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TITLE YIELD INDEX BASED ON CONCEPT OF SIX- SIGMA SSC$_{pk}$ FOR ASSESSMENT PROCESS PERFORMANCE IN INDUSTRIES

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A B S T R A C T

Process quality is a key factor in facilitating product sales, (PCIs), which determine the relationship between the manufacturing specifications and the actual process performance by quantifying process potential. Although C_p, C_{pk} indices are the most popular and important criteria used in manufacturing industries to measure process performance reported extensively in the literature, in the literature, the majority of previous studies neglected the idea of Six Sigma (SS). However, existing PCIs based on SS presented only a range of quality levels rather than a specific quality level value. The purpose of this study is to measure and enhance the precision of performance evaluation for processes industrial, through the use of (PCIs) and (SS) concept. In light of this, this study introduces new performance index based on the idea SS which are SSC_{pk} by extending the indices C_p, C_{pk} and calculating the sigma process level directly to measure yield process based on idea SS. To demonstrate the effectiveness applicability of the SSC_{pk} index, this study presents an industrial case study to assess the process performance of Aden oil refinery in Yemen. Toward this end, the data for essential quality characteristic of petroleum products namely octane number was collected randomly from Aden refinery. The findings of this study indicated that the proposed index SSC_{pk} outperformed on the existing indices C_{pk} as the shows at the results and discussion. Finally, the proposed process yield index based on SS concept is a promising approach and thus can be utilized by other industries and practitioners to assess process performance in the aspect of precision and quality control.

1. INTRODUCTION

The paper must be written In today's competitive and globalized markets, industries are obligated to produce high-quality and cost-effective products that consistently meet the consumers and engineering design specifications (Felipe & Benedito, 2017; Goswami & Dutta, 2013; Krolczyk et al., 2015). Subsequently, quality level and process capability have become indispensable attributes and key issues among producers to achieve competitive advantage particularly in the world of knowledgeable consumers (Krolczyk et al., 2015; Leiva, Marchant, Saulo, Aslam, & Rojas, 2014;

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Lupo, 2015). Over the years, manufacturers have consistently attempted to identify the sources of variations in order to develop control measures for eliminating or minimizing process variabilities whenever possible(Goodwin, 2015; Rosa & Broday, 2018). Process capability refers to the ability of a process to produce products that will consistently meet customer expectations and the design requirements (Felipe & Benedito, 2017; Krolczyk et al., 2015). More specifically, it is a scientific and a systematic procedure that uses control charts and capability indices to detect and eliminate the unnatural causes of variation until a state of statistical control is reached. According to (Shahriari & Abdollahzadeh, 2009) and (Kotz & Lovelace, 1998), the enemy of high process capability and perfect output is variation. The authors further stated that "since process variation can never be totally eliminated, the control of such variation is the key to achieve product quality". Hence, in order to reach high process capability and perfect output, variation must be identified, controlled and eliminated (Goodwin, 2015).

Process Capability Analysis is a statistical technique used to determine how well a process meet a set of specification limits (Felipe & Benedito, 2017; Kargar, Mashinchi, & Parchami, 2014; Lupo, 2015; Montgomery, 2009). PCA can be used if the process has reasonable statistical control, stable, and does not produce acceptable products that meet pre-specified targets (F. Ali & Ahmed, 2016; Leiva et al., 2014). The procedure of capability analysis involves taking a sample data from a process to estimate the Defects Per Million Opportunities (DPMO), Process Capability Indices (PCIs) and Sigma Quality Estimates(F. Ali & Ahmed, 2016; Srinivasan, Muthu, Devadasan, & Sugumaran, 2016). In fact, PCA provides numerical statistical measures including PCIs, six sigma, process expected loss and process yield to measure process capability, reduce variability and defects and consistently produce products and services that meet the pre-specified control limits(Chen, Yu, & Sheu, 2006). PCIs are powerful statistical tools utilized by industries to assess manufacturing process performance and to measure the variability of a process relative to its specification limits (Chakraborty & Chatterjee, 2016). They are typically a set of formulas which uses the mean and variance of a particular product characteristic to determine whether the process that makes that product is capable of meeting production tolerance. In addition to providing numerical measures of whether or not a manufacturing process is capable of producing consistent products based on predetermined specification limits, PCIs are also convenient and an effective tools to facilitate communication among engineers (Allam, Becker, Baudouin, Bigot, & Krumpipe, 2014; Pan, Li, & Shih, 2016; Parchami, Sadeghpour, Nourbakhsh, & Mashinchi, 2014; Pearn, Wu, & Chia, 2014; Pham, 2015; Srinivasan et al., 2016).

