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Optimal nonlinear filtering of stochastic processes in rescue radar

Subject and Purpose. *Smoke, fog, avalanches, debris of collapsed structures and other optically opaque obstacles in both natural and man-made disasters make optical sensors useless for detecting victims. Electromagnetic waves of the decimeter range penetrate well almost all obstacles, reflect from the trapped people and return to the radar receiver. Due to the breathing and heartbeat, the human-reflected sounding signals get the Doppler phase modulation, which is an information signal. These information signals and their properties provide the subject matter for the present research with the aim to create optimal methods and algorithms of random event processing for the prompt location of survivors by rescuers.*

Method and Methodology. *The method of stochastic analysis of the fluctuation Doppler spectra of reflected sounding signals shows that the information signals have properties of conditional Markov processes.*

Results. *The problem of optimal nonlinear filtering of conditional Markov processes entering the radar signal processing unit has been examined closely. An optimal adaptive filter has been proposed to reduce the masking effect of interferences caused by non-stationary noises and sounding signal reflections from stationary objects. The optimality criterion is the minimum mean square error function whose current value is evaluated in real time during the filtering process as the statistics is accumulated. The filter coefficients are calculated by the recurrent, steepest descent algorithm. The real-time work is carried out through the use of fast Fourier transform algorithms.*

Conclusion. *The structure of the optimal adaptive filter to be built into the radar signal processing unit has been developed. Real radar signals have shown that the optimal filtering during the signal processing in systems designed for detecting live people by their breathing and heartbeat facilitates the interpretation of the observed signals. Some spectra of real signals generated by human breathing and heartbeat are presented. Fig. 3. Ref.: 13 items.*

Key words: *stochastic process, optimal filter, algorithm, sounding signals, noise, Doppler shift, digital signal processing, radar, spectral function, sampling frequency, conditional Markov processes, mean square error criterion.*

Rescue radars [1, 2] are used to search injured people in various man-made and natural disasters. Such devices can be built on a variety of technical principles and use video pulse [3—5] or continuous [6—8] signals. The main advantages of video pulse radars are their high range resolution and a relative simplicity of the design. However, reliable estimates of Doppler responses are difficult to obtain with this method of sounding. The only information about people trapped under debris is Doppler fluctuations that are caused in the phase of the reflected signal by the chest movements while breathing and heartbeat. These ultra-low-frequency Doppler fluctuations generated in the phase of the reflected signal by the processes of breathing and heartbeat of human beings are in the frequency range 0.1...1.2 Hz. Reliable estimates of these processes are possible to obtain by using continuous signals of sufficient duration. At the same time, a required range resolution is achieved using a pseudo-random phase shift manipulation of the sounding signal [9, 10].

In general, Doppler fluctuations in the phase of the reflected signal can be considered as a stochastic periodically correlated process whose detection and identification in the environment of noise and interference is possible with the help of adaptive filters. Because of nonstationarity of the information process and *a priori* ambiguity of the probability density function of noise, the correlation-extreme method of signal processing can be ineffective. Therefore, it seems expedient to construct an algorithm for the output radar signal filtering in such a way that the filter changes its characteristics as information on the observed signal and noise is accumulated. The immediate goal of the paper is to solve the problem of optimal non-linear radar signal filtering by the conditional Markov approach using the minimum mean square error criterion. A creation of a generalized method of radar signal filtering in an effort to improve the detection of sufferers under optically opaque obstacles is in sight.

