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TRANSFER FUNCTION OF SYSTEM PROTECTED FROM VIBRATION AND ITS MINIMUM

Abstract: *The problem of determining the transfer function and the conditions of its minimization of a rod with an elastic dissipative characteristic of the hysteresis type in conjunction with a liquid section dynamic absorber under the influence of harmonic excitations is considered. The transfer function is determined depending on the system parameters. The conditions for having a minimum are given in the form of a theorem. The elastic dissipative characteristics of the rod hysteresis type are based on the Pisarenko - Boginich hypothesis.*

Key words: *transfer function; mode shapes of vibrations; absolute acceleration; vibrations; hysteresis.*

Language: *English*

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Introduction

It is important to study the dynamics of harmful vibrations in mechanical systems, to identify the factors that hinder their long-term perfect operation, and to address their elimination. In this regard, the study of the dynamics of the rod protected from vibrations is a urgent problem.

Theoretical and experimental studies have been conducted to investigate the dynamics of nonlinear motion of complex mechanical systems, including transverse vibrations of the rod and its minimization, taking into account the elastic dissipative properties of dynamic absorbers and rods.

Works [1-3] show the method of internal resonances in kinematic motion. In particular, the motion of a rod with variable and constant cross-section under the influence of harmonic excitations was obtained using the Lagrage equations, and the transfer function and amplitude-frequency characteristics were analyzed.

In works [4-7], the vibrational forms of vibration-protected rods have been studied experimentally. Graphs of shape modes with frequency variations were used to get conclusions and make recommendations.

In works [8 - 12], the theoretical basis for determining the shape modes and frequencies of vibrations involved in the transfer function of rods has been developed taking into account the effects of various external loads, and the results of the experiment have been presented.

The work [13,14] analyzed the dynamics of a rod with elastic dissipative characteristics of the hysteresis type, which is protected from vibrations, and liquid section dynamic absorber on the basis of transfer functions, conclusions and recommendations for the selection of system parameters.

Although each of these works has its advantages and disadvantages, they are all widely used in the development of theoretical research and in solving practical problems.

The results of the analysis showed that there is a need for a large-scale study to determine the transfer functions of rods with elastic dissipative characteristics of the hysteresis type, protected from vibration. Therefore, solving such problem is one of the current problems.

Materials and methods

In this paper, we consider the problem of determining the transfer function of the rod with an elastic dissipative characteristic of the hysteresis type, which is protected from vibrations under the influence of harmonic excitations, and the conditions for its minimum.

Harmonic excitations consist of forces $F_L(t)$ and $F_R(t)$ applied to both ends of the rod.

The differential equations of motion of the system under consideration can be obtained using the bond graph method, and it is as follows:

$$A_*\ddot{Q} + B\dot{Q} + CQ = F, \tag{1}$$

where

$$\ddot{Q} = \begin{bmatrix} \ddot{q}_i \\ \ddot{q}_3 \\ \ddot{q}_4 \end{bmatrix}; \dot{Q} = \begin{bmatrix} \dot{q}_i \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix}; Q = \begin{bmatrix} q_i \\ q_3 \\ q_4 \end{bmatrix}; \ F = \begin{bmatrix} u_m(0)F_L + u_m(L)F_R \\ 0 \\ 0 \end{bmatrix};$$

$$A_* = \begin{bmatrix} m_i & 0 & 0 \\ (m_{13}+m_{2*})u_m(x_1) & m_{13}+m_{2*} & m_{2*}+m_v \\ (m_{2*}-m_v)u_m(x_1) & m_{2*}-m_v & m_{2*}+m_{4*} \end{bmatrix};$$

$$B = \begin{bmatrix} 0 & -u_m(x_1)b_F & 0 \\ 0 & b_F & 0 \\ 0 & 0 & b_S \end{bmatrix}; \ C = \begin{bmatrix} c_i & -u_m(x_1)c_{1*} & 0 \\ 0 & c_{1*} & 0 \\ 0 & 0 & 2c_{2*} \end{bmatrix};$$