PCIs are essential indicators for evaluating process performance in industries through calculating process yield. Process yield refers to the capability of a process to produce consistent products and services according to pre-defined control limits. According to (Tano & Vannman, 2012) , performance criteria are particularly evaluated by process yield index. Indeed, PCIs, C_p, C_{pk}, C_{pm} and S_{pk} are acknowledged as capability measures, quality assurance and capability analysis based on various criteria including consistency of process, the departure of a process from the target, process yield, and process loss (Chakraborty & Chatterjee, 2016). Besides that, the quality yield index of a process can be described as the conventional process yield minus the expected relative loss within the specifications. Thus, the quality index is a vital measure for evaluating process performance and process quality. (PCIs) have been proposed for the manufacturing industry to provide numerical measures on how well a process is capable of reproducing items within the present specification limits in the factory. Numerous process capability indices, including C_p, C_{pk} and C_{pm} have been used to evaluate process performance for cases with single quality characteristics, which are important tools for quality/reliability assurance (Chakraborty & Chatterjee, 2016; Coetzer & de Jongh, 2016; Dianda, Quaglino, & Pagura, 2017; Felipe & Benedito, 2017; Lupo, 2015; Pearn, Shiau, Tai, & Li, 2011). the existing PCIs have not assessed the quality of a product or service while taking into consideration the possibility of the process mean to shift by as much as 1.5σ from the target (Bothe, 2002; Hsu, Pearn, & Wu, 2008; Nourelfath, Aldowaisan, & Hassan, 2016). Moreover, in the context of PCIs based on six sigma implementation in industries, the literature indicates that implementing process capability indices based on six sigma for assessing the process performance of oil refinery has not been sufficiently explored (Agina-Obu, 2015; Alkubaisi, 2013). In particular, the implementation of six sigma in the petroleum industry in Yemen is still lacking(F. A. M. Ali & Ahmed, 2017). Nevertheless, there are studies in the field of oil in different countries particularly in the field of quality control measurement such as (Alkubaisi, 2013; Bhanpurkar, Bangar, Goyal, & Agrawal, 2012; He, Lin, Li, Sui, & Xu, 2015; James Dhinakaran, Maharaja Ganapathy, Kodeswaran, Muthu Kannan, & Murugan, 2012; Kindi & Lawati, 2014) But those studies used control charts to monitor process data such as $\bar{X}$, $\overline{\text{R}}$ and $\bar{X}_s$ charts and tools of Ishikawa (Alkubaisi, 2013).

2. EFFECT OF SIGMA ESTIMATION ON RELATIONSHIP BETWEEN Σ LEVELS AND PCIS

Leave one clear line before and after a main or secondary heading and after each paragraph. Sigma estimation (standard deviation) is a crucial topic in the field of statistical control and process capability. This is due to fact that the estimation of capability process indices relies on the estimation of process variability. As a result, process variation impacts the process capability indices PCIs and leads to incapable process. At the same time, it affects levels of sigma process calculated indirectly from the outcome of PCIs. Six sigma is important to minimize defects and error range and increase the product quality. Six sigma originates from process capability, which is a statistical measure used to ensure that the manufacturing process meets the predetermined specification limit (Gupta, 2015; Şenvar & Tozan, 2010). PCIs are indicators or measures employed for evaluating the capability of the process for producing products according to predetermined tolerance limits and to achieve excellent quality (Montgomery, 2009). Process capability can be defined as a function of the process variations $(i.e.\ 6\sigma)$. Statistically, PCIs comprise of many indices. At the present, the most widely applied PCI s are unilateral specification indices C_{pu}, C_{pl} and bilateral specification indices C_p and C_{pk}. The basic index include C_p (Juran, 1974). Statistically, C_p is the result of comparing the based curve with the normal distribution of six sigma. The C_{pk} index was introduced by Kane (1986) to measure one side of the curve.

The first index C_p is called the capability index which signifies the tolerance width divided by the process capability even though the process is at the center.