1. Generalized mathematical formulation of the problem. To solve the problem of nonlinear filtering in terms of the theory of conditional Markov processes one needs to solve the generalized stochastic differential equation [11] for the posterior probability density $P(\vec{X};t)$ in the form

$$\dot{P}(\vec{X};t) = 0.5\sum_{i=1}^{n}\sum_{j=1}^{n}\left[\frac{\partial^2}{\partial x_i \partial x_j}\left[B_{ij}(\vec{X})P(\vec{X};t)\right] + \left[C(\vec{X}) - E\{C(\vec{X})\}\right]P(\vec{X};t)\right] - \sum_{i=1}^{n}\frac{\partial}{\partial x_i}\left[A_i(\vec{X})P(\vec{X};t)\right], \quad (1)$$

where $A(\vec{X},t), B(\vec{X},t), C(\vec{X},t)$ are the priory defined functions of time t and of the coordinate vector $\vec{X} = (x_1, x_2, ..., x_n)$ specifying a point in an n-sized space, $E\{\cdot\}$ is the expectation symbol.

A rigorous solution to (1) can be obtained in Gaussian approximation as in [11].

Unfortunately, the well-known method in which the filtration equation is substituted with a chain of partial differential equations for posterior probability density parameters, the so-called non-Gaussian solution, is unrealizable in this case. However, the main specific non-linear filtering effects, such as signal cut-off, non-linear gain, etc., clearly point to the non-Gaussian case. Nevertheless, a rigorous solution of equation (1) is still possible under certain limiting assumptions. Specifically, they are: (a) the elements of matrix $B(\vec{X},t) = \vec{B}(\vec{X})$ do not depend on time t and (b) the function $A(\vec{X},t)$ linearly depends on $\vec{X}$ like $A_i(\vec{X}) = A_i + A_{ij}x_j$.

If conditions (a) and (b) are true, the Fourier transform of (1) can be obtained as an equation for the characteristic function $G(\omega;t)$ of the posterior process. Namely,

$$\dot{G}(\omega;t) = \left[j\sum_{k=1}^{n}A_i\omega_i - 0,5\sum_{k=1}^{n}\sum_{m=1}^{n}B_{km}\omega_k\omega_m\right] \times G(\omega;t) + \sum_{k=1}^{n}\sum_{m}^{n}A_{mn}\omega_k\frac{\partial G(\omega;t)}{\partial\omega_m} + c(\omega;t), \quad (2)$$

where j is the imaginary unit and $c(\omega;t) = E\left\{\left[C(\vec{X}) - E\{C(\vec{X})\}\right]\exp\left[j\sum_{k=1}^{n}x_k\omega_k\right]\right\}$.

The substitution of the Gaussian model

$$P^{(1)}(\vec{X};t) = N\exp\left\{-0.5\sum_{k=1}^{n}\sum_{m=1}^{n}A_{km}(x_k - x_k^0)(x_m - x_m^0)\right\}$$

in equation (1) yields the first-order approximation equation

$$\dot{x}_k^0 = A_k + \sum_{m=1}^{n}A_{km}x_m^0 + \sum_{m=1}^{n}K_{km}^{(1)}\frac{\partial C(x^0)}{\partial x_m}, \quad k = 1, 2, ..., n, \quad (3)$$

where $K_{km}^{(1)}$ are the priori process cumulants [12, 13] calculated as

$$\dot{K}_{km}^{(1)} = \sum_{i=1}^{n}(A_{ki}K_{im}^{(1)} + A_{mi}K_{ik}^{(1)}) + B_{km} + \sum_{i=1}^{n}\sum_{l=1}^{n}K_{ki}^{(1)}K_{lm}^{(1)}\frac{\partial^2 C(x^0)}{\partial x_i \partial x_l}. \quad (4)$$

Calculate the function $c^{(1)}(\omega;t)$ in the first approximation

$$c^{(1)}(\omega;t) = \int_{-\infty}^{\infty}...\int_{-\infty}^{\infty}\left[C(x) - E\{C(x)\}\right]P^{(1)}(x;t) \times \exp\left\{j\sum_{k=1}^{n}x_k\omega_k\right\}d\vec{X}, \quad (5)$$

and obtain the characteristic function in the second approximation

$$\dot{G}(\omega;t) = c^{(0)}(\omega;t)G(\omega;t) + \sum_{m=1}^{n}\left[\sum_{n=1}^{n}A_{nm}\omega_n\right]\frac{\partial G(\omega;t)}{\partial\omega_m} + c^{(1)}(\omega;t), \quad (6)$$