m_{1*} is the inert dimension (mass) of the outer body of the dynamic absorber, which surrounds the liquid; $m_{2*}$is the inert dimension (mass) of a solid of dynamic absorber; m_{3*} is inert dimension of liquid (mass); m_{4*} is inert dimension (mass) of the fluid adhering to the body 2 with mass m_{2*}; b_F is coefficient of resistance of damper (coefficient of viscosity); c_{1*}^{-1} and c_{2*}^{-1} are compliances (coefficients of elasticity); q_3 and q_4 are displacements of masses m_{1*} and m_{2*}, respectively; $m_{13} = m_1 + m_3$; m_i and $c_i = c_{1i} + jc_{2i}$ are modal mass and modal stiffness corresponding to i-sets of the rod, respectively; q_i are displacement of i-sets of the rod; $u_i(x_1), u_i(0)$ and $u_i(L)$ are the values of mode shapes of the rod at the point $x = x_1$ where the dynamic absorber installed and at the points $x = 0$ and $x = L$ where the external forces exist, respectively; m_v is the mass of liquid squeezed out by body of mass m_{2*}; b_S is viscosity coefficient of the liquid;

$$c_{1i} = [\int_0^L \rho A(1 - C_0\eta_1)u_m^2 dx - \frac{3EI}{\omega_{*m}^2}\eta_1 \sum_{i^*=1}^{n} C_{i^*} q_{ma}^{i^*} \frac{h^{i^*}}{2^{i^*}(i^*+3)} \int_0^L u_m \frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 u_m}{\partial x^2}\left|\frac{\partial^2 u_m}{\partial x^2}\right|^{i^*}\right) dx]\omega_{*m}^2;$$

$$c_{2i} = [\int_0^L \rho A C_0 \eta_2 u_m^2 dx + \frac{3EI}{\omega_{*m}^2}\eta_2 \sum_{i^*=1}^{n} C_{i^*} q_{ma}^{i^*} \frac{h^{i^*}}{2^{i^*}(i^*+3)} \int_0^L u_m \frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 u_m}{\partial x^2}\left|\frac{\partial^2 u_m}{\partial x^2}\right|^{i^*}\right) dx]\omega_{*m}^2;$$

A and ρ are the cross-sectional area and density of the rod; $C_0, C_1, \dots, C_n$ are experimentally determined coefficients of the hysteresis loop, depending on the damping properties of the rod material [15]; E is Yong's module; I is moment of inertia; q_{ma} are amplitude values of rod vibration forms; h and ω_{*m} are the thickness and natural frequency of the rod; u_m are mode shapes; $\eta_1, \eta_2 = sign(\omega)\eta_{22}$ are constant coefficients depending on the dissipative properties of the rod material, determined from the hysteresis loop, $sign(\omega)$ is the sign of ω, η_{22} is constant coefficient; L is length of the rod; $j^2 = -1$.

(1) Using a system of differential equations, we determine the transfer function and the condition of its minimum, which allows to analyze the dynamics of the system under consideration.

Result and discussion

Using the system of differential equations (1), the system under consideration can be reduced to a system of algebraic equations by the differential operator $S = \frac{d}{dt}$, and from this system of algebraic equations the variables q_i, q_3, q_4 are defined as follows [14]:

$$q_i(S) = \frac{a_3(b_2d_3 - b_3d_2)}{a_1(b_2d_3 - b_3d_2) + a_2(b_3d_1 - b_1d_3)};$$

$$q_3(S) = \frac{a_3(b_3d_1 - b_1d_3)}{a_1(b_2d_3 - b_3d_2) + a_2(b_3d_1 - b_1d_3)}; \tag{2}$$

$$q_4(S) = \frac{a_3(b_1d_2 - b_2d_1)}{a_1(b_2d_3 - b_3d_2) + a_2(b_3d_1 - b_1d_3)};$$

