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FREQUENCY ANALYSIS OF LONGITUDINAL-RADIAL VIBRATIONS OF A CONICAL SHELL

Abstract: *The article presents the equations of torsional vibrations of a three-layer conical shell. For this, a surface is selected at a certain distance from the middle layer of the conical shell. Shear heads on this surface are selected as searchable objects. Then, the equations of torsional vibrations of a three-layer conical shell with respect to these sought-for functions were formed.*

Key words: *Conical shell, middle layer, three-layer shell, torsional vibration, stress, deformation, displacement.*

Language: *English*

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Introduction

The solution of applied problems of the dynamics of layered conical shells is based on the well-known Kirchhoff-Love, Hermann-Mirsky and other refined theories of vibration [1-2]. These theories are developed for single-layer, homogeneous shells [3-4], and therefore, their application to study the dynamics of layered structural elements is accompanied with certain difficulties of a mathematical nature and ensuring the fulfillment of the contact conditions between the layers [5-6]. Therefore, in the last few decades, theories of vibrations of layered elements of structures began to be developed [7-8]. The number of works devoted to the development of new theories of vibration of structural elements, taking into account various rheological, temperature, anisotropic and other properties of materials, is large. Despite this, at present, the study of non-stationary oscillations of such elements continues on the basis of new theories and equations of oscillation [9-11].

This article is devoted to the study of the equations of torsional vibrations of layered conical elastic shells following from the general equations of vibrations of a three-layer elastic shell as limiting cases.

Formulation of the problem.

In a cylindrical coordinate system (r,θ,z), the problem of torsional vibrations of a homogeneous and isotropic conical shell made of an elastic material is considered, the inner r_1 and outer r_2 radii of which are linear functions of the longitudinal coordinate, i.e.

$$r_1 = r_0 + kz;\; r_2 = r_0 + kz + d;$$

where $r_0 = const$, d - толщина оболочки; $k = tg\alpha$ (рис.1). When deriving the equations of oscillation, it is assumed that the conical shell, as a three-dimensional body, strictly obeys the mathematical theory of elasticity and, in its exact formulation, is described by its equations. In a cylindrical coordinate system, we consider a three-layer conical shell of an elastic material that is inhomogeneous in thickness.

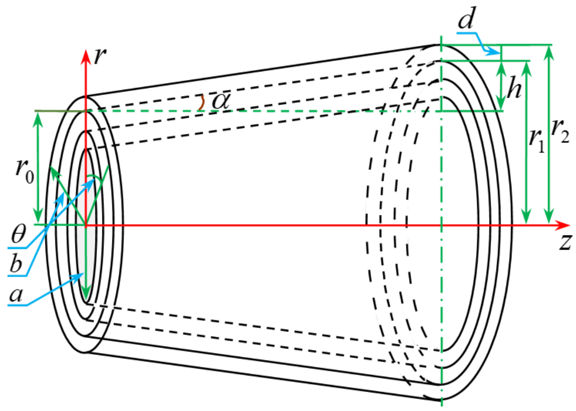

Figure 1.

We will assume that the rational design of the conical shell from the point of view of its work on the action of dynamic loads will be such when the bulk of the rigid material in the form of two layers, hereinafter called load-bearing layers [1], is spaced at some distance by means of a thin wall or a third layer. The third layer can be of the same material as the carrier layers. When the space between two rigid layers is filled with a lighter, and therefore less rigid material, it is hereinafter called a filler. The third layer or filler keeps the bearing layers at a distance equal to its thickness and carries out their joint work.

Moreover, if problems are considered that are different from the problems of transverse vibration of the shell, then it is easy to guess that the joint work of the bearing layers depends on the ability of the filler to resist their relative shear. Based on these considerations, we will assume that the contacts between the bearing layers and the filler are rigid.

We direct the Oz axis of the coordinate system along the symmetry axis of the shell and number the layers as shown in Fig. 1. Through *a* and *b* we denote the inner and outer radii of the shell, and through r_1 and r_2 the inner and outer radii of the middle layer (filler). When deriving the equations of oscillation, we will assume that both the cylindrical shell as a whole, and the bearing layers and the filler, strictly obey the mathematical theory of elasticity and in the exact setting are described by its three-dimensional equations in a linear formulation.

The components of the vectors of displacement of the points of the layers along the coordinate axes, which are considered small, will be denoted by $u_{rm}(r,\theta,z,t)$, $u_{\theta m}(r,\theta,z,t)$, $u_{zm}(r,\theta,z,t)$. Here and below, the index takes on the values $0,1,2$. Therefore, in the future, we will not emphasize this every time, implying that this is always the case.

