.$$
$$+\sigma_{21})y_\psi'^{(p)}-(\sigma_{12}+\sigma_{22})x_\psi'^{(p)}-x_\varphi'^{(p)}-y_\varphi'^{(p)}\Big)^2+\Big((\sigma_{11}-$$
$$\left.-\sigma_{21})y_\psi'^{(p)}+(\sigma_{22}-\sigma_{12})x_\psi'^{(p)}-x_\varphi'^{(p)}+y_\varphi'^{(p)}\Big)^2\right); \quad (9)$$

$$\lambda_1>0,\ \lambda_2>0, \quad (10)$$

where η is regularization parameter.

We write the corresponding difference analogues in the mesh domains $G_z^{\gamma(p)}$ when $z_{i,j}^{(p)}=z^{(p)}(\varphi_i^{(p)},\psi_j^{(p)})$ similar to [2–4]. Here $G_z^{\gamma(p)}=\left\{z_{i,j}^{(p)}=(x_{i,j}^{(p)},y_{i,j}^{(p)})\right\}$, $G_\omega^{\gamma(p)}=\left\{(\varphi_i^{(p)},\psi_j^{(p)}):\right.$ $\varphi_i^{(p)}=\varphi_*^{(p)}+i\Delta\varphi^{(p)},$ $i=\overline{0,m^{(p)}+1};$ $\psi_j^{(p)}=j\Delta\psi^{(p)},$

$j=\overline{0,n^{(p)}+1}$; $\gamma^{(p)}=\Delta\varphi^{(p)}/\Delta\psi^{(p)}$; $\Delta\psi^{(p)}=Q^{(p)}/(n^{(p)}+1)$, $\Delta\varphi^{(p)}=(\varphi^{*(p)}-\varphi_*^{(p)})/(m^{(p)}+1)$, $m^{(p)},\ n^{(p)}\in\mathbf{N}\}$, $\gamma^{(p)}$ are quasiconformal invariants for the corresponding domains.

The algorithm for solving the initial problem consists in the alternate parametrization of internal nodes of the mesh domains $G_z^{\gamma(p)}$, the CT and in use of ideas of the block iteration method [3, 12]. Namely: we set the number of injections $\tilde{p}$, domains border $G_z^{(p)}$ (by functions $x=\tilde{x}(\tau)$, $y=\tilde{y}(\tau)$), parameters $\tau_A^{(p)}$, $\tau_B^{(p)}$, $\tau_C^{(p)}$, $\tau_D^{(p)}$ and ε_1, ε_2 (accuracy), q ($q>1$ is responsible for the number of iterations of refinement of internal nodes having a specific CT), quasipotentials $\varphi_*^{(p)}$, $\varphi^{*(p)}$ and discharges $Q^{(p)}$, parameters $m^{(p)}$, $n^{(p)}$ of domains partition $G_\omega^{\gamma(p)}$ (in order to improve the accuracy of the calculations, it is desirable to select this values so that $\frac{Q^{(p)}}{\varphi^{*(p)}-\varphi_*^{(p)}}\frac{n^{(p)}+1}{m^{(p)}+1}\approx 1$) [2, 3], regularization parameter η and the angles distribution function of extreme value directions of the conductivity coefficient $\theta=\theta(x,y)$. Then we calculate the coordinates of the angular points A_p, B_p, C_p, D_p on $\partial G_z^{(p)}$, steps of partitioning the complex quasipotential domains $\Delta\psi^{(p)}=Q^{(p)}/(n^{(p)}+1)$, $\Delta\varphi^{(p)}=(\varphi^{*(p)}-\varphi_*^{(p)})/(m^{(p)}+1)$ and the quasiconformal invariants values $\gamma^{(p)}=\Delta\varphi^{(p)}/\Delta\psi^{(p)}$.