where $a_1 = m_iS^2 + c_i$; $a_2 = -u_m(x_1)(b_FS + c_{1*})$; $a_3 = u_m(0)F_L + u_m(L)F_R$;

$$b_1 = M_1 u_m(x_1)S^2; b_2 = M_1 S^2 + b_F S + c_{1*}; b_3 = M_2 S^2;$$
$$d_1 = M_3 u_m(x_1)S^2;\ d_2 = M_3 S^2;\ d_3 = M_4 S^2 + b_S S + 2c_{2*};$$
$$M_1 = m_{13} + m_{2*}; M_2 = m_{2*} + m_v; M_3 = m_{2*} - m_v;\ M_4 = m_{2*} + m_{4*}.$$

Expressions (2) allow us to study the dynamics of transversal vibrations of liquid section dynamic absorber and hysteresis-type elastic dissipative characteristic rod. Absolute accelerations are also important in studying the dynamics of systems. For this purpose, we determine the absolute acceleration of the system under consideration.

Suppose that the external forces F_L and F_R acting on the base by placing the left and right ends of the rod give the system acceleration W_0. In that case

$$F_L = F_R = m_i W_0. \qquad (3)$$

Absolute acceleration of the rod

$$\ddot{W}_a = \ddot{w}_i + W_0. \qquad (4)$$

We put the expression for the forces (3) in the system of equations (1) and use them and the expression for the absolute acceleration (4) to obtain the ratio of the expression for the acceleration to the base acceleration as follows:

$$W_i(S,x) = 1 + \frac{u_i(x)S^2 q_i(S)}{W_0}. \qquad (5)$$

(5) represents the transfer function of the system under consideration.

We set the first equation of the system of equations (2) to the transfer function (5) and change from variable S to variable $j\omega$.

$$W_i(j\omega, q_{ma}, x) = \frac{E_0 + jE_1}{N_0 + jN_1}, \qquad (6)$$

where

$$E_0 = \mu_0 - \mu_1\omega - \mu_2\omega^2 + \mu_3\omega^3 + \mu_4\omega^4 - \mu_5\omega^6; \qquad (7)$$
$$E_1 = \mu_6 + \mu_7\omega - \mu_8\omega^2 - \mu_9\omega^3 + \mu_{10}\omega^4 + \mu_{11}\omega^5; \qquad (8)$$
$$N_0 = \alpha_0 - \alpha_1\omega - \alpha_2\omega^2 + \alpha_3\omega^3 + \alpha_4\omega^4 - \alpha_5\omega^6; \qquad (9)$$
$$N_1 = \alpha_6 + \alpha_7\omega - \alpha_8\omega^2 - \alpha_9\omega^3 + \alpha_{10}\omega^4 + \alpha_{11}\omega^5; \qquad (10)$$
$$\mu_0 = \alpha_0 = 2c_{1*}c_{2*}c_{1i};\ \mu_1 = \alpha_1 = (2b_F c_{2*} + b_S c_{1*})c_{2i}; \mu_2$$
$$= (c_{1*}M_4 + b_F b_S + 2c_{2*}M_1)c_{1i} + 2c_{1*}c_{2*}(m_i + u_i^2(x_1)M_1) - u_i(x)m_i(u_i(0) + u_i(L)));$$
$$\mu_3 = \alpha_3 = (b_F M_4 + b_S M_1)c_{2i}; \mu_4 = \Delta c_{1i} + M_1 u_i^2(x_1)b_F b_S + u_i^2(x_1)\Delta c_{1*} +$$
$$+(1 + u_i(x)(u_i(0) + u_i(L)))m_i(M_4 c_{1*} + b_F b_S + 2c_{2*}M_1); \mu_5 = \Delta m_i(1 + u_i(x)(u_i(0) + u_i(L)));$$
$$\mu_6 = \alpha_6 = 2c_{1*}c_{2*}c_{2i}; \mu_7 = \alpha_7 = (2b_F c_{2*} + b_S c_{1*})c_{1i}; \mu_8 = \alpha_8 = (c_{1*}M_4 + b_F b_S + 2c_{2*}M_1)c_{2i};$$
$$\mu_9 = (b_F M_4 + b_S M_1)c_{1i} + (m_i + u_i^2(x_1)M_1 + u_i(x)m_i(u_i(0) + u_i(L)))(c_{1*}b_S + 2c_{2*}b_F); \mu_{10} = \alpha_{10} = \Delta c_{2i};$$
$$\mu_{11} = u_i^2(x_1)b_F\Delta + (1 + u_i(x)(u_i(0) + u_i(L)))m_i(M_4 b_F + b_S M_1);$$
$$\alpha_2 = (c_{1*}M_4 + b_F b_S + 2c_{2*}M_1)c_{1i} + 2c_{2*}c_{1*}(m_i + u_i^2(x_1)M_1);$$
$$\alpha_4 = \Delta c_{1i} + m_i(M_4 c_{1*} + b_F b_S + 2c_{2*}M_1) + u_i^2(x_1)c_{1*}\Delta + u_i^2(x_1)b_F b_S M_1.$$
$$\alpha_5 = m_i\Delta; \alpha_9 = (b_F M_4 + b_S M_1)c_{1i} + (2b_F c_{2*} + b_S c_{1*})(m_i + u_i^2(x_1)M_1);$$
$$\alpha_{11} = m_i(b_F M_4 + b_S M_1) + u_i^2(x_1)b_F\Delta; \Delta = M_1 M_4 - M_2 M_3.$$