The dependencies between the components of stresses and strains at the points of the layers of a conical three-layer shell are considered to be given in the form [2].

$$\sigma_{ij}^{(m)}(r,\theta,z,t) = \lambda_m\left(\varepsilon^{(m)}\right) + 2\mu_m\left(\varepsilon_{ij}^{m}\right);\; (i,j = r,\theta,z), \quad (1)$$

λ_m, μ_m – Lame coefficients of the materials of the layers.

The equations of motion of points of layers, as conical three-dimensional bodies, in the absence of volumetric forces have the form [3].

$$\frac{\partial\sigma_{rr}^{(m)}}{\partial r} + \frac{1}{r}\frac{\partial\sigma_{r\theta}^{(m)}}{\partial\theta} + \frac{\partial\sigma_{rz}^{(m)}}{\partial z} + \frac{\sigma_{rr}^{(m)} - \sigma_{\theta\theta}^{(m)}}{r} = \rho\frac{\partial^2 u_{rm}}{\partial t^2};$$

$$\frac{\partial\sigma_{r\theta}^{(m)}}{\partial r} + \frac{1}{r}\frac{\partial\sigma_{\theta\theta}^{(m)}}{\partial\theta} + \frac{\partial\sigma_{z\theta}^{(m)}}{\partial z} + \frac{2}{r}\sigma_{r\theta}^{(m)} = \rho\frac{\partial^2 u_{\theta m}}{\partial t^2}; \quad (2)$$

$$\frac{\partial\sigma_{rz}^{(m)}}{\partial r} + \frac{1}{r}\frac{\partial\sigma_{\theta z}^{(m)}}{\partial\theta} + \frac{\partial\sigma_{zz}^{(m)}}{\partial z} + \frac{\sigma_{zr}^{(m)}}{r} = \rho\frac{\partial^2 u_{zm}}{\partial t^2}.$$

Further, following [4], the potentials of longitudinal $\varphi^{(m)}$ and transverse $\vec{\phi}^{(m)}$ waves are introduced by the formula

$$\vec{U}^{(m)} = grad\,\varphi_m + rot\left[\vec{e}_z\psi_m + rot\left(\vec{e}_z\chi_m\right)\right]. \quad (3)$$

Note that when the vector potentials $\vec{\phi}^{(m)}$ are represented in the form

$$\vec{\phi}_m = \vec{e}_z\psi_m + rot\left(\vec{e}_z\chi_m\right), \quad (4)$$

where $\vec{e}_z$ – 111- unit vector along the axis z, solenoid conditions for the range of vector fields $\vec{\phi}^{(m)}$, $div\vec{\phi}^{(m)} = 0$ are performed automatically [5].

Substitution of expressions (3) into the equations of motion (2) allows us to write them through the wave equations with respect to the wave potentials [6]

$$\Delta\varphi^{(m)} = \frac{\rho_m}{\lambda_m + 2\mu_m}\ddot{\varphi}_m; \quad \Delta\vec{\phi}_m = \frac{\rho_m}{\mu_m}\ddot{\vec{\phi}}_m; \qquad (5)$$

where ρ_m – density of layer materials; Δ – Laplace operator.

$$\Delta = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} + \frac{\partial^2}{\partial z^2}.$$

From expressions (3), it is easy to determine the components of the displacement vectors of the layers of the shell u_{rm}, $u_{\theta m}$, u_{zm} through the potentials of the longitudinal φ_m and transverse ψ_m, χ_m waves

$$u_{rm} = \frac{\partial\varphi_m}{\partial z} - \frac{1}{r}\frac{\partial\chi_m}{\partial r} - \frac{\partial^2\chi_m}{\partial r^2} - \frac{1}{r}\frac{\partial^2\chi_m}{\partial\theta^2};$$
$$u_{\theta m} = \frac{1}{r}\frac{\partial\varphi_m}{\partial\theta} - \frac{\partial\psi_m}{\partial r} + \frac{1}{r}\frac{\partial^2\chi_m}{\partial z\partial\theta}; \qquad (6)$$
$$u_{zm} = \frac{\partial}{\partial r}\left[\varphi_m + \frac{\partial\chi_m}{\partial z}\right] + \frac{1}{r}\frac{\partial\psi_m}{\partial\theta}.$$