Table 1. PCIs and grading descriptio

Capability value	Grading
<1	Inadequate
$1 \le C_{pk} < 1.33$	Capable
$1.33 \le C_{pk} < 1.5$	Satisfactory
$1.50 \le C_{pk} < 2.00$	Excellent
≥ 2	Super

According Juran (1974), the C_p index is obtained using Equation (1).

$$C_p = \frac{USL - LSL}{6\sigma} \tag{1}$$

Where, *LSL* and *USL* are the lower and upper tolerance limits respectively. The relationship between capability process index and sigma level can be described using Equation (2).

$$\text{Sigma levels } \sigma_L = 3 \times C_p \tag{2}$$

In Equation (1) does not allow checking whether the process is centered (which is desirable). This index was proposed to evaluate the overall variations of a process with respect to the tolerance limits and indicate the Potential performance of the process. The index C_{pk} represents a process with poor proximity on mean and small variability. Thus, multiple indices can be used to integrate a target to measure the process capability. According Kane (1986) these indices are presented as follows:

$$C_{pk} = \min\left[\frac{(\overline{\overline{X}} - LSL)}{3\sigma}, \frac{(USL - \overline{\overline{X}})}{3\sigma}\right] = \min\ [C_{pu}, C_{pl}] \tag{3}$$

Indices C_p, C_{pk} are employed to deal with two sided normally distributed process. Moreover, indices C_{pu} and C_{pl} are intended precisely for one sided processes, where the guideline for traditional indices PCIs annotated in Table No (1) and Table (2) shows the capability process index and sigma level with regard to the impact on the advanced sigma levels on both industrial and service operations (Ali & Ahmed, 2017).

Table 2. PCIs with sigma levels

Capability value index C_{pk}	Sigma level
0.50	1.5
0.67	2.0
0.83	2.5
1.00	3.0
1.17	3.5
1.33	4.0
1.50	4.5
1.67	5.0
1.83	5.5
2.00	6.0

Estimating sigma is a crucial aspect and represents the basis for statistical study of process capability. Capability indices estimated from sample statistics are subject to statistical variability; consequently, this variability affects the estimated indices(Ali & Ahmed, 2017). There several models can be used to estimate the Sigma estimation (standard deviation). These models can be categorized as follows:

$$\left.\begin{aligned} \hat{C}_{pl} = \frac{(\overline{X} - L)}{3\hat{\sigma}_R}, \hat{C}_{pu} = \frac{(U - \overline{X})}{3\hat{\sigma}_R}, \\ \hat{C}_{pk} = \min\ (\hat{C}_{Pl}, \hat{C}_{pu}) \end{aligned}\right\} \tag{4}$$

Where $R = \max(x) - \min(x)$ $\hat{\sigma}_R = \overline{R} / d_2(n)$ is one of the ways to estimated Standard deviation using control charts, $\overline{\overline{R}} = \sum_{i=1}^{m} \overline{R} / d_2(n)$ is the average of the sample ranges as; d_2 is control chart constant values formulated based on the sample size *n*. $\hat{\sigma}_R$ is used to estimate standard deviation

$$\left.\begin{aligned}\hat{C}_{pl} &= \frac{(\overline{X}-L)}{3\hat{\sigma}_S},\ \hat{C}_{pu} = \frac{(U-\overline{X})}{3\hat{\sigma}_S},\\ \hat{C}_{pk} &= \min(\hat{C}_{Pl},\hat{C}_{pu})\end{aligned}\right\} \quad (5)$$

Where $\hat{\sigma}_s = \bar{S}/C_4(n)$ is one of the ways to estimated standard deviation using control charts, and $\bar{S} = \sum_{i=1}^{m} \bar{S} / N$, $S_i = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}\left(x_{ij}-\bar{x}_i\right)^2}$ and C_4 control chart constant values formulated based on the sample size n and

$$\left.\begin{aligned}\hat{C}_{pl} &= \frac{(\overline{X}-L)}{3\hat{\sigma}_{S_i}},\ \hat{C}_{pu} = \frac{(U-\overline{X})}{3\hat{\sigma}_{S_i}},\\ \hat{C}_{pk} &= \min(\hat{C}_{Pl},\hat{C}_{pu})\end{aligned}\right\} \quad (6)$$

Where σ_{Si} is used for pooled sigma estimation. Here, $V = \left(\sum_{i=1}^{m} n\right) - m + 1$, and $S_i = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}\left(x_{ij}-\bar{x}_i\right)^2}$ are unbiased (Luko, 1996).

$$\left.\begin{aligned}\hat{C}_{pl} &= \frac{(\overline{X}-L)}{3\hat{\sigma}_{w_i}},\hat{C}_{pu} = \frac{(U-\overline{X})}{3\hat{\sigma}_{w_i}},\\ \hat{C}_{pk} &= \min(\hat{C}_{Pl},\hat{C}_{pu})\end{aligned}\right\} \quad (7)$$