Since it is of practical importance that the absolute accelerations of the rod points, determined from the expression of the transfer function (6) in the dynamic reducing of vibrations, reach a minimum value, we will test this function to a minimum.

The absolute value of the transfer function (6) depends on the variables ω and q_{ma}.

$$|W_i(j\omega, q_{ma}, x)| = \Phi_i(\omega, q_{ma}, x) = \sqrt{\frac{E_0^2 + E_1^2}{N_0^2 + N_1^2}}. (11)$$

The following equations can be generated that allow the determination of stationary points:

$$\frac{\partial\Phi_i}{\partial q_{ma}} = \Phi_i[\frac{E_0(E_0)'_{q_{ma}} + E_1(E_1)'_{q_{ma}}}{E_0^2 + E_1^2} - \frac{N_0(N_0)'_{q_{ma}} + N_1(N_1)'_{q_{ma}}}{N_0^2 + N_1^2}]. \qquad (12)$$
$$\frac{\partial\Phi_i}{\partial\omega} = \Phi_i[\frac{E_0(E_0)'_{\omega} + E_1(E_1)'_{\omega}}{E_0^2 + E_1^2} - \frac{N_0(N_0)'_{\omega} + N_1(N_1)'_{\omega}}{N_0^2 + N_1^2}]. \qquad (13)$$

We define second-order partial derivatives from (12) and (13).

$$\frac{\partial^2\Phi_i}{\partial q_{ma}\partial\omega} = \frac{\partial^2\Phi_i}{\partial\omega\partial q_{ma}} = \Phi_i[\frac{((E_0)'_{\omega}(E_0)'_{q_{ma}} + E_0(E_0)''_{q_{ma}\omega} + (E_1)'_{\omega}(E_1)'_{q_{ma}} +}{(E_0^2 + E_1^2)^2}$$
$$\underline{+E_1(E_1)''_{q_{ma}\omega})(E_0^2 + E_1^2) - 2(E_0(E_0)'_{q_{ma}} + E_1(E_1)'_{q_{ma}})(E_0(E_0)'_{\omega} + E_1(E_1)'_{\omega})}$$

