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A METHOD OF APPROXIMATE CALCULATION BY SUBSTITUTING SOME DEFINITE INTEGRALS USING INTERPOLATION POLYNOMIALS

***Abstract**: In this work, the function under the integral was replaced by a higher-level algebraic function for the approximate calculation of some definite integrals, and a system of linear equations was formed. In doing so, more emphasis is placed on the use of soda integrals, and the sequence of calculations is shown.*

The algorithm for the approximate calculation of the integral considered at the end of the work is fully studied.

***Key words**: exact integral, approximate calculation, a system of linear equations, interpolation polynomial, substitution, interval, ascending, descending, unknown coefficients, Chebyshev's formula.*

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Introduction

Let us be given a function that satisfies conditions $f(x)\in C^1(a;b)$ and $f(0)=0$. Consider the following integral:

$$\int_0^b \frac{f(x)dx}{\left(x^m+c\right)^p} \quad (1)$$

In it was $p>0,\ p\neq 1,\ p\neq 2,\ m\geq 2,\ \forall m\in N$.

Let us consider the approximate calculation of the exact integral (1).

First, let's divide the interval $(a;b)$ into n equal parts and denote by $h=\frac{b-a}{n}$ and $a_i=a+ih$, resulting in $(a;b)=\bigcup_{i=0}^{n-1}(a_i;a_{i+1})$.

(1) can be written as follows:

$$\int_a^b \frac{f(x)dx}{\left(x^m+c\right)^p}=\sum_{i=0}^{n-1}\int_{a_i}^{a_{i+1}}\frac{f(x)dx}{\left(x^m+c\right)^p}. \quad (2)$$

(2) On the right side of the equation, the function $f(x)$ is in the arbitrary interval $(a_i;a_{i+1})$

$$f(x)\approx p_i x^{2m-1}+q_i x^{m-1} \quad (3)$$

(3) Let's do a polynomial substitution [1-7].

Let us be given a function that grows between $f(x)\in C^1(a;b)$ and $(a;b)$ and satisfies the conditions $f(0)=0$.

Let's consider the following exact integral approximation:

$$\int_a^b \frac{f(x)dx}{\left(x^{2m}+c\right)^p},$$

In it was $p>0$, $p\neq 1$, $p\neq 2$, $\forall m\in N$, $c>0$. [8-19].

Where p_i and q_i are arbitrary constant coefficients. x^{2m-1} and x^{m-1} are incremental, and if the coefficients p_i and q_i are positive, (1) the integral function $f(x)$ also increases, and conversely, if the coefficients p_i and q_i are negative, (1) the $f(x)$ function in the integral also decreases. Replacement will be appropriate [21-37].

Also

$$\sum_{i=0}^{n-1}\int_{a_i}^{a_{i+1}}\frac{f(x)dx}{(x^m+c)^p}\approx\sum_{i=0}^{n-1}\int_{a_i}^{a_{i+1}}\frac{p_ix^{2m-1}+q_ix^{m-1}dx}{(x^m+c)^p} \quad (4)$$

(4) is formed. Let us divide the integral into two parts in (4)

$$\sum_{i=0}^{n-1}\int_{a_i}^{a_{i+1}}\frac{p_ix^{2m-1}+q_ix^{m-1}dx}{(x^m+c)^p}=\sum_{i=0}^{n-1}\int_{a_i}^{a_{i+1}}\frac{p_ix^{2m-1}dx}{(x^m+c)^p}+\sum_{i=0}^{n-1}\int_{a_i}^{a_{i+1}}\frac{q_ix^{m-1}dx}{(x^m+c)^p} \quad (5)$$

and

$$I_1=\int\frac{x^{2m-1}}{(x^m+c)^p}dx \quad (6)$$

$$I_2=\int\frac{x^{m-1}}{(x^m+c)^p}dx \quad (7)$$

Let us form the integrals (6) and (7).

First, let's calculate (6), we first get the following result by simple fractional integration [27-39]:

$$I_1=\int\frac{x^{2m-1}}{(x^m+c)^p}dx=\left\{\begin{array}{ll} u=x^m; & du=mx^{m-1}dx \\ dv=\dfrac{x^{m-1}}{(x^m+c)^p}dx; & v=\dfrac{1}{(p-1)(x^m+c)^{p-1}}\end{array}\right\}=$$

$$=\frac{x^m}{m(p-1)(x^m+c)^{p-1}}-\frac{1}{p-1}\int\frac{x^{m-1}}{(x^m+c)^{p-1}}dx=$$

$$=\frac{x^m}{m(p-1)(x^m+c)^{p-1}}-\frac{1}{m(p-2)(p-1)(x^m+c)^{p-2}}.$$

So,

$$I_1=\frac{x^m}{m(p-1)(x^m+c)^{p-1}}-\frac{1}{m(p-2)(p-1)(x^m+c)^{p-2}} \quad (8)$$

Equation (8) is valid.