Specify the local velocity value $\Psi_{*j}^{(p)}$, $\Psi_j^{*(p)}$ (and therefore, stream functions $\psi_{*j}^{(p)}$, $\psi_j^{*(p)}$) and potentials $\overline{\varphi}_i^{(p)}$, $\underline{\varphi}_i^{(p)}$ having some arguments $\tau_{*j}^{(p)}$, $\tau_j^{*(p)}$, $\overline{\tau}_i^{(p)}$, $\underline{\tau}_i^{(p)}$ (results of physical measurements), respectively, then by the way of interpolation we build the functions $\tau=\tau_*^{(p)}(\psi)$, $\tau=\overline{\tau}^{(p)}(\varphi)$, $\tau=\tau^{*(p)}(\psi)$, $\tau=\underline{\tau}^{(p)}(\varphi)$ ($\varphi_*^{(p)}\le\varphi\le\varphi^{*(p)}$, $0\le\psi\le Q^{(p)}$), whereupon, similarly to [2], we find the coordinates $x_{0,j}^{(p)}$, $y_{0,j}^{(p)}$, $x_{i,n^{(p)}+1}^{(p)}$, $y_{i,n^{(p)}+1}^{(p)}$, $x_{m^{(p)}+1,j}^{(p)}$, $y_{m^{(p)}+1,j}^{(p)}$, $x_{i,0}^{(p)}$, $y_{i,0}^{(p)}$ ($0\le i\le m^{(p)}+1$, $0\le j\le n^{(p)}+1$, $p=\overline{1,\tilde{p}}$) on $\partial G_z^{\gamma(p)}$. After that, we begin the iterative reconstruction process, which consists of the following steps: apply a difference representation of Laplace type equations [3] (taking into account the injectivity) for finding the coordinates of the internal nodes when $p=\overline{1,\tilde{p}}$, $i=\overline{1,m^{(p)}}$, $j=\overline{1,n^{(p)}}$ q times; solve the problem of conditional minimization, constructed on the basis of (9), (10) (by one of the local optimization methods, e.g., [13]) relative to $a_{k_a-r_a,r_a}^{(l)}$, $b_{k_b-r_b,r_b}^{(l)}$ (here $l=0,1,...$ is step iteration number, $k_a=\overline{1,s_a}$, $r_a=\overline{0,k_a}$, $k_b=\overline{1,s_b}$, $r_b=\overline{0,k_b}$); check the conditions for the completion of the iterative process, among which may be [3]: stabilization of border nodes, CT, quasiconformality degree parameter, discharge values etc. ($1\le p\le\tilde{p}$, $1\le i\le m^{(p)}$, $1\le j\le n^{(p)}$). In case of not fulfilling at least one of these conditions, the iteration process is restarted, otherwise we build the corresponding reconstructed image and, if necessary, electrodynamic meshes, domains of complex quasipotential or calculate the current densities fields etc.

Note that the algorithm will be similar if instead of the angles distribution function of extreme value directions of the conductivity coefficient, known is the distribution either λ_1 or λ_2 for the desired θ and either λ_1 or λ_1, respectively. However, solving the nonlinear programming problem thus formed will require the application of the global optimization method.

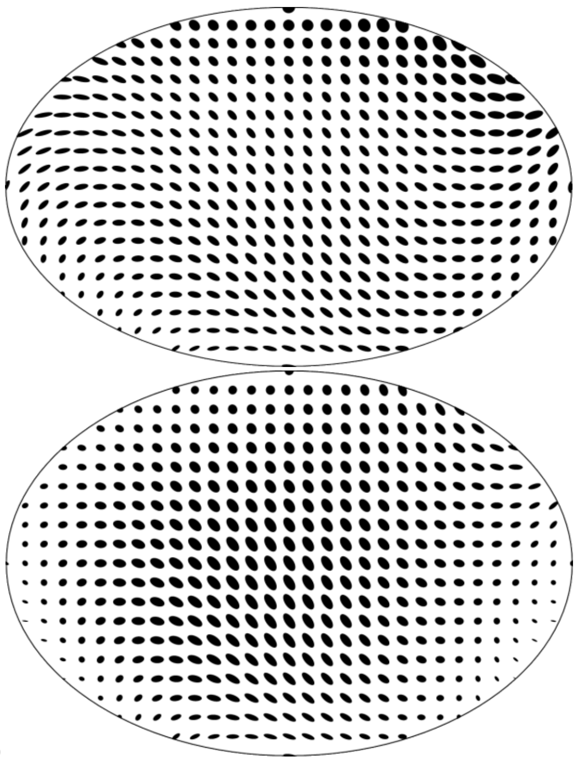
It should be noted that here, as in [2] (in opposite to [3, 4]), there is no need to use formulas to refine the boundary nodes (these coordinates are a priori known), which, of course, accelerates the reconstruction process of the image. In order to save machine time, it is also allowed to use formulas constructed on the basis of (9) and (10) for selected points only. In particular (if it allows to do the chosen optimization algorithm) condition (10) should not require all mesh nodes of the meshes, but only in the coordinates of the extreme values of the minimum of functions (4). In other cases, it makes sense to specify a series of control points inside the investigated domain. Such in many cases can be nodes of meshes of arbitrary injection.