$$-\frac{\left((N_0)'_\omega(N_0)'_{q_{ma}}+N_0(N_0)''_{q_{ma}\omega}+(N_1)'_\omega(N_1)'_{q_{ma}}+N_1(N_1)''_{q_{ma}\omega}\right)(N_0^2+N_1^2)-}{(N_0^2+N_1^2)^2}$$
$$\frac{-2\left(N_0(N_0)'_{q_{ma}}+N_1(N_1)'_{q_{ma}}\right)\left(N_0(N_0)'_\omega+N_1(N_1)'_\omega\right)}{}]+$$
$$+\frac{\partial\Phi_i}{\partial\omega}\left[\frac{E_0(E_0)'_{q_{ma}}+E_1(E_1)'_{q_{ma}}}{E_0^2+E_1^2}-\frac{N_0(N_0)'_{q_{ma}}+N_1(N_1)'_{q_{ma}}}{N_0^2+N_1^2}\right]; \quad (14)$$

$$\frac{\partial^2\Phi_i}{\partial q_{ma}^2}=\Phi_i\left[\frac{\left(((E_0)'_{q_{ma}})^2+E_0(E_0)''_{q_{ma}q_{ma}}+((E_1)'_{q_{ma}})^2+E_1(E_1)''_{q_{ma}q_{ma}}\right)\times}{(E_0^2+E_1^2)^2}\right.$$
$$\frac{\times\,(E_0^2+E_1^2)-2\left(E_0(E_0)'_{q_{ma}}+E_1(E_1)'_{q_{ma}}\right)^2}{}-\frac{\left(((N_0)'_{q_{ma}})^2+N_0(N_0)''_{q_{ma}q_{ma}}\right.}{(N_0^2+N_1^2)^2}$$
$$\left.\frac{+\left((N_1)'_{q_{ma}}\right)^2+N_1(N_1)''_{q_{ma}q_{ma}}\right)(N_0^2+N_1^2)-2\left(N_0(N_0)'_{q_{ma}}+N_1(N_1)'_{q_{ma}}\right)^2}{}\right]+$$
$$+\frac{\partial\Phi_i}{\partial q_{ma}}\left[\frac{E_0(E_0)'_{q_{ma}}+E_1(E_1)'_{q_{ma}}}{E_0^2+E_1^2}-\frac{N_0(N_0)'_{q_{ma}}+N_1(N_1)'_{q_{ma}}}{N_0^2+N_1^2}\right]; \quad (15)$$

$$\frac{\partial^2\Phi_i}{\partial\omega^2}=\Phi_i\left[\frac{\left(((E_0)'_\omega)^2+E_0(E_0)''_{\omega\omega}+((E_1)'_\omega)^2+E_1(E_1)''_{\omega\omega}\right)(E_0^2+E_1^2)-}{(E_0^2+E_1^2)^2}\right.$$
$$\frac{-2\left(E_0(E_0)'_\omega+E_1(E_1)'_\omega\right)^2}{}-\frac{\left(((N_0)'_\omega)^2+N_0(N_0)''_{\omega\omega}+((N_1)'_\omega)^2+\right.}{(N_0^2+N_1^2)^2}$$
$$\left.\frac{\left.+N_1(N_1)''_{\omega\omega}\right)(N_0^2+N_1^2)-2\left(N_0(N_0)'_\omega+N_1(N_1)'_\omega\right)^2}{}\right]+$$
$$+\frac{\partial\Phi_i}{\partial\omega}\left[\frac{E_0(E_0)'_\omega+E_1(E_1)'_\omega}{E_0^2+E_1^2}-\frac{N_0(N_0)'_\omega+N_1(N_1)'_\omega}{N_0^2+N_1^2}\right]. \quad (16)$$

The stationary values of the variables q_{ma}va ω are determined from the following system of equations:

$$\frac{\partial\Phi_i}{\partial q_{ma}}=0;\ \frac{\partial\Phi_i}{\partial\omega}=0. \quad (17)$$