Where $\hat{\sigma}_{wi}$ is also known as a minimum variance linear unbiased estimator (MVLUE). This estimator is the weighted average of (N) unbiased estimates of $\hat{\sigma}$ in the form $R / d_{2(n)}$ and $w_i = \left(d_{2(n)}\right)^2 \Big/ 1-\left(d_{2(n)}\right)^2$, and is intended for situations with varied pooled sample sizes .

$$\left.\begin{aligned}C_{pl} &= \frac{(\overline{X}-L)}{3\hat{\sigma}_{h_i}},C_{pu} = \frac{(U-\overline{X})}{3\hat{\sigma}_{h_i}},\\ C_{pk} &= \min(C_{Pl},C_{pu})\end{aligned}\right\} \quad (8)$$

MVLUE method is based on the subgroup standard $\hat{\sigma}_{h_i}$ this estimate is a weighted average of N unbiased estimates of σ of the form s_i / C_4 where $h_i = \dfrac{\left[C_{4(n_i)}\right]^2}{1-\left[C_{4(n_i)}\right]^2}$.

3. PROPOSE PROCESS ACTUAL YIELD INDEX BASED ON 6Σ

Since the μ and σ are not sufficient to evaluate the performance of industrial process, Juran, (1974) combined product specifications as well as the process parameters and brought up the idea of PCIs. Similarly, Kane V, (1986) introduced the C_{pk} index for measuring the actual process capability. C_{pk} gives the mean of the process some influences on the overall capability of the process and it is expressed as follows:

$$\begin{aligned}C_{pk} &= \min\left[\frac{(\overline{\overline{X}} - LSL)}{3\sigma},\ \frac{(USL - \overline{\overline{X}})}{3\sigma}\right]\\ &= \min\ [C_{pu}, C_{pl}]\end{aligned} \quad (9)$$

Where U and L are the upper and lower specification limits, respectively, σ is the process standard deviation and μ is the process mean, and. Also, the index C_{pk} can be defined as follows:

$$\begin{aligned}C_{pk} &= C_p\left[1 - \frac{|\mu - T|}{U - L / 2}\right]\\ &= \frac{U-L}{6\sigma}\left[1 - \frac{|\mu - T|}{U - L / 2}\right]\end{aligned} \quad (10)$$

Since it provides bound yield process for normal distributed processes, the index C_{pk} is considered as a yield index. For process with two-sided tolerance limits, the yield process can be obtained as $\%Y = F(U) - F(L)$ where $F(.)$ is CDF of the process characteristic. Alternatively, for normally distributed characteristics the yield process can be calculated as $\%\ Yield = Y = \Phi(U - \mu)/\sigma\ ) - \Phi(\mu - L / \sigma)$. Where $F(.)$ is the CDF of the standard normal distribution. Based on the above the $L\sigma$ values of C_{pl}, C_{pu} and C_{pk} which shift by as much as 1.5σ can be computed as follows:

$$\begin{aligned}C_{pu} &= \frac{U-\mu}{3\sigma} = \frac{6\sigma - 1.5\sigma}{3\sigma} = 1.5,\\ C_{pl} &= \frac{\mu - L}{3\sigma} = \frac{6\sigma - 1.5\sigma}{3\sigma} = 1.5,\\ C_{pk} &= \frac{d - |\mu - T|}{3\sigma} = \frac{6\sigma - 1.5\sigma}{3\sigma} = 1.5\end{aligned} \quad (11)$$

If the values of C_{pl}, C_{pu} and C_{pk} can be more than 1.5 when the mean process shifts by as much as 1.5σ off the target, the process still reach the standard of 6σ. In this case, a process is at K_c level sigma process if specification limits interval is two times of $K\sigma$. When a 1.5σ shift is introduced into the calculation, the C_{pk} based on Six Sigma assuming that.

$U - L = D = D_u + D_l = 2k\,\sigma$ and $|\mu - T| = \delta\sigma$ This means $U - L = 2K\sigma$ and that the δ is equal to a constant value which is 1.5, thus based on Equation (10), C_{pk} will become as follows:

$$SSC_{pk} = \frac{2k_c\sigma}{6\sigma}(1-\frac{2\delta\sigma}{2k_c\sigma}) = = \frac{k_c-\delta}{3} = \frac{k_c}{3} - 0.5 \tag{12}$$

Where $K_C = \min\left(\frac{1-\delta}{\lambda}+1.5, \frac{1+\delta}{\lambda}+1.5\right)$