Now if we calculate (7),

$$I_2=\int\frac{x^{m-1}}{(b^m-x^m)^p}dx=\frac{1}{m(p-1)(x^m+c)^{p-1}} \quad (9)$$

Equation (9) is also valid.

As a result, from equations (8) and (9), the following equation holds:

$$\begin{aligned}&\sum_{i=0}^{n-1}\int_{a_i}^{a_{i+1}}\frac{p_ix^{2m-1}+q_ix^{m-1}dx}{(x^m+c)^p}=\\&=\sum_{i=0}^{n-1}\int_{a_i}^{a_{i+1}}\frac{p_ix^{2m-1}dx}{(b^m-x^m)^p}+\sum_{i=0}^{n-1}\int_{a_i}^{a_{i+1}}\frac{q_ix^{m-1}dx}{(b^m-x^m)^p}\approx\\&\approx\sum_{i=0}^{n-1}\left[p_i\left(\frac{x^m}{m(p-1)(b^m-x^m)^{p-1}}-\frac{1}{m(p-2)(p-1)(b^m-x^m)^{p-2}}\right)+q_i\frac{1}{m(p-1)(b^m-x^m)^{p-1}}\right]\Bigg|_{a_i}^{a_{i+1}}\end{aligned} \quad (4^*)$$

Now let's look at the unknown coefficients and the problem of finding. Substitution (3) gives the following system of linear equations:

$$\begin{cases} f(a_i)\approx p_ia_i^{2m-1}+q_ia_i^{m-1} \\ f(a_{i+1})\approx p_ia_{i+1}^{2m-1}+q_ia_i^{m-1}\end{cases} \quad (10)$$

The system of linear equations (10) has a unique solution, because

$$\begin{vmatrix} a_i^{2m-1} & a_i^{m-1} \\ a_{i+1}^{2m-1} & a_{i+1}^{m-1}\end{vmatrix}\neq 0 \quad (11)$$

Since (11) is appropriate, the solution of the system of linear equations (10) [40-45]:

$$\begin{cases} q_i \approx \dfrac{a_i^{2m-1} f(a_{i+1}) - a_{i+1}^{2m-1} f(a_i)}{a_i^{m-1} a_{i+1}^{m-1}\left(a_i^m - a_{i+1}^{m-1}\right)} \\ p_i \approx \dfrac{a_i^{m-1} f(a_{i+1}) - a_{i+1}^{m-1} f(a_{i+1})}{a_i^{m-1} a_{i+1}^{m-1}\left(a_{i+1}^{m-1} - a_i^m\right)} \end{cases} \quad (12)$$

(12) came out.

(4) approximate substitution, (4 *) and (12) result from (1) the approximate value of the integral.

If (1) $p=1$, $m \geq 2$, $\forall m \in N$ in the integral, then

$$\int_0^b \frac{f(x)dx}{x^m + c} \quad (1^*)$$

(1 *) is formed. Then integrals (5) and (6)

$$I_1^* = \int \frac{x^{m-1}}{x^m + c} dx = -\frac{x^m + 1}{m} \ln(x^m + c) \quad (6^*)$$

$$I_2^* = \int \frac{x^{m-1}}{x^m + c} dx = -\frac{1}{m} \ln(x^m + c) \quad (7^*)$$

(6 *) and (7 *) appear. According to the substitution (3) above

$$\sum_{i=0}^{n-1} \int_{a_i}^{a_{i+1}} \frac{p_i x^{2m-1} + q_i x^{m-1} dx}{x^m + c} =$$

$$= \sum_{i=0}^{n-1} \int_{a_i}^{a_{i+1}} \frac{p_i x^{2m-1} dx}{x^m + c} + \sum_{i=0}^{n-1} \int_{a_i}^{a_{i+1}} \frac{q_i x^{m-1} dx}{x^m + c} = \quad (4^{**})$$

$$= \sum_{i=0}^{n-1} \left[-p_i \frac{x^m + 1}{m} \ln(x^m + c) - q_i \frac{1}{m} \ln(x^m + c) \right] \Bigg|_{a_i}^{a_{i+1}}.$$

(4) approximate substitution, (4**) and (12) result in the approximate value of the integral (1*).