The last expressions for the components of the displacement vectors of the points of the shell layers make it possible to express the deformation component in terms of the wave potentials

$$\varepsilon_{rr}^{(m)} = \frac{\partial^2\varphi_m}{\partial r^2} - \frac{1}{r^2}\frac{\partial\varphi_m}{\partial\theta} + \frac{1}{r}\frac{\partial^2\psi_m}{\partial r\partial\theta} + \frac{\partial^3\chi_m}{\partial z\partial r^2},$$
$$\varepsilon_{zz}^{(m)} = \frac{\partial^2\varphi_m}{\partial z^2} - \left(\Delta - \frac{\partial^2}{\partial z^2}\right)\frac{\partial\chi_m}{\partial z},$$
$$\varepsilon_{\theta\theta}^{(m)} = \frac{1}{r}\left(\frac{1}{r}\frac{\partial^2}{\partial\theta^2} + \frac{\partial}{\partial r}\right)\left(\varphi_m + \frac{\partial\chi_m}{\partial z}\right) + \left(\frac{1}{r} - \frac{\partial}{\partial r}\right)\frac{\partial\psi_m}{\partial\theta},$$
$$\gamma_{r\theta}^{(m)} = \frac{1}{r}\left(\frac{\partial}{\partial r} - \frac{1}{r}\right)\frac{\partial\varphi_m}{\partial\theta} + \frac{1}{r}\left(\frac{\partial}{\partial r} - \frac{1}{r}\right)\frac{\partial^2\chi_m}{\partial\theta\partial z} +$$
$$+\frac{1}{2r^2}\frac{\partial^2\psi_m}{\partial\theta^2} - \frac{r}{2}\frac{\partial}{\partial r}\left(r\frac{\partial\psi_m}{\partial r}\right), \qquad (7)$$
$$\gamma_{\theta z}^{(m)} = \frac{1}{2}\frac{\partial^2\varphi_m}{\partial\theta\partial z} - \frac{1}{2}\frac{\partial^2\psi_m}{\partial z\partial r} - \frac{1}{2r}\left(\Delta - 2\frac{\partial^2}{\partial z^2}\right)\frac{\partial\chi_m}{\partial\theta},$$
$$\gamma_{rz}^{(m)} = \frac{1}{2}\frac{\partial}{\partial r}\left(2\frac{\partial^2}{\partial z^2} - \Delta\right)\chi_m + \frac{\partial^2\varphi_m}{\partial r\partial z} + \frac{1}{2}\frac{\partial^2\psi_m}{\partial\theta\partial z}.$$

Moreover, it is easy to check by these formulas the validity of the equality

$$\varepsilon^{(m)} = \varepsilon_{rr}^{(m)} + \varepsilon_{\theta\theta}^{(m)} + \varepsilon_{zz}^{(m)} = \Delta\varphi_m$$

If problems are considered symmetric with respect to the axis, then the components of the displacement vectors of the shell layers do not depend on the angular coordinate θ and, expressions (6) take the form

$$u_{rm} = \frac{\partial\varphi_m}{\partial z} - \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\chi_m}{\partial r}\right), u_{\theta m} = -\frac{\partial\psi_m}{\partial r},$$

$$u_{zm} = \frac{\partial\varphi_m}{\partial r} + \frac{\partial^2\chi_m}{\partial r\partial z}. \qquad (8)$$

In this case, the formulas for the deformation components are also simplified, which can be written in the form

$$\varepsilon_{rr}^{(m)} = \frac{\partial^2}{\partial r^2}\left(\varphi_m + \frac{\partial\chi_m}{\partial z}\right),$$
$$\varepsilon_{zz}^{(m)} = \frac{\partial^2\varphi_m}{\partial z^2} - \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial^2\chi_m}{\partial r\partial z}\right), \varepsilon_{\theta\theta}^{(m)} = \frac{1}{r}\frac{\partial}{\partial r}\left(\varphi_m + \frac{\partial\chi_m}{\partial z}\right)$$
$$\gamma_{r\theta}^{(m)} = -\frac{r}{2}\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\psi_m}{\partial r}\right), \ \gamma_{\theta z}^{(m)} = -\frac{1}{2}\frac{\partial^2\psi_m}{\partial z\partial r} \qquad (9)$$
$$\gamma_{rz}^{(m)} = \frac{\partial^2\varphi_m}{\partial r\partial z} + \frac{1}{2}\frac{\partial}{\partial r}\left[\frac{\partial^2}{\partial z^2} - \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)\right]\chi_m$$

It is assumed that at $t < 0$ the conical shell is at rest, and at the moment $t = 0$, stresses are applied to its boundary surfaces, causing its torsional vibrations, i.e. it is believed that the boundary conditions have the form

$$\text{at } r = a, \ \sigma_{r\theta}^{(1)}(a,z,t) = f_{r\theta}^{(1)}(z,t)$$
$$\text{at } r = b, \ \sigma_{r\theta}^{(2)}(b,z,t) = f_{r\theta}^{(2)}(z,t) \qquad (10)$$

In addition, according to the conditions of rigid contact on the boundary surfaces between the layers, the conditions of equality of mixing and stresses must be fulfilled, i.e.

at $r = r_1$,

$$u_{\theta 0}(r_1, z, t) = u_{\theta 1}(r_1, z, t);$$
$$\sigma_{r\theta}^{(0)}(r_1, z, t) = \sigma_{r\theta}^{(1)}(r_1, z, t). \qquad (11)$$

at $r = r_2$

$$u_{\theta 0}(r_2, z, t) = u_{\theta 2}(r_2, z, t);$$
$$\sigma_{r\theta}^{(0)}(r_2, z, t) = \sigma_{r\theta}^{(2)}(r_2, z, t). \qquad (12)$$

The initial conditions of the problem are assumed to be zero.