$\hat{\delta} = (\mu - T)/d$ and $\lambda = \sigma/d$, Hence, based on Six Sigma concept, if the level sigma = 6 then the value $SSC_{pk} = 1.5$ And if the level sigma = 3 and the mean shift of target $\delta = 1.5$, then the value of $SSC_{pk} = 0.5$ Apart from this, when $SSC_{pk} > 1.5$ the $L\sigma > 6$ Note that, $|\mu - T| = \delta\sigma(\delta \geq 0)$ however this study assumes that δ has a fixed value which is 1.5. Besides that, the δ can be estimated from process centering μ as follows:

$L_i + \sqrt{X_{v,\alpha}}.\sigma \leq \mu_i \leq U_i - K\sqrt{X_{v,\alpha}}.\sigma$ Based on six sigma $T - 1.5\sigma \leq \hat{\mu} \leq T + 1.5\sigma.$

$$\begin{aligned} &-D_l + K\sqrt{X_{v,\alpha}}.\sigma \leq \delta \leq D_u - K\sqrt{X_{v,\alpha}}.\sigma \\ &or \quad \delta = D * x_1 - \frac{C_{pk}}{2C_p} \\ &\text{where } D_u = x_1 D \ , \ D_l = x_2 D \ , \\ &x_1 = \frac{\delta}{D} + \frac{C_{pk}}{2C_p} \text{ or } x_1 = mix\frac{D_u}{D}, \frac{D_l}{D} \end{aligned} \tag{13}$$

According Kuen Chen & Chang (2017) Yield with assuming Motorola and the idea of Six Sigma can be calculated as follows:

$$Y = \int_{L*}^{U*} N(\hat{\mu}, \hat{\sigma}^2)dx \tag{14}$$

Where $U^* = U + 1.5*\sigma = T + d + 1.5\sigma$, and $L^* = L - 1.5\sigma = T - d - 1.5\sigma$ Then, according the Equations (10) and (12) there is a correspondence between SSC_{pk} index and process yield for δ values $|\mu - T|$. the estimation, the process yield can be calculated while taking into consideration sigma level and the assumptions of the shift of process mean from the target by 1.5 sigma as follows:

$$SSY = \Phi(1.5 + L\sigma) - \Phi(1.5 - L\sigma) \tag{15}$$

On the assumption that $L\sigma = K_c$ here in the light of Equations (11), (14) and (15) the Sigma process level SPL can be calculated as follows:

$$\left.\begin{aligned} &\hat{\delta}' = 1.5/k_c \text{ and } \lambda = 1/k_c \text{,thus} \\ &SPL_{pu} = \frac{1-(1.5/k_c)}{1/k_c} + 1.5 = k_c \\ &\hat{\delta}' = -1.5/k_c \text{ and } \lambda = 1/k_c \text{ thus} \\ &SPL_{pl} = \frac{1-(1.5/k_c)}{1/k_c} + 1.5 = k_c \\ &= SPL_{pk_c} = \min(SPL_{pu}, SPL_{pl}) = k_c \end{aligned}\right\} \tag{16}$$

Where the $SPL_{pu} = \frac{1-\delta'}{\lambda} + 1.5$ and

$SPL_{pl} = \frac{1+\delta'}{\lambda} + 1.5$ thus

$SPL_{pk_c} = \min\left(\frac{1-\delta'}{\lambda}+1.5, \frac{1+\delta'}{\lambda}+1.5\right)$ On the assumption that $L\sigma = k_c$ when the process is at k_c Sigma process level, and a 1.5σ is introduced the process yield can be further calculated with estimated and based on Equation (14) the process yield can be further calculated with estimated specifications limits as follows:

$$SSY = \int_{L-1.5\hat{\sigma}}^{U+1.5\hat{\sigma}} N(\hat{\mu}, \hat{\sigma})dx = = \Phi\ (1.5 + k_C) - \Phi\ (1.5 - k_C) \tag{17}$$

Way based on $SS\hat{C}_{pk}$ can be expressed of the yield process as following:

$$2\Phi(3 * SSC_{pk}) - 1 \leq SS\hat{Y} \leq \Phi(3 * SSC_{pk}) \tag{18}$$

Here $SS\hat{C}_{pk}$ index or $L\sigma$ and yield process SSY have one to one relationship when $\delta = 1.5.$ In light of value $SS\hat{C}_{pk}$ index there is a guide line to interpreting the results of this index and the yield process, for instance, if $SS\hat{C}_{pk} = 0.5$ that means the process is capable and guarantees that the level sigma equals there sigma $L\sigma = 3$ and when the $SS\hat{C}_{pk} = 1.5$ that means is process super the level sigma equals there sigma $L\sigma = 3$ and assurances that the yield process will be not less than 0.999996602268 equivalently not more than 3.5 DPMO.