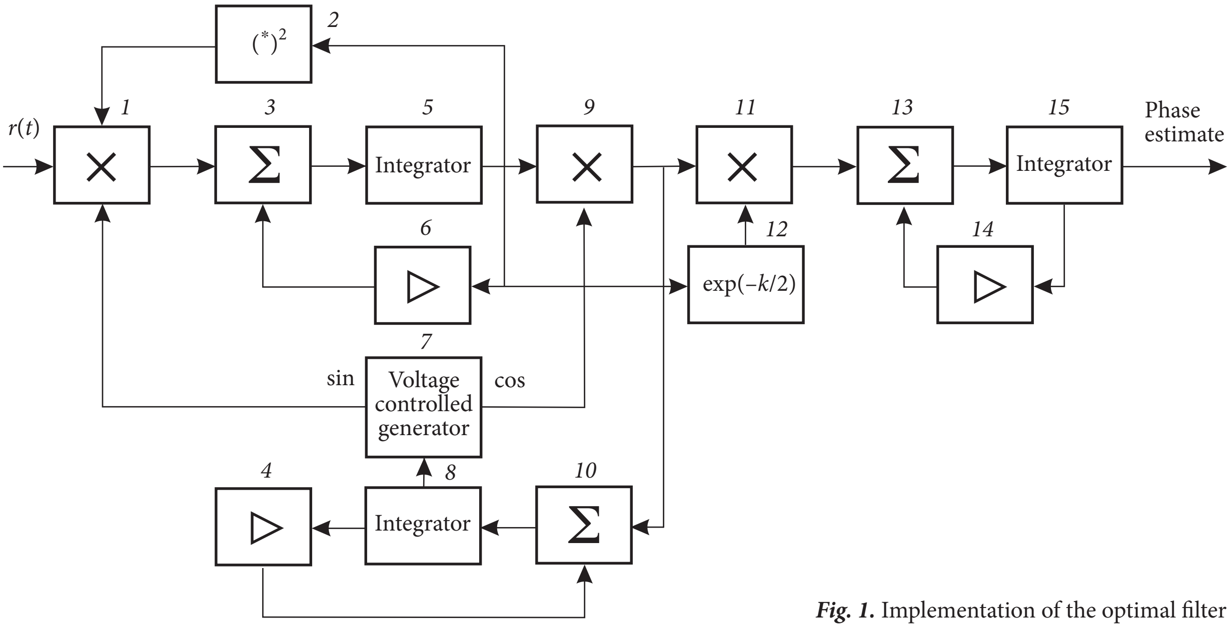

Fig. 1. Implementation of the optimal filter

where

$$c^{(0)}(\omega;t) = j\sum_{k=1}^{n} A_k\omega_k - 0.5\sum_{k=1}^{n}\sum_{m=1}^{n} B_{km}\omega_k\omega_m.$$

To solve the problem of integration, linear inhomogeneous first-order partial differential equation (6) is reduced to the characteristic equations

$$\dot{\omega}_m = -\sum_{k=1}^{n} A_{km}\omega_k; \tag{7}$$

$$\dot{G}(\omega;t) = c^{(0)}(\omega;t)G(\omega;t) + c^{(1)}(\omega;t). \tag{8}$$

Thus, the algorithm for calculating the characteristic function of the posterior process consists in the integration of equations (3) and (4) of the first Gaussian approximation and in the recursive calculation of the filter weights by algorithm (7) and (8).