For $\Phi_i\neq 0$,

$$\frac{E_0(E_0)'_{q_{ma}}+E_1(E_1)'_{q_{ma}}}{E_0^2+E_1^2}-\frac{N_0(N_0)'_{q_{ma}}+N_1(N_1)'_{q_{ma}}}{N_0^2+N_1^2}=0. \quad (18)$$

$$\frac{E_0(E_0)'_\omega+E_1(E_1)'_\omega}{E_0^2+E_1^2}-\frac{N_0(N_0)'_\omega+N_1(N_1)'_\omega}{N_0^2+N_1^2}=0. \quad (19)$$

Based on the above results and the *theorem* that a function of two known variables has a minimum can be defined for the absolute value of the transfer function as follows:

If the variables q_{ma} and ω satisfy system of equations (17) and

$$\frac{\partial^2\Phi_i}{\partial q_{ma}^2}>0,\frac{\partial^2\Phi_i}{\partial\omega^2}>0, \quad (20)$$

along with satisfying inequalities

$$\frac{\partial^2\Phi_i}{\partial q_{ma}^2}\frac{\partial^2\Phi_i}{\partial\omega^2}-\left(\frac{\partial^2\Phi_i}{\partial q_{ma}\partial\omega}\right)^2>0, \quad (21)$$

satisfy the inequalities, then the absolute value of the transfer function reaches a minimum at these values of the variable.

The dynamics of hysteresis type elastic dissipative characteristic rod and the liquid section dynamic absorber were studied on the basis of the following numerical values of the design parameters:

the rod material is an alloy of aluminum AL 19, the mechanical characteristic of which is taken to be $E=6964119\cdot 10^4\ \frac{N}{m^2},\rho=2780\ \frac{kg}{m^3}$. Geometric dimensions of the rod: height is $z=h=5\cdot 10^{-4}\ m$, length is $L=120\cdot 10^{-2}m$, cross-sectional area is $A=12\cdot 10^{-6}m^2$. In this case $I=\frac{Ah^2}{12}=25\cdot 10^{-14}m^4$.

Instructions for selecting the parameters of liquid section dynamic absorber are available in the work [13]. Based on this, the parameters accepted the following values when performing calculations:$b_S=102.9\cdot 10^3\frac{N\cdot s}{m};m_{4*}=4.1\cdot 10^{-3}\ kg;m_v=4.1\cdot 10^{-6}\ kg;m_{2*}=4.5\ \cdot 10^{-6}kg;\ m_1=1.3\cdot 10^{-3}\ kg;\ m_3=2.7\ \cdot 10^{-3}kg$.

In order to determine the coefficients related to the dissipative properties, taking into account the relationship between stress and strain in the AL19 aluminum alloy rod material under consideration, based on the numerical values given in the handbooks and using the method given in [16].

$$\delta(\xi_*=q_{ma})=\chi_1 q_{ma}+\cdots+\chi_n q_{ma}^n, \quad (22)$$

We first determine the following three terms of the logarithmic decrement coefficients [17]:

$$\chi_1=10.6662475;\chi_2=-55.22539871\cdot 102;$$
$$\chi_2=10.43466067\cdot 10^5.$$

This defined logarithmic decrement expression (22) allows us to determine the variability in the absorption factor. Based on this, we write the absorption factor follows [17]:

$$\psi(\xi_* = q_{ma}) = C_0 + C_1 \left|\frac{\partial^2 u_m}{\partial x^2}\right| q_{ma} z + \cdots + C_n \left(\left|\frac{\partial^2 u_m}{\partial x^2}\right| q_{ma} z\right)^n . \qquad (23)$$

We know that there is the following relationship between the absorption factor and the vibration decrement [17]: $\psi(\xi_*) = 2\delta(\xi_*)$.

For the values of the second-order derivative obtained from the first mode shape at the point which liquid section dynamic absorber is set, these coefficients assume the following values: $C_0 = 0; C_1 = 48.12119136 \cdot 10^2; C_2 = -56.20284398 \cdot 10^7$;
$C_3 = 23.95479624 \cdot 10^{12}$.

We get the coefficients η_1 and η_{22} as follows [18]: $\eta_1 = \frac{3}{4}; \eta_{22} = \frac{1}{\pi}$.