4. IMPLEMENTATION AND RESULTS, DISCUSSIONS IN ADEN REFINERY OF OIL

Measurement is an essential goal for implementing Six Sigma. Six Sigma can be successfully implemented using statistical tools and methods and focusing on the following themes.

4.1 Measurement Evaluation of Current Performance in Aden Refinery

Measuring the current performance is the first step toward determining the process performance status of any industry. There are many indicators through which the current process performance can be revealed. Most of these indicators are subjected to a variety of estimation methods which leads to different outcomes. Therefore, it is vital to utilize the appropriate estimation methods and measurement tools for assessing process performance. This is basically the major attention for many studies and this study aims to develop precise indicators for measuring and evaluating process performance in industries. Hence, this study presents a case study to measure and evaluate the process performance of oil refinery in Yemen.

4.1.1 Data Acquisition and Collection

The octane number characteristic of oil is very important, the octane number of petroleum represents the degree of explosion (combustion) in the machines and it has various impacts on the overall quality of the oil. For instance, if the octane number is lower than 90 this causes instability of consumption and leads to increasing the temperature of the engine and affects the speeds of cars. Usually, during the distillation process, the octane number is low due to the presence of paraffins and aromatic hydrocarbons (naphthalence) and etc. which leads to instability of oil against the fugitive (its explosive stability). This phenomenon can be modified by adding materials such as the lead material to the liquid or by mixing the liquid with lower or higher-octane number. For this purpose, the quality control of oil petroleum users in Aden refinery is constantly testing the oil to ensure that it is produced according to the specification limits and the international standards. The accepted octane number values should be between the upper and lower specification limits which are 100-90 respectively (Aden Refinery 2016). To acquire the relative data of octane number, this procedure is considered: At first, a sample of oil is taken randomly using (hydrometer) from the oil tanks (the big tanks) at three distinctive locations which are the upper, the middle and the bottom sections of the tank. The sample is then mixed together because the values of the octane number vary at every location of the tank. After mixing the sample, it is then taken to the laboratory to be tested to obtain the characteristic octane number, 50 samples were acquired, each consisting of four items, from the product in even intervals (every 8 h) after the random data was collected to 200 size samples has been done important statistical tests associated with the validation of the data for further analysis. It comprises of basic statistical tests which are normality test, stationary test, autocorrelation and heteroscedasticity tests (autoregressive model) and the process capability test.
The results of the tests of the gasoline characteristic octane number is normality and the data of octane number stationary where results conclude that the tested series do not have a unit root. Also, the data of octane number does not have autoregressive and the results indicate that the process is capable but at lower levels, therefore, the results concluded that normality, stationary, not have autoregressive and capable that mean on the tested one octane number characteristic are statistically reliable for further analysis we implementation the process actual yield proposed index SSC_{pk} based on six sigma concept for octane number characteristic of oil based on the shift of process mean from the target by 1.5σ. The upper and lower specification limits of those characteristic are (90,100) respectively.

4.1.2 Process Capability Estimation

Using equations numbers (1) to (8) we can evaluating the current performance in oil refineries by using the traditional indices and for octane number characteristic where the estimation for the characteristic of octane number based on sigma estimation as show in Table (3).

4.1.3 Effect Of Sigma Estimation On Relationship Between Lσ Levels Sigma And PCIs

Sigma estimation (standard deviation) is a crucial topic in the field of statistical control and process capability. This is due to fact that the estimation of capability process indices relies on the estimation of process variability. As a result, process variation impacts the process capability indices PCIs and leads to incapable process. At the same time, it affects levels of sigma process calculated indirectly from the outcome of PCIs. Six sigma is important to minimize defects and error range and increase the product quality. Six sigma originates from process capability, which is a statistical measure used to ensure that the manufacturing process meets the predetermined specification limit (Gupta, 2015; Şenvar & Tozan, 2010). PCIs are indicators or measures employed for evaluating the capability of the process for producing products according to predetermined tolerance limits and to achieve excellent quality (Montgomery, 2009). Process capability can be defined as a function of the process variations $(i.e.\ 6\sigma)$. Statistically, PCIs comprise of many indices. At the present, the most widely applied PCI s are unilateral specification indices C_{pu}, C_{pl} and bilateral specification indices C_p and C_{pk}. The basic index include C_p (Juran, 1974). Statistically, C_p is the result of comparing the based curve with the normal distribution of six sigma. The C_{pk}. index was introduced by Kane (1986) to measure one side of the curve.