If in the integral $p=2$, $m \geq 2$, $\forall m \in N$,

$$\int_a^b \frac{f(x)dx}{\left(x^m + c\right)^2} \quad (1^{**})$$

(1*) is formed. In this case, the integrals (5) and (6) look like

$$I_1^* = \int \frac{x^{2m-1}}{\left(x^m + c\right)^2} dx = -\frac{x^m}{m\left(x^m + c\right)} + \frac{1}{m} \ln\left(x^m + c\right)$$

(5**)

$$I_2^* = \int \frac{x^{m-1}}{\left(x^m + c\right)^2} dx = -\frac{1}{2m\left(x^m + c\right)} \quad (6^{**})$$

(5 **) and (6 **) appear. According to the substitution (*) above

$$\sum_{i=0}^{n-1} \int_{a_i}^{a_{i+1}} \frac{p_i x^{2m-1} + q_i x^{m-1} dx}{x^m + c} =$$

$$= \sum_{i=0}^{n-1} \int_{a_i}^{a_{i+1}} \frac{p_i x^{2m-1} dx}{x^m + c} + \sum_{i=0}^{n-1} \int_{a_i}^{a_{i+1}} \frac{q_i x^{m-1} dx}{x^m + c} \approx \quad (4^{***})$$

$$\approx \sum_{i=0}^{n-1} \left[p_i \left(\frac{1}{m} \ln\left(x^m + c\right) - \frac{x^m}{m\left(x^m + c\right)} \right) - q_i \frac{1}{m\left(x^m + c\right)} \right] \Bigg|_{a_i}^{a_{i+1}}.$$

(4) approximate substitution, (4 ***) and (11) result in (1) the approximate value of the integral. Now let's move on to the numerical method of approximate calculation using the Chebyshev formula.

In this $x = \dfrac{a + b + (b-a)t}{2}$ and $g(t) = \dfrac{P_{n-1}(x) P_{n-1}^2(x)}{g_1(x) P_{n-1}^1(x)}$ by substituting we get:

$$\int_L \frac{f(x)dl}{(a-x)^{1-p_1}(b-x)^{1-p_2}} \approx \int_a^b \frac{P_{n-1}(x) P_{n-1}^2(x)}{g_1(x) P_{n-1}^1(x)} dx = \frac{b-a}{2} \int_{-1}^1 g(t) dt.$$

Thus, the approximate calculation of the last integral can be done using the following Chebyshev formula:

$$\frac{b-a}{2} \int_{-1}^1 g(t) dt = \frac{2}{n} \left[f(t_1) + f(t_2) + \ldots + f(t_n) \right],$$

then none of n 3,4,5,6,7,9, t_1, $t_2, \ldots, t_n$ are Chebyshev's values in section $[-1;1]$ [3].

References:

1. Sattorov, A. M., & Xujaxonov, Z. Z. (2019). Approach calculation of certain specific integrals by interpolating polynomials. *Scientific Bulletin of Namangan State University*, *1*(3), 10-12.
2. Abdurazakov, A., Makhmudova, N., & Mirzamakhmudova, N. (2021). *On one method for solving degenerating parabolic systems by the direct line method with an appendix in the theory of filtration.*
3. Abdurazakov, A., Mahmudova, N., & Mirzamahmudova, N. (2020). Chislennoe reshenie metodom prjamyh integrala differencirovanija uravnenij, svjazannyh s zadachami fil`tracii gaza. *Universum: tehnicheskie nauki*, (7-1 (76)), 32-35.
4. Abdurazakov, A., Mahmudova, N., & Mirzamahmudova, N. (2019). Reshenija mnogotochechnoj kraevoj zadachi fil`tracii gaza v mnogoslojnyh plastah s uchetom relaksacii. *Universum: tehnicheskie nauki*, (11-1 (68)).
5. Mirzamahmudov, T., & Umarova, G. (2014). *Nekotorye voprosy osnov mestnogo samoupravlenija*. In Teorija i praktika razvitija jekonomiki na mezhdunarodnom, nacional`nom, regional`nom urovnjah (pp. 222-224).
6. Mirzamahmudov, T. M., Rahimov, N. R., Musaev, Je. S., Gafurov, U. A., Butaev, T. B., & Zokirov, R. Z. (1991). *Datchik-zond dlja opredelenija vlazhnosti.*
7. Shadimetov, K., & Daliyev, B. (2021, July). Composite optimal formulas for approximate integration of weight integrals. In *AIP Conference Proceedings*, Vol. 2365, No. 1, p. 020025. AIP Publishing LLC.
8. Shadimetov, H. M., & Daliev, B. S. (2020). Kojefficienty optimal`nyh kvadraturnyh formul dlja priblizhennogo reshenija obshhego integral`nogo uravnenija Abelja. *Problemy vychislitel`noj i prikladnoj matematiki*, (2 (26)), 24-31.
9. Hayotov, A. R., Bozarov, B. I., & Abduganiev, A. (2018). Optimal formula for numerical integration on two dimensional sphere. *Uzbek Mathematical Journal*, *3*, 80-89.
10. Bozarov, B. I. (2019). An optimal quadrature formula with sinx weight function in the Sobolev space. *Uzbekistan Academy Of Sciences Vi Romanovskiy Institute Of Mathematics*, 47.
11. Hayotov, A., & Bozarov, B. (2021, July). Optimal quadrature formulas with the trigonometric weight in the Sobolev space. In *AIP Conference Proceedings*, Vol. 2365, No. 1, p. 020022. AIP Publishing LLC.
12. Hayotov, A., & Bozarov, B. (2021, July). Optimal quadrature formulas with the trigonometric weight in the Sobolev space. In *AIP Conference Proceedings*, Vol. 2365, No. 1, p. 020022. AIP Publishing LLC.
13. Hayotov, A., & Bozarov, B. (2021, July). Optimal quadrature formulas with the trigonometric weight in the Sobolev space. In *AIP Conference Proceedings* (Vol. 2365, No. 1, p. 020022). AIP Publishing LLC.
14. Alimjonova, G. (2021). Modern Competencies In The Techno-Culture Of Future Technical Specialists. *Current Research Journal Of Pedagogics (2767-3278)*, *2*(06), 78-84.
15. Karimov, Sh. T., & Hozhiakbarova, G. (2017). Analog zadachi gursa dlja odnogo neklassicheskogo uravnenija tret`ego porjadka s singuljarnym kojefficientom. *Toshkent Shahridagi Turin Politexnika Universiteti*, 121.
16. Tillabayev, B., & Bahodirov, N. (2021). Solving the boundary problem by the method of green's function for the simple differential equation of the second order linear. *Academicia: An International Multidisciplinary Research Journal*, *11*(6), 301-304.
17. Kosimov, H., & Tillabaev, B. (2018). Mixed fractional order integral and derivatives for functions of many variables. *Scientific journal of the Fergana State University*, *1*(2), 5-11.
18. Ahmedova, G. A., & Fajzullaev, Zh. I. (2014). Upravlenie innovacionnoj aktivnost`u promyshlennyh predprijatij na osnove jeffektivnyh metodov ee ocenki i stimulirovanija. *Aktual`nye problemy gumanitarnyh i estestvennyh nauk*, (4-1).
19. Fayzullaev, J. (2020). A systematic approach to the development of mathematical competence among students of technical universities. *European Journal of Research and Reflection in Educational Sciences, Vol*, *8*(3).
20. Fayzullayev, J. I. (2020). Mathematical competence development method for students through solving the vibration problem with a maple system. *Scientific Bulletin of Namangan State University*, *2*(8), 353-358.
21. Mirzakarimov, E. M., & Faizullaev, J. I. (2019). Method of teaching the integration of information and educational technologies in a heterogeneous parabolic equation. *Scientific*

Bulletin of Namangan State University, *1*(5), 13-17.