The torsional vibrations of the conical shell are axisymmetric, and therefore the displacements and deformations of the points of the layers, and, consequently, the stresses, do not depend on the angular coordinate. Only displacements $u_{\theta m}$, stresses $\sigma_{r\theta}^{(m)}$, $\sigma_{\theta z}^{(m)}$ and deformations $\varepsilon_{r\theta}^{(m)}$, $\varepsilon_{\theta z}^{(m)}$ will be nonzero, [7]. In this case, displacements and deformations are determined by formulas (8) and (9), from which it follows that they depend only on the potentials ψ_m,

In this case, the equations of motion (2) take the form

$$\frac{\partial\sigma_{r\theta}^{(m)}}{\partial r} + \frac{\partial\sigma_{z\theta}^{(m)}}{\partial z} + \frac{2}{r}\sigma_{r\theta}^{(m)} = \rho\frac{\partial^2 u_{\theta m}}{\partial t^2} \qquad (13)$$

Which, after applying (5), go over to wave equations with respect to potentials ψ_m, i.e., the equations of motion of a conical elastic three-layer shell, with its torsion will be

$$\Delta\psi_m = \frac{\rho_m}{\mu_m}\frac{\partial^2\psi_m}{\partial t^2}; \qquad (14)$$

where

$m=1$ at $a \le r \le r_1$, $m=0$ at $r_1 \le r \le r_2$ and $m=2$ at $r_2 \le r \le b$.

Thus, the problem of torsional vibrations of a three-layer conical shell is reduced to the integration of equations (14) at boundary - (10), contact - (11), (12), and also zero at $t=0$

$$\psi_m = \frac{\partial\psi_m}{\partial t} = 0; \qquad (15)$$

initial conditions.

The solution of the problem.

To solve this problem, the functions of external influences can be represented in the form

$$f_{r\theta}^{(i)}(z,t) = \int_0^\infty \left.\begin{matrix}\sin kz\\ \cos kz\end{matrix}\right\} dk \int_{(l)} \tilde{f}_{r\theta}^{(i)}(k,p)e^{pt}dp, \ (i=1,2). \qquad (16)$$

In accordance with the accepted representations for the external action function - (16), the solution of problem (5) (14), (11), (12) and (15) will be sought in the form

$$[\varphi_m, \psi_m] = \int_0^\infty \left.\begin{matrix}\sin kz\\ \cos kz\end{matrix}\right\} dk \int_{(l)} [\tilde{\varphi}_m, \tilde{\psi}_m]e^{pt}dp;$$

$$\chi_m = \int_0^\infty \left.\begin{matrix}\cos kz\\ \sin kz\end{matrix}\right\} dk \int_{(l)} \tilde{\chi}_m e^{pt}dp. \qquad (17)$$

Substituting transformations (14) for potential functions ψ_m into the wave equations, we will have

$$\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \beta_m^2\right)\tilde{\psi}_m = 0, \qquad (18)$$

where

$$\beta_m^2 = k^2 + \rho_m / \mu_m^{-1}; \qquad (18^*)$$

General solutions of equations (18) have the form

$$\tilde{\psi}_m(r) = C_m^{(1)} I_0(\beta_m r) + C_m^{(2)} K_0(\beta_m r); \qquad (19)$$

where $I_0(r)$, $K_0(r)$, - modified Bessel functions [1].