3. Numerical results

We present the results of numerical experiments of imitative restoration of medium structure for the following input data: $m^{(p)}=100$, $\tilde{x}(\tau)=150\cos\tau$, $\tilde{y}(\tau)=100\sin\tau$, $s_a=s_b=3$, $a_{0,0}=b_{0,0}=1$, $a_{k_a-r_a,r_a}=b_{k_b-r_b,r_b}=0$ ($k_a=\overline{1,s_a}$, $r_a=\overline{0,k_a}$, $k_b=\overline{1,s_b}$, $r_b=\overline{0,k_b}$), $\eta=0.01$, $\tilde{p}=20$, $\tau_A^{(p)}=\frac{9\pi}{8}+(p-1)\frac{\pi}{\tilde{p}}$, $\tau_B^{(p)}=\tau_A^{(p)}-\frac{\pi}{4}$, $\tau_C^{(p)}=\tau_A^{(p)}-\pi$, $\tau_D^{(p)}=\tau_C^{(p)}-\frac{\pi}{4}$, $\varphi_*^{(p)}=0$, $\varphi^{*(p)}=1$, $Q^{(p)}$, $\Psi_{*j}^{(p)}$, $\Psi_j^{*(p)}$, $\overline{\varphi}_i^{(p)}$, $\underline{\varphi}_i^{(p)}$ ($1\le p\le\tilde{p}$), $\varepsilon_1=\varepsilon_2=10^{-2}$, $\theta(x,y)=1+12\cdot10^{-4}x-10^{-4}x^2+16\cdot10^{-6}xy-36\cdot10^{-6}y^2-10^{-7}x^3-7\cdot10^{-7}x^2y+45\cdot10^{-8}xy^2-54\cdot10^{-8}y^3$), $q=200$. A visual representation of received CT distribution is carried out similar to [10] by application of a specially developed procedure. According to it, the researched domain is divided into square sections by lines, which are parallel to the coordinate axes. The CT is characterized in the center of this figures as an ellipses (axes and radii of which correspond to the directions of eigenvectors and proportional to the values of eigenvalues, respectively) of the form $\kappa_{11}x^2+\kappa_{22}y^2+2\kappa_{12}xy=1$, where $\kappa_{11}=sin^2\theta/\lambda_2^2+cos^2\theta/\lambda_1^2$, $\kappa_{22}=cos^2\theta/\lambda_2^2+sin^2\theta/\lambda_1^2$, $\kappa_{12}=sin\theta cos\theta(1/\lambda_1^2-1/\lambda_2^2)$. Fig. 2b presents the reconstructed image of the CT distribution in comparison with the given theoretically (Fig. 2a).

4. Acknowledgment

The numerical quasiconformal mapping method for solving the problem of finding the eigenvalues of the CT having information about its directions in anisotropic medium and using AQT data is generalized. The algorithm for solving the corresponding problem is based on the application of the idea of quasiconformal similarity in the small for construction of curvilinear quadrilaterals (quasiparallelograms), that are components of dynamic mesh in the physical domain and the corresponding squares in the domain of complex quasipotential and alternate parametrization of internal nodes of dynamic meshes (constructed for each of the injections) and the desired CT. The developed algorithm is characterized by comparatively fast computer convergence (since, unlike many useful methods, it does not require finding the derivatives of the distribution function of the CT at the specified points and specifying the boundary nodes at each iteration step), the possibility of relatively easy its parallelization and the premature stop of the calculation procedure only if some of the conditions for the completion of the process are fulfilled.



a)

b)

Fig. 2. CT distribution: exact (when $a_0 = 1.5$, $a_1 = 5 \cdot 10^{-4}$, $a_2 = 5 \cdot 10^{-3}$, $a_3 = 0$, $a_4 = 3 \cdot 10^{-6}$, $a_5 = -10^{-5}$, $a_6 = -5 \cdot 10^{-8}$, $a_7 = -10^{-6}$, $a_8 = 3 \cdot 10^{-7}$, $a_9 = -4 \cdot 10^{-7}$, $b_0 = 0.7$, $b_1 = 0$, $b_2 = 12 \cdot 10^{-4}$, $b_3 = -27 \cdot 10^{-7}$, $b_4 = -8 \cdot 10^{-6}$, $b_5 = 18 \cdot 10^{-6}$, $b_6 = 4 \cdot 10^{-8}$, $b_7 = -12 \cdot 10^{-9}$, $b_8 = 27 \cdot 10^{-8}$, $b_9 = -6 \cdot 10^{-7}$) *(a) and approximated (b) solutions*

The latter may occur in areas with large computational errors (so-called "stagnant zones" and "large gradient zones") that arise near the singular points of the non-smooth boundary lines and the critical points of the interior of the corresponding domains. But we note that the considerably new algorithm is to take into account the conditions of "anisotropic quasiorthogonality" along the boundary equipotentials and flow lines (instead of orthogonality in cases of isotropic media), which causes additional substantially new constructions. Also, the anisotropy tensor on orders affects the deterioration of accuracy, stability, which in particular requires the creation of new structures, procedures of regularization Tikhonov type.It is also worth noting that, unlike the traditional approaches to the formulation and solving the problems of electrically impedance tomography [1, 6 – 10], we determine the distribution of local velocities of a substance (fluid, current) and averaged potentials on sites the contact of the plate and body, and in other sections the distribution of the potential (according to experimental data). This, of course, provides greater mathematical conformity, and therefore a certain gain in the iterative reconstruction process.

We plan to extend this algorithm to the following cases: spatial reconstruction possibility, formation the several sections of apply the quasipotential at the initial stream, parameters identification of the CT of piecewise-homogeneous and piecewise-inhomogeneous media and filtration-convection-diffusion type processes (see, for example, [5, 11]).

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