As a result, the expressions c_{1i} and c_{2i} are as follows: $c_{11} = \rho A \omega_{*1}^2 \int_0^{0.6} u_1^2 dx - 3EI\eta_1(G_1 + G_2 + G_3); c_{21} = 3EI\eta_{22}(G_1 + G_2 + G_3)$,
where

$$G_1 = C_1 \frac{h}{8} q_{1a} \int_0^{0.6} u_1 \frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 u_1}{\partial x^2}\left|\frac{\partial^2 u_1}{\partial x^2}\right|\right) dx;$$

$$G_2 = C_2 \frac{h^2}{20} q_{1a}^2 \int_0^{0.6} u_1 \frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 u_1}{\partial x^2}\left|\frac{\partial^2 u_1}{\partial x^2}\right|^2\right) dx;$$

$$G_3 = C_3 \frac{h^3}{48} q_{1a}^3 \int_0^{0.6} u_1 \frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 u_1}{\partial x^2}\left|\frac{\partial^2 u_1}{\partial x^2}\right|^3\right) dx.$$

If we calculate the integrals G_1, G_2, G_3, they are: $G_1 = 230.3833425 q_{1a}; G_2 = -69232.74335 q_{1a}^2$; $G_3 = 8019069.423 q_{1a}^3. c_{11} = 1.031053156 - 9.024845694 q_{1a} + 2712.06598 q_{1a}^2 - 314132.3646 q_{1a}^3; c_{21} = 3.830263476 q_{1a} - 1151.036552 q_{1a}^2 + 133321.9163 q_{1a}^3$.

Based on the determined values, we will plot a graph of the transfer function.

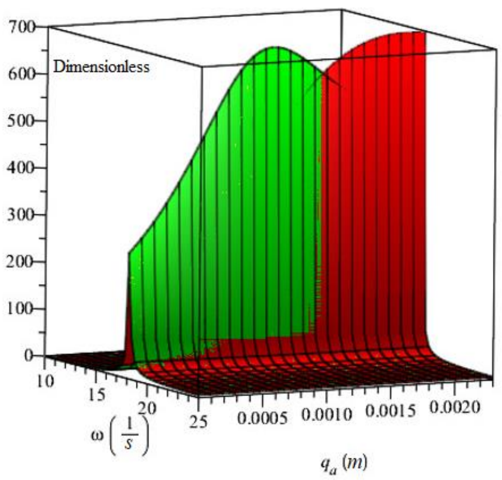

Figure 1. Graph of the transfer function

Analyzing these results, the graphs c_{1*} stiffness and b_F damping coefficient in the graph in Fig. 1 are infinitely large (red) and $c_{1*} = 10^5 \frac{N}{m}; b_F = 10^5 \frac{N \cdot s}{m}$ (green), changing of transfer function depending on amplitude and frequency is described. The results of the analysis show that when c_{1*} stiffness and b_F damping coefficient are infinitely large, spring with c_{1*} stiffness and damper can be considered as a solid.

This means that in this case the spring and damper are removed from the physical model of liquid section dynamic absorber. The result obtained therefore represents a graph of the transfer function of a new physical model of liquid section dynamic absorber mounted directly on rod with body surrounding liquid.

From this graph, when evaluating the efficiency of both liquid section dynamic absorbers above, it can be seen that the liquid section dynamic absorber with c_{1*} stiffness and damper is effective.

Conclusion

1. The defined transfer function allows to analyze the dynamics of the vibration of hysteresis type elastic dissipative characteristic rod in conjunction with liquid section dynamic absorber under the influence of harmonic excitations at different values of the system parameters.

2. From the given theorem the variables and system parameters for which the absolute value of the transfer function is minimized are necessary. As a result, it allows the practical design of a system

consisting of hysteresis type elastic dissipative characteristic rod and liquid section dynamic absorber.