The further task is to express the components of the displacement vectors and stress tensors at the points of all three layers through the obtained solutions (19). For this purpose, we will first do this for the movements. Therefore, the displacements $u_{\theta m}$ can also be represented as

$$u_{\theta m}(r,z,t) = \int_0^\infty \left.\begin{matrix}\sin kz\\ \cos kz\end{matrix}\right\} dk \int_{(l)} \tilde{u}_{\theta m}(r,k,p)e^{pt}dp. \qquad (20)$$

Substituting (17) and (20) into (8) for the transformed values of displacements $\tilde{u}_{\theta m}(k,p)$, we obtain

$$\tilde{u}_{\theta m}(r,k,p) = -\frac{d}{dr}\tilde{\psi}_m; \qquad (21)$$

Let us represent voltages $\sigma_{r\theta}$ as well as (16)

$$\sigma_{r\theta}^{(m)}(r,z,t) = \int_0^\infty \left.\begin{matrix}\sin kz\\ \cos kz\end{matrix}\right\} dk \int_{l} \tilde{\sigma}_{r\theta}^{(m)}(r,k,p)e^{pt}; \qquad (22)$$

and substitute representations (16) and (22) into boundary conditions (10). We get

$$\int_0^\infty \left.\begin{matrix}\sin kz\\ \cos kz\end{matrix}\right\} dk \int_{(l)} \tilde{\sigma}_{r\theta}^{(i)}(r,k,p)e^{pt}dp =$$

$$= \int_0^\infty \left.\begin{matrix}\sin kz\\ \cos kz\end{matrix}\right\} dk \int_{(l)} f_{r\theta}^{(i)}(k,p)e^{pt}dp \qquad (23)$$

From here

$$\tilde{\sigma}_{r\theta}^{(1)}(a,k,p) = \tilde{f}_{r\theta}^{(1)}(k,p),$$

$$\tilde{\sigma}_{r\theta}^{(2)}(b,k,p) = \tilde{f}_{r\theta}^{(2)}(k,p). \qquad (24)$$

On the other hand, based on the fundamentals, we have

$$\tilde{\sigma}_{r\theta}^{(m)}(r) = \tilde{\mu}_m\left(\frac{1}{r} - \frac{d}{dr}\right)\frac{d\tilde{\psi}_m}{dr}. \qquad (25)$$

Substituting (25) into (24), we obtain

$$\left(\frac{1}{r} - \frac{d}{dr}\right)\frac{d\tilde{\psi}_1}{dr}\bigg|_{r=a} = \tilde{\mu}_1^{-1}[\tilde{f}_{r\theta}^{(1)}];$$

$$\left(\frac{1}{r} - \frac{d}{dr}\right)\frac{d\tilde{\psi}_2}{dr}\bigg|_{r=b} = \tilde{\mu}_2^{-1}[\tilde{f}_{r\theta}^{(2)}]. \qquad (26)$$

Similarly transformed contact conditions (11) and (12), based on expressions (20), (21) and (25), will have the following forms:

at $r = r_1$

$$\frac{d}{dr}\tilde{\psi}_1 = \frac{d}{dr}\tilde{\psi}_0, \qquad (27)$$

$$\left(\frac{1}{r} - \frac{d}{dr}\right)\frac{d\psi_0}{dr} = \mu_0^{-1}\mu_1\left(\frac{1}{r} - \frac{d}{dr}\right)\frac{d\psi_1}{dr}, \qquad (28)$$

and at $r = r_2$

$$\frac{d}{dr}\tilde{\psi}_0 = \frac{d}{dr}\tilde{\psi}_2, \qquad (29)$$

$$\left(\frac{1}{r} - \frac{d}{dr}\right)\frac{d\tilde{\psi}_0}{dr} = \mu_0^{-1}\mu_2\left(\frac{1}{r} - \frac{d}{dr}\right)\frac{d\psi_2}{dr}. \qquad (30)$$

General solutions (19) for all three layers have the same structure, taking into account the boundedness of solutions at $r \to 0$ and $r \to \infty$ simultaneously. In this case, the boundaries of the first layer are equal to a and r_1, $a \le r \le r_1$. It is bounded from below (from the inside) by surface $r = a$, which in the limit can tend to zero, i.e. $a \to 0$ but cannot exceed r_1 in any way, i.e. cannot strive for infinity.

Therefore, when writing a general solution to the potential function of the first layer- $\varphi_1(r)$, one can restrict oneself to taking into account its limitations only at $r \to 0$. Based on this, we take the general solution (19) for the first layer in the form

$$\tilde{\psi}_1(r) = C_1^{(1)} I_0(\beta_1 r); \;\; (a \le r \le r_1), \qquad (31)$$

where $C_1^{(1)}$ - constant of integration.

Similarly, the boundaries of the second, outer layer are cylindrical surfaces $r = r_2$ and $r = b$; $r_2 \le r \le b$.

It is bounded from above (from the outside) by surface $r = b$, the radius of which can tend to infinity, i.e. $b \to \infty$. On the other hand, the inner surface of this layer cannot be pulled to a straight line, b.c. this would lead to a homogeneous round bar with a radius $r = b$.

Therefore, in the general solution for the potential function of the second layer- $\varphi_2(r)$, one can restrict oneself to taking into account its limitations only at $r \to \infty$.