22. Ernazarov, A. A. (2020). The relevance of the use of computer-aided design systems for teaching students of higher educational institutions. *Scientific Bulletin of Namangan State University*, *2*(8), 348-353.
23. Mirzakarimov, E. M., & Fayzullaev, J. S. (2020). Improving the quality and efficiency of teaching by developing students* mathematical competence using the animation method of adding vectors to the plane using the maple system. *Scientific Bulletin of Namangan State University*, *2*(9), 336-342.
24. Abdurazakov, A., Mahmudova, N., & Mirzamahmudova, N. (2020). Chislennoe reshenie metodom prjamyh integrala differencirovanija uravnenij, svjazannyh s zadachami fil`tracii gaza. *Universum: tehnicheskie nauki*, (7-1 (76)), 32-35.
25. Abdurazakov, A., Makhmudova, N., & Mirzamakhmudova, N. (n.d.). *The numerical solution by the method of direct integrals of differentiation of equations have an application in the gas filtration theorem.*
26. Nazarova, G. (2021). Methods of directing economics to scientific research activities. *Current Research Journal Of Pedagogics (2767-3278)*, *2*(06), 90-95.
27. Nazarova, G. (2021). Modern pedagogical factors for the development of analytical thinking in future economists. *Academicia: An International Multidisciplinary Research Journal*, *11*(3), 511-517.
28. Atroshchenko, P. V., & Yusupova, N. I. (2007). On an approach to risk forecasting in leasing activities. *Problemy Upravleniya*, *6*, 35-40.
29. Azizov, M. S., & Rustamova, S. T. (2017). Yuqori tartibli differensial tenglamalarni bernulli tenglamasiga keltirib yechish. *Toshkent shahridagi turin politexnika universiteti*, 61.
30. Abdulkhaev, Z. E. (2021). Protection of Fergana City from Groundwater. *Euro Afro Studies International Journal*, (6), 70-81.
31. Qo’ziyev, S. (2021, April). Methods, tools and forms of distance learning. In *Konferencii.*
32. Kuziev, S. S. (2019). Practical and methodological bases of technology in creating electronic educational resources reserves. *Scientific Bulletin of Namangan State University*, *1*(3), 326-329.
33. Kuziev, Sh. A. (2017). Aktual`noe chlenenie kak osobaja harakteristika sintaksicheskogo urovnja. *Molodoj uchenyj*, (1), 528-530.
34. Karimov, Sh. T., & Jylbarsov, H. A. (2021). *Zadacha gursa dlja odnogo psevdoparabolicheskogo uravnenija tret`ego porjadka s operatorom besselja*. BBK 22.161. p.56, 176.
35. Nazarova, G. A., & Arziqulov, Z. O. (2019). Determinig the intervention for privatization of parablical digestive differencial testing in maple system. *Scientific Bulletin of Namangan State University*, *1*(11), 19-26.
36. Kosimov, K., & Mamayusupov, J. (2019). Transitions melline integral of fractional integrodifferential operators. *Scientific Bulletin of Namangan State University*, *1*(1), 12-15.
37. Husanov, J. J., Tashtanov, H. N., & Sattorov, A. M. (2021). Mashina detallarni parmalab ishlov beriladigan notehnologik uzalar turlari. *Scientific progress*, 2 (1), 1322-1332.
38. Xujaxonov, Z. Z. (2019). Approximate computation by the interpolation polynomial method some curvilinear integrals with singular coefficients. *Scientific Bulletin of Namangan State University*, *1*(6), 22-25.
39. Sattorov, A. M., & Xujaxonov, Z. Z. (2019). Approach calculation of certain specific integrals by interpolating polynomials. *Scientific Bulletin of Namangan State University*, *1*(3), 10-12.
40. Shaev, A. K., & Nishonov, F. M. (2018). Singuljarnye integral`nye uravnenija so sdvigom Karlemana s racional`nymi kojefficientami. *Molodoj uchenyj*, (39), 7-12.
41. Azizov, M., & Rustamova, S. (2019). The Task of Koshi for ordinary differential equation of first order which refer to equation of Bernoulli. *Scientific journal of the Fergana State University*, 2(1), 13-16.
42. Hayotov, A. R., & Rasulov, R. G. (2019). The order of convergence of an optimal quadrature formula with derivative in the space $ W_2^{(2, 1)} $. *arXiv preprint arXiv:1908.00450*.
43. Hayotov, A., & Rasulov, R. (2021, July). Improvement of the accuracy for the Euler-Maclaurin quadrature formulas. In *AIP Conference Proceedings* (Vol. 2365, No. 1, p. 020035). AIP Publishing LLC.
44. Hajotov, A. R., Rasulov, R. G., & Sajfullaeva, N. B. (2020). Extension of the Euler-Maclaurin quadrature formula in a Hilbert space. *Problemy vychislitel`noj i prikladnoj matematiki*, (2 (26)), 12-23.
45. Haetov, A. R., & Rasulov, R. G. (2020). Rasshirenie kvadraturnoj formuly Jejlera-Maklorena v prostranstve W. *Matematika Instituti Byulleteni Bulletin of the Institute of Mathematics Bulleten` Instituta*, (3), 167-176.