2. Synthesis of the optimal filtering algorithm. As mentioned above, some kinds of rescue radar use pseudo-random phase-modulated signals [2, 4, 6—8]. For the optimality criterion we take the criterion of the minimum root mean square (*RMS*) error of receiving a phase-modulated signal of the type

$$s(t) = A_0\sin(\omega_0 t + \varphi(t)), \tag{9}$$

which is mixed with the white noise $n(t)$. The noise has zero mean, $E\{n(t)\} = 0$, and its correlation function is the delta function, $E\{n(t)n^*(t-\tau)\} = N\delta(\tau)$, where asterisk (*) stands for the complex conjugate. Thus, the signal that arrives at the processing device is

$$r(t) = s(t) + n(t). \tag{10}$$

The differential equation for the signal phase can be written as follows

$$\dot{\varphi}(t) = -\gamma\varphi(t) + \xi(t), \tag{11}$$

where $E\{\xi(t)\} = 0$ and $E\{\xi(t)\xi^*(t-\tau)\} = \chi\delta(\tau)$, γ and χ are constants.

According to generalized differential equation (1), the equation for the posterior probability density is

$$\dot{P}(\varphi;t) = \gamma\frac{\partial}{\partial\varphi}(\varphi(t)P(\varphi;t)) + \frac{\chi}{2}\frac{\partial^2 P(\varphi;t)}{\partial\varphi^2} + \left[C(\varphi(t)) - E\{C(\varphi(t))\}\right]P(\varphi;t), \tag{12}$$

where

$$C(\varphi(t)) = \frac{A_0}{N}r(t)\sin(\omega_0 t + \varphi(t)).$$

The solution of equation (12) by the general method for solving equations of type (1) yields the following relations

$$\dot{\varphi}(t) = -\gamma\varphi^{(0)}(t) + K(t)\frac{A_0}{N}r(t)\cos(\omega_0 t + \varphi^{(0)}(t)); \tag{13}$$

$$\dot{K} = -2\chi K + \chi + K^2 \frac{A_0}{N} r(t) \sin(\omega_0 t + \varphi^{(0)}(t)); \quad (14)$$

$$E\{C(\varphi(t))\} = \frac{1}{\sqrt{2\pi K}} \frac{A_0}{N} r(t) \times$$
$$\times \int_{-\infty}^{\infty} \sin(\omega_0 t + \varphi(t)) \exp\left\{ -\frac{(\varphi(t) - \varphi^{(0)}(t))^2}{2K} \right\} d\varphi =$$
$$= e^{-K/2} \frac{A_0}{N} r(t) \sin(\omega_0 t + \varphi^{(0)}); \quad (15)$$

$$E\{\varphi(t) C(\varphi(t))\} = \frac{1}{\sqrt{2\pi K}} \frac{A_0}{N} r(t) \times$$
$$\times \int_{-\infty}^{\infty} \varphi(t) \sin(\omega_0 t + \varphi(t)) \times$$
$$\times \exp\left\{ -\frac{(\varphi(t) - \varphi^{(0)}(t))^2}{2K} \right\} d\varphi =$$
$$= e^{-K/2} \frac{A_0}{N} r(t) \cos(\omega_0 t + \varphi^{(0)}) \times$$
$$\times e^{-K/2} \frac{A_0}{N} \varphi^{(0)} r(t) \sin(\omega_0 t + \varphi^{(0)}). \quad (16)$$

The solution of equations (13)—(16) specifies equation (3) for the observed signal phase

$$\dot{\varphi}^{(0)} = -\gamma \varphi^{(0)} + K e^{-K/2} \frac{A_0}{N} r(t) \cos(\omega_0 t + \varphi^{(0)}). \quad (17)$$

Equations (13), (14), and (17) determine the structure of the filter that is optimal in terms of the minimum mean square error for a phase-modulated signal. The structure of the optimal filter is shown in Fig. 1.