Based on this, we take the general solution (19) for the second layer in the form

$$\tilde{\psi}_2(r) = C_2^{(2)} K_0(\beta_2 r); \;\; (r_2 \le r \le b) \qquad (32)$$

For the middle layer, we will accept the general solution (19), taking into account that our solution, in the absence of two outer layers, should transform into the known solution for a homogeneous cylindrical layer, limited at $r \to 0$ and $r \to \infty$, i.e.

$$\tilde{\psi}_0(r) = C_0^{(1)} I_0(\beta_0 r) + C_0^{(2)} K_0(\beta_0 r), \;\; r_2 \le r \le b. \qquad (33)$$

Thus, the number of integration constants to be determined from the contact conditions is reduced to two, $C_1^{(1)}$ and $C_2^{(2)}$. Therefore, there is no need for four contact conditions (11) and (12). Taking this circumstance into account, we restrict ourselves to only two contact conditions, leaving in (11) - (12) only the conditions of equality of displacements,

at $r = r_1$

$$u_{\theta 1}(z,t) = u_{\theta 0}(z,t) \qquad (34)$$

and at $r = r_2$

$$u_{\theta 2}(z,t) = u_{\theta 0}(z,t) \qquad (35)$$

Conditions (34) and (35) are equivalent to conditions (27) and (29), respectively. Substituting solutions (31), (32), and (33) into transformed boundary conditions (26) and contact conditions (27) and (29), we obtain

$$\left[\frac{2\beta_1}{a} I_1(\beta_1 a) - \beta_1^2 I_0(\beta_1 a)\right] C_1^{(1)} = \tilde{\mu}_1^{-1}[\tilde{f}_{r\theta}^{(1)}(k,p)], \qquad (36)$$

$$-\left[\frac{2\beta_2}{b} K_1(\beta_2 b) + \beta_2^2 K_0(\beta_2 b)\right] C_2^{(2)} = \tilde{\mu}_2^{-1}[\tilde{f}_{r\theta}^{(2)}(k,p)], \qquad (37)$$

$$\beta_1 I_1(\beta_1 r_1) C_1^{(1)} = \beta_0 I_1(\beta_0 r_1) C_0^{(1)} - \beta_0 K_1(\beta_0 r_1) C_0^{(2)},$$

$$-\beta_2 K_1(\beta_2 r_2) C_2^{(2)} = \beta_0 I_1(\beta_0 r_2) C_0^{(1)} - \beta_0 K_1(\beta_0 r_2) C_0^{(2)}.$$

From the last two equations we find

$$C_1^{(1)} = \frac{\beta_0 I_1(\beta_0 r_1) C_0^{(1)} - \beta_0 K_1(\beta_0 r_1) C_0^{(2)}}{\beta_1 I_1(\beta_1 r_1)}; \qquad (38)$$

$$C_2^{(2)} = -\frac{\beta_0 I_1(\beta_0 r_2) C_0^{(1)} - \beta_0 K_1(\beta_0 r_2) C_0^{(2)}}{\beta_2 K_1(\beta_2 r_2)}. \qquad (39)$$

Substituting (38) and (39) into boundary conditions (36), (37) and obtain the following system of equations

$$\frac{\frac{2}{a} I_1(\beta_1 a) - \beta_1 I_0(\beta_1 a)}{I_1(\beta_1 r_1)} \left[\beta_0 I_1(\beta_0 r_1) C_0^{(1)} - \beta_0 K_1(\beta_0 r_1) C_0^{(2)}\right] = \tilde{\mu}_1^{-1}[\tilde{f}_{r\theta}^{(1)}(k,p)], \qquad (42)$$

$$\frac{\frac{2}{b} K_1(\beta_2 b) + \beta_2 K_0(\beta_2 b)}{K_1(\beta_2 r_2)} \left[\beta_0 I_1(\beta_0 r_2) C_0^{(1)} - \beta_0 K_1(\beta_0 r_2) C_0^{(2)}\right] = \tilde{\mu}_2^{-1}[\tilde{f}_{r\theta}^{(2)}(k,p)], \qquad (43)$$

Let us express the transformed displacements of layers $\tilde{u}_{\theta m}$ in terms of solutions (31), (32) and (33). To do this, it is enough to recall formulas (20) for $\tilde{u}_{\theta m}(r,k,p)$ i.e.

$$\tilde{u}_{\theta m}(r,k,p) = -\frac{\partial \psi_m}{\partial r}, \qquad (44)$$