References:

1. ElBassiouny A.F. Vibrations and dynamical stability of nonlinear system of rods to biharmonic excitations. Physica Scripta. Volume 74 (1), 2006, Pp. 28-38.
3. Yardimoglu B., Aydin L. Exact longitudinal vibration characteristics of rods with variable cross-sections. Shock and Vibration. Volume 18, 2011, Pp. 555–562, DOI 10.3233/SAV-2010-0561
4. Mkrtchyan K.S. Study of Forced Transverse Vibrations of Elastic Hinge-Operated Rod Taking Into Account Rotative Movement. Mech. Solids. Volume 54, 2019, Pp. 112–121. https://doi.org/10.3103/S0025654419010096
5. Zainulabidin M.H., Jaini N. Vibration Analysis of a Beam Structure Attached with a Dynamic Vibration Absorber. Applied Mechanics and Materials. Volume 315, 2013, Pp. 315-319.
6. Bonsel J. H., Fey R. H. B., Nijmeijer H. Application of a Dynamic Vibration Absorber to a Piecewise Linear Beam System. Nonlinear Dynamics. Volume 37, 2004, Pp. 227–243.
7. Adil E.B., Fulgence R. Transverse vibration analysis of Euler-Bernoulli beam carrying point masse submerged in fluid media. International Journal of Engineering & Technology, Science Publishing Corporation. Volume 4 (2), 2015, Pp.369-380.
8. Polukoshko S., Kononova O., Schukin I., Smirnova R. Experimental research of dynamic damping of lateral vibrations of a rigid cantilever beam. Journal of Vibroengineering. Volume 15 (1), 2013, Pp. 265-270.
9. Jue X., Yuan-jie Ch., Yong-li M. Transverse vibrations of a thin loaded rod: theory and experiment. European Journal of Physics. Volume 36, 2015, http://iopscience.iop.org/0143-0807/36/5/055035
10. Chaohe Ch., Yong H., Guangfan L. Calculation of the Natural Frequencies of Transverse Vibration of Complex Beams Using the Differential-Matrix Equations. Advanced Materials Research. Volume 243-249, 2011, Pp. 284-289, https://doi.org/10.4028/www.scientific.net/AMR.243-249.284
11. Mohamed G., Sinan M. Transverse vibration of two axially moving beams connected by an elastic foundation. Proceedings of 2005 ASME International Mechanical Engineering Congress and Exposition, November 11-15, 2005, Orlando, Florida. DOI:10.1115/IMECE2005-80377
12. Banerjee J., Su. H., Gunawardana W. D. Dynamic Stiffness Formulation and Free Vibration Analysis of a Moving Timoshenko Beam. Proceedings of 49th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, April 1-8, 2008, Schaumburg. DOI:10.2514/6.2008-2078
13. Kong L., Parker R.G. Approximate eigensolutions of axially moving beams with small flexural stiffness. Journal of Sound and Vibration. Volume 276, 2004, Pp. 459–469.
14. Dusmatov O.M. Dynamic damping of elastic systems with lumped and distributed parameters // Diss. Cand. physical-mat. Sciences / Kiev Polytechnic. ins. Kiev, 1988.
15. Mirsaidov M.M., Dusmatov O.M., Khodjabekov M.U. The problem of mathematical modeling of a vibration protected rod under kinematic excitations, in Proceedings of VII International Scientific Conference Integration, Partnership and Innovation in Construction Science and Education, 11-14 November, 2020, Tashkent, Uzbekistan (2020), doi:10.1088/1757-899X/1030/1/012069
16. Pisarenko G.S., Yakovlev A.P., Matveev V.V. Vibration-damping properties of construction materials. Reference book. - Kiev: Nauk. dumka, 1971, P. 327.
17. Pisarenko G.S., Boginich O.E. Vibrations of kinematically excited mechanical systems taking into account energy dissipation (Kiev: Nauk. Dumka) 1981, p 220
18. Pavlovsky M.A. et al Nonlinear problems of the dynamics of vibration protection systems (Kiev:Technique) 1997, p 204