The input signal $r(t)$ is fed to correlator *1*. Signals at the other two inputs of correlator 1 come to correlator *9* from voltage-controlled generator *7* and from block *2*. The circuit consisting of adder *3*, integrator *5* and linear amplifier *6* with gain γ forms the offset for correlator *1*. The inertial circuit involving adder 10, integrator *8* and amplifier *4* eliminates phase jumps caused by the fluctuation component of the input signal. The instantaneous value of the phase estimate of the signal reflected from the target is formed using functional converter *12* at the output of correlator *11*. The inertial circuit consisting of storage device *13*, integrator *15* and amplifier *14* with gain γ forms the output estimate of the signal phase.

As already noted, the information signals have a very low frequency. The Doppler frequencies due

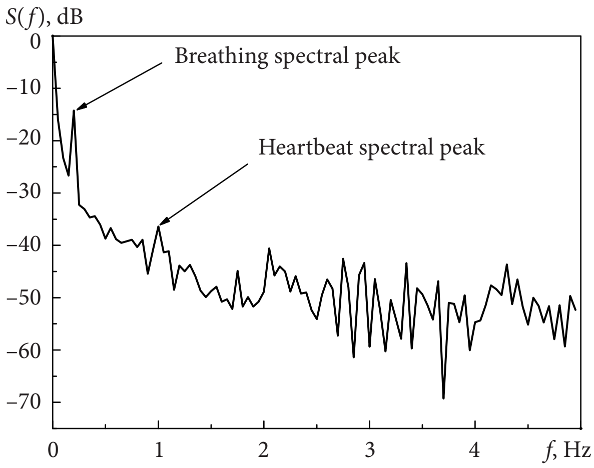

Fig. 2. The signal spectrum of human breathing and heartbeat without filtering

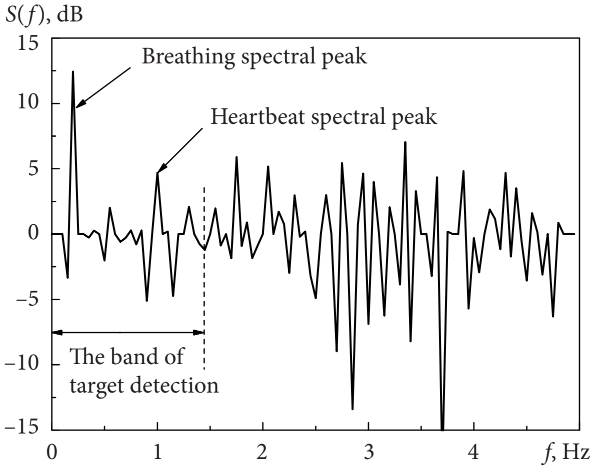

Fig. 3. The signal spectrum from Fig. 2 after the optimal adaptive filtering

to the human breathing are within 0.1...0.25 Hz, and the Doppler frequencies due to the heartbeat are within 0.9...1.1 Hz. So, to get correct expectation estimates, the interval of observations should be longer than 10...20 sec. If the carrier frequency of the Doppler radar is 1.8 GHz with a quasi-continuous phase-code-manipulated signal of $N = 2^{16} - 1$ elements in each sounding sequence, the duration of the sequence (sequence period) is around $T = N \cdot \tau \approx 0.3$ ms at $\tau \approx 5.0$ ns, which corresponds to the range resolution $\delta R \approx 0.75$ m. Thus, the number of accumulated signal samples is approximately 10^5, which makes it possible to obtain statistically acceptable estimates of the filter weights.

A sample of numerical results is shown in Fig. 2. The spectrum of the target signal clearly shows the breathing and heartbeat responses of a human being. Here the obstacle distance is 2 m, the spacing between the target (human being) and the obstacle (a block of clad "half-brick" reinforced concrete with a 150 × 150 mm reinforcing cell) is 0.5 m, and the obstacle is approximately 0.3 m thick. The transmitter radiated power is 100 mW and the receiver sensitivity is –165 dB/W. Notice that the difference between the amplitudes of the spectral components of breathing and heartbeat is more than 20 dB. This is because the trend of the spectral function is the sum of the spectral components of the signal reflected from the target by itself, the signal reflected from non-moving obstacles, the flicker noise and other interfering signals. The signal spectrum in Fig. 3 is the spectrum from Fig. 2 after its optimal adaptive filtering.