Substituting (31), (32), and (33) into (44) at $m=0;\, m=1;$ and $m=2$, we obtain, respectively

$$\begin{cases} \tilde{u}_{\theta 0}(r,k,p) = -\beta_0 I_1(\beta_0 r) C_0^{(1)} + \beta_0 K_1(\beta_0 r) C_0^{(2)}; \\ \tilde{u}_{\theta 1}(r,k,p) = -\beta_1 I_1(\beta_1 r) C_1^{(1)}; \\ \tilde{u}_{\theta 2}(r,k,p) = \beta_2 K_1(\beta_2 r) C_2^{(2)}. \end{cases} \qquad (44^*)$$

In the expression for the torsional displacement $\tilde{u}_{\theta 0}(r,k,p)$ of the middle layer, we expand the Bessel functions $I_1(\beta_0 r)$ and $K_1(\beta_0 r)$ in power series in the argument $(\beta_0 r)$, or, to put it another way, we use the standard expansions in power series of Bessel functions $I_1(\beta_0 r)$ and $K_1(\beta_0 r)$ in powers $(\beta_0 r)$. We get

$$\tilde{u}_{\theta 0}(r,k,p) = -\beta_0 C_0^{(1)} \sum_{n=0}^{\infty} \beta_0^{2n+1} \frac{(r/2)^{2n+1}}{n!(n+1)!} + \frac{1}{2} C_0^{(2)} +$$

$$+ \beta_0 C_0^{(2)} \sum_{n=0}^{\infty} \left\{ \ln \frac{\beta_0 r}{2} - \frac{1}{2}[\gamma(n+1) + \gamma(n+2)] \right\} \beta^{2n+1} \frac{(r/2)^{2n+1}}{n!(n+1)}$$

Combining the amounts here, we get finally

$$\tilde{u}_{\theta 0}(r,k,p) = \frac{1}{r} C_0^{(2)} + \sum_{n=0}^{\infty} \left\{ -C_0^{(1)} + C_0^{(2)} \left[\ln \frac{\beta_0 r}{2} - \frac{1}{2}(\gamma(n+1) + \gamma(n+2)) \right] \right\} \bullet \beta_0^{2n+2} \frac{(r/2)^{2n+1}}{n!(n+1)!} \qquad (45)$$

Here $\gamma(n)$ – is the logarithmic derivative of the Gamma function

$$\gamma(z) = \frac{\Gamma'(z)}{\Gamma(z)}.$$

Following the work [23] for the unknown values for the values of displacement and stress, calculated at the points of a certain “intermediate” surface of the middle layer. The radius of this surface is defined in the interval $\xi \in [r_1, r_2]$. At $\xi = r_1$ and $\xi = r_2$, this "intermediate" surface turns into contact surfaces between the layers, and at. It passes into the median surface of the filler. At $r_1 = r_2$, the radius of the intermediate surface passes into the radius of the contact surface between the bearing layers.

We put $r = \xi$ in the expression of the transformed displacement (2.1.31) and select its main parts, assuming that they are determined as the first terms of a converging power series, we obtain

$$\tilde{u}_{\theta 0}(\xi) = \frac{1}{\xi} C_0^{(2)} + \left\{ -C_0^{(1)} + C_0^{(2)} \left[\ln \frac{\beta_0 \xi}{2} - \gamma(1) - \frac{1}{2} \right] \right\} \beta_0^2 \left(\frac{\xi}{2}\right);$$

We introduce the following notation [23]

$$\begin{cases} \tilde{u}_{\theta 0}^{(1)} = \dfrac{1}{\xi} C_0^{(2)}; \\ \tilde{u}_{\theta 0}^{(0)} = \left\{ C_0^{(1)} - C_0^{(2)} [\ln \dfrac{\beta_0 \xi}{2} - \gamma(1) - \dfrac{1}{2}] \right\} \beta_0^2 \left(\dfrac{\xi}{2}\right). \end{cases} \quad (46)$$

where

$$C_0^{(2)} = \xi \tilde{u}_{\theta 0}^{(1)};$$

$$C_0^{(1)} = \tilde{u}_{\theta 0}^{(1)} \xi \left[\ln \frac{\beta_0 \xi}{2} - \frac{1}{2} - \gamma(1) \right] - \frac{2}{\beta_0^2} \tilde{u}_{\theta 0}^{(0)};$$

In order to express the transformed displacement $\tilde{u}_{\theta 0}(r,k,p)$ through the introduced new functions $\tilde{u}_{\theta 0}^{(0)}$ and $\tilde{u}_{\theta 0}^{(1)}$ transform (47) as follows

$$\tilde{u}_{\theta 0}(r,k,p) = \frac{1}{r} C_0^{(2)} + \sum_{n=0}^{\infty} \{2\beta_0^{2n} (-\frac{1}{2}\beta_0^2) \cdot [C_0^{(1)} - C_0^{(2)} (\ln \frac{\beta_0 \xi}{2} - \gamma(1) - \frac{1}{2})]\} \frac{(r/2)^{2n+1}}{n!(n+1)!} + \sum_{n=0}^{\infty} \beta_0^{2n+2} \cdot C_0^2 \left[\ln \frac{r}{\xi} - \frac{1}{2}\gamma(n+1) - \frac{1}{2}\gamma(n+2) + \frac{1}{2}\gamma(1) + \frac{1}{2} \right] \frac{(r/2)^{2n+1}}{n!(n+1)!} \quad (47)$$