The first index C_p is called the capability index which signifies the tolerance width divided by the process capability even though the process is at the center.

According Juran (1974), the C_p index is obtained using Equation (1). Restrict figures to single-column width unless this would make them illegible. If necessary for the purpose of clarity they can be spread over both columns. Figures, numbered consecutively with captions, should be incorporated into the main body of the text. Place the centered figures after they are mentioned in the main text (Figure 1). may span both columns (Table 2).

Table 3. The obtained PCIs with different ways for estimating S.D of oil octane characteristic

Proposed indices	$\hat{\sigma}_{LT}$	$\hat{\sigma}_{R}$	$\hat{\sigma}_{S}$	$\hat{\sigma}_{Si}$	$\hat{\sigma}_{wi}$	$\hat{\sigma}_{hi}$
$\hat{C}_p$	0.5385	0.9107396	0.906191	0.963248	0.910741	0.906191
$\hat{C}_{pk}$	0.3547	0.592313	0.589317	0.59690	0.5923131	0.589317
S.D	3.0946	1.8300146	1.8391998	1.730257	1.800146	1.83998

Besides that, the value of $\hat{C}_p, \hat{C}_{pk}$ have changed dramatically with different ways to estimate the standard deviation $(\hat{\sigma}_{LT}), (R), (S), (S_i), (w_i)$ and (h_i). Thus, it can be clearly observed that different estimation of process standard deviation has influenced the values of C_p, C_{pk} In addition, sigma level for octane characteristic can be obtained using the relationship between capability process and sigma level as the following:

$$L\sigma = 3 * C_p = 0.96324 * 3 = 2.8896$$
$$L\sigma = 3 * C_p = 0.53 * 3 = 1.59 \tag{19}$$

The displayed results are based on process capability indices namely C_p, C_{pk}. It can be observed that the process performance of oil gasoline refinery for octane characteristic does not conform to the predefined specifications. This observation is based on the C_p, C_{pk} values as shown in Table 3. For example, the value of C_p for long term is 0.538564, which is less than 1. In line with this is the value of C_{pk} which is less than 1 for all estimation. The level of sigma process very weak through estimation of indirect way and based on the assumption $\mu = T$.

4.2 Evaluation and Measurement of Six Sigma Process

This section elaborates the evaluation and measurement of the potential and actual yield process indices based on Six Sigma concept, standard deviation and magnitude of variation coefficient δ as well as the shift of process mean from the target by 1.5 sigma. More specifically in this section, indexes process yield for different probability functions PDF is evaluated using the proposed indices SSC_{pk} and SPL important process capability indices namely, the actual yield based on six sigma concept. To demonstrate the applicability of the new tools we presented a real-world case in Aden refinery of oil in Yemen using the most important characteristic of petroleum namely, octane number, as flowing.

4.2.1 Process Actual Yield SSC_{pk} Based Six Sigma

The actual yield index for octane number characteristic, based on the shift of process mean from the target by 1.5σ can be calculated using the extended SSC_{pk} in Equation (12) as follows:

$$SSC_{pk} = \frac{2K_C\sigma}{6\sigma}(1 - \frac{2\delta\sigma}{2K_C\sigma}) =$$
$$= \frac{K_C - \delta}{3} = \frac{K_C}{3} - 0.50 = \tag{20}$$
$$= \frac{3.415 - 1.5}{3} = \frac{3.415}{3} - 0.50 = 0.636$$

Where

$$K_c = \min\left(\frac{1-\delta}{\sigma / d} + 1.5, \frac{1+\delta}{\sigma / d} + 1.5\right) =$$
$$= \left(\frac{1-0.34}{0.346} + 1.5, \frac{1+0.34}{0.346} + 1.5\right) =$$
$$= (3.415,\ 5.3731)$$

In SSC_{pk} calculation, the sigma process level can be identified based on the extended SPL_{pk} using Equation (16) as follows:

$$\hat{\delta} = 1.5 / 3.415 = 0.439$$
$$\lambda = 1 / 3.43529 = 0.2928 thus$$
$$SPL_{pu(k_c)} = \frac{1 - (1.5 / 3.415)}{0.2928} + 1.5 =$$
$$= \frac{0.5607}{0.2928} + 1.5 = 3.415,$$

$$SPL_{pl(k_c)} = \frac{1 - (1.5 / 3.415)}{1 / k_c} + 1.5$$
$$= \frac{0.636191}{0.2928} + 1.5 = 3.415$$
$$SPL_{PK(k_c)} = \min\ (SPL_{pu(k_c)}, SPL_{pl(k_c)})$$
$$= \min = (3.415,\ 3.415\ ) \tag{21}$$