Let us compare the spectral functions in Figs. 2 and 3 obtained by the adaptive filtering procedure. The adaptive filter processor calculates the filter coefficients so as to smooth the trend of the spectral function and then subtract it from the measurement result. It gives a useful advantage for the signal analysis in the spectrum band 0.1...1.5 Hz. It is the band where informative spectral components exist when the target gets into the radar responsibility zone.

Conclusion. Thus, the use of the theory of optimal filtering makes it possible to synthesize simple high-speed phenomenological-class algorithms for the analysis of non-stationary information signals. The optimization of the filter performance is carried out by minimizing the *RMS* criterion. The calculation of the efficiency indicator is carried out continuously in real time as the information on the statistics of the observed processes is accumulated. The Fast Fourier Transform algorithm made it possible. The validity of this approach has been verified experimentally on the existing model of Doppler radar for rescuers. In addition, it can be argued that the use of an optimal filter can reduce the influence of interferences caused by the sounding signal reflections from stationary obstacles and by flicker noise effects.

In addition, it can be concluded that the optimal filtering during the signal processing in systems designed to detect live people by their breathing and heartbeat signals improves the interpretation of the observed signals and, importantly, works in real time.

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ОПТИМАЛЬНА НЕЛІНІЙНА ФІЛЬТРАЦІЯ
СТОХАСТИЧНИХ ПРОЦЕСІВ У РАДАРІ ДЛЯ РЯТУВАЛЬНИКІВ

Предмет і мета роботи. Використанню оптичних датчиків для виявлення і рятування людей, які постраждали під час природних та техногенних катастроф, часто заважають оптично непрозорі перешкоди — дим, туман, сніг, завали цегляних та бетонних уламків. Електромагнітні хвилі дециметрового діапазону добре проникають крізь ці перешкоди, відбиваються від тіла постраждалої людини і повертаються до приймача радара. Завдяки процесам дихання та серцебиття фаза відбитого зондувального сигналу отримує доплерівську модуляцію, яка є інформаційним сигналом. Предметом дослідження є інформаційні сигнали та їх властивості. Мета роботи — створення оптимальних методів і алгоритмів оброблення випадкових процесів задля оперативного виявлення та ідентифікації постраждалих людей при рятувальних заходах.

Метод і методологія роботи. Методом стохастичного аналізу флуктуаційних доплерівських спектрів відбитих зондувальних сигналів встановлено, що інформаційні сигнали мають властивості умовних марковських процесів.

Результати роботи. Досліджено проблему оптимальної нелінійної фільтрації умовних марковських процесів у радарі для рятувальників. Для зменшення маскуючого ефекту завад, спричинених нестаціонарними шумами та відбиттями зондувального сигналу від нерухомих об'єктів, запропоновано оптимальний адаптивний фільтр. Як критерій оптимальності використано функцію мінімальної середньої квадратичної помилки, поточне значення якої обчислюється в режимі реального часу у міру накопичення статистики. Коефіцієнти фільтрів обчислюються за рекурентним алгоритмом найскорішого спуску.

Висновок. На реальних радіолокаційних сигналах було показано, що оптимальна фільтрація під час обробки сигналів у системах, призначених для виявлення живих людей за їх диханням та сигналами серцевого ритму, полегшує інтерпретацію спостережуваних сигналів.

Ключові слова: *стохастичний процес, оптимальний фільтр, алгоритм, зондувальні сигнали, шум, доплерівський зсув, цифрове оброблення сигналів, радар, спектральна функція, частота дискретизації, умовні марковські процеси, критерій середньої квадратичної помилки.*