Based on Equation (21) it is possible to calculate the sigma level directly. Apart from this, the process actual yield for the octane number of oil, can be further calculated based on the extended SSY index using Equation (15) as follows:

i. For octane number characteristic, the actual process yield is:

$$SSY = \Phi(1.5 + L\sigma) - \Phi(1.5 - L\sigma) = \Phi(1.5 + 3.415) - \Phi(1.5 - 3.415) = 0.97542570 \quad (21)$$

Likewise the yield process can be expressed based on Equation (15) as follows:

$$\begin{aligned} &2\Phi(3 * SSC_{pk}) - 1 \le SSY \le \Phi(3 * SSC_{pk}) \\ &2\Phi(3 * 0.633) - 1 \le SSY \le \Phi(3 * 0.6363) \\ &0.99463075 \le SSY \le 0.97542570 \end{aligned} \quad (22)$$

Note, the using Equation (22) and Equation (23), the same results of yield process estimation can be obtained based on the idea of Six Sigma when $\delta = 1.5,$. Thus, there is direct correspondence one to one relationship between $L\sigma, SSC_{pk}$ and yield process SSY when $\delta = 1.5,$. For instance, if $SSC_{pk} = 0.5,$ then the yield process is approximately 0.9973000 which means the process is capable and the level of sigma is equals 3, $L\sigma = 3.$ In addition, if the $SSC_{pk} = 1.5,$ then the process is super and that the yield process is approximately 0.999996602268 which is equivalent to not more than 3.4 DPMO. In this regard, for octane number characteristic the $SSC_{pk} = 0.636$, which means the process is Satisfactory and the level of sigma is not less than 4.

As explained previously using the Equations (15) and (18) there are two different ways for estimating the process yield SSY based on Six Sigma, and the rate of the process yield for Octane number characteristic.

According to the above the, it must be noted that in the new way of estimation, the process yield index SSC_{pk} was calculated while taking into consideration the assumptions of the shift of process mean from the target by 1.5 sigma that leads to better results of the traditional index where the value index for $SSC_{pk} = 0.6363$ while the values of C_{pk} Shows in table (3) it's between 0.3547 and 0.604491 through different ways for estimating. The guideline for C_{pk} annotated in Tables (1) and (2) while the index SSC_{pk} provide a guide to interpretate the output of the process yield index and is explained in following Table 4.

Table 4. SSC_{pk} grading description with sigma levels

yield index value	Grading	Sigma levels
$SSC_{pk} < 0.5$	Inadequate	SL<2.5
$0.5 \le SSC_{pk} < 0.833$	Capable	$3 \le$ SL< 4
$0.833 \le SSC_{pk} < 1.17$	Satisfactory	$4 \le$ SL< 5
$1.17 \le SSC_{pk} < 1.5$	Excellent	$5 \le$ SL < 6
$SSC_{pk} \ge 1.5$	Super	SL $\ge$ 6

Hence, based on Six Sigma concept, if the level sigma = 6 and δ =1.5 then the value $SSC_{pk} = 1.5$ as well as, there is direct correspondence or one to one relationship between SSC_{pk} index, $L\sigma$ and yield process SSY.

5. CONCLUSION

Six-sigma is a quality improvement tool, in which the values of the indices indicate the level of sigma achieved for a given quality characteristic. Quality control personnel and engineers can utilize the indices to determine sigma process levels. The most popular yield-based index C_{pk} for processes has been investigated intensively but was comparatively neglected for processes with idea six sigma this paper, suggest the effectiveness yield measure index based on six sigma concept SSC_{pk} and provided statistical procedures as a important tool for decision making on the process performance. To demonstrate the applicability of the new tool, we presented a real-world case in Aden refinery using characteristic octane number of oil, after the random data was collected to 200 size samples has been done important statistical tests associated are the limitations in research to ensure an effective assessment of process quality. The results of statistical tests for the octane number characteristic concluded that the data normality, stationary, do not have autoregressive and stable process. The main returns of this study indicated that the SS SSC_{pk} index outperformed previous index C_{pk} and provided better results and sensitivity for measuring process performance in industries. The tools established in this study is a practical method to assess process yield and could provide a reference for engineers in manufacturing or achieving quality control for the evaluation of yield processes and level of quality.

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