In the last expression, we introduce the notation

$$\eta_1(n,r) = \ln \frac{r}{\xi} - \frac{1}{2}\gamma(n+1) - \frac{1}{2}\lambda(n+2) + \lambda(1) + \frac{1}{2}$$

or

$$\eta_1(n,r) = \ln \frac{r}{\xi} + \frac{n}{2(n+1)} - \sum_{k=1}^{n} \frac{1}{k} \quad (48)$$

Taking into account (48) and (46), expression (2.1.31) takes the form.

$$\tilde{u}_{\theta 0}(r,k,p) = \frac{\xi}{r} \tilde{u}_{\theta 0}^{(1)} + 2 \sum_{n=0}^{\infty} \beta_0^{2n} \tilde{u}_{\theta 0}^{(0)} \frac{(r/2)^{2n+1}}{n!(n+1)!} + \xi \sum_{n=0}^{\infty} \beta_0^{2n+2} \cdot \tilde{u}_{\theta 0}^{(1)} \eta_1(n,r) \frac{(r/2)^{2n+1}}{n!(n+1)!} \quad (49)$$

Note that if the middle layer is thin (for example, a thin layer of epoxy glue, usually applied between the layers), then we can assume that $r = \xi$. Then

$$\eta_1(n) = \frac{n}{2(n+1)n} - \sum_{k=1}^{n} \frac{1}{k} \quad (50)$$

which is a number, for example, for

$$n = 0 \quad \eta_1(0) = 0 \text{ а при } \quad n = 1 \quad \eta_1(1) = -\frac{3}{4}$$

In order to express the boundary conditions (42) (43) in terms of the main parts of the transformed displacement $\tilde{u}_{\theta 0}$ introduced by formulas (46), consider the following formula

$$\beta_0 [I_1(\beta_0 r_i) B_1 - K_1(\beta_0 r_i) B_2] = -\tilde{u}_{\theta 0} \quad (51)$$

Then, based on (49) for $\beta_0 [I_1(\beta_0 r_i) B_1 - K_1(\beta_0 r_i) B_2]$ will have (51)

$$\beta_0 [I_1(\beta_0 r_i) B_1 - K_1(\beta_0 r_i) B_2] = \frac{\xi}{r_i} \tilde{u}_{\theta 0}^{(1)} + \sum_{n=0}^{\infty} [2\tilde{u}_{\theta 0}^{(0)} + \xi \cdot \eta(n, r_i) \beta_0^2 \tilde{u}_{\theta 0}^{(1)}] \cdot \beta_0^{2n} \frac{(r_i/2)^{2n+1}}{n!(n+1)!}$$

Taking into account (51), equations (42) and (43) can be written in the form

$$\frac{\frac{2}{a} I_1(\beta_1 a) - \beta_1 I_0(\beta_1 a)}{I_1(\beta_1 r_1)} \cdot \left\{ \sum_{n=0}^{\infty} \beta_0^{2n} \left[2\tilde{u}_{\theta 0}^{(0)} + \xi \eta_1(n, r_1) \cdot \beta_0^2 \tilde{u}_{\theta 0}^{(1)} \right] \frac{(r_1/2)^{2n+1}}{n!(n+1)!} + \frac{\xi}{r_1} \tilde{u}_{\theta 0}^{(1)} \right\} = \tilde{\mu}_1^{-1} [\tilde{f}_{r\theta}^{(1)}(k,p); \quad (53)$$

$$\frac{\frac{2}{b} K_1(\beta_2 b) + \beta_2 K_0(\beta_2 b)}{K_1(\beta_2 r_2)} \cdot \left\{ \sum_{n=0}^{\infty} \beta_0^{2n} \left[2\tilde{u}_{\theta 0}^{(0)} + \xi \eta_1(n, r_2) \cdot \beta_0^2 \tilde{u}_{\theta 0}^{(1)} \right] \frac{(r_2/2)^{2n+1}}{n!(n+1)!} + \frac{\xi}{r_2} \tilde{u}_{\theta 0}^{(1)} \right\} = \tilde{\mu}_2^{-1} [\tilde{f}_{r\theta}^{(2)}(k,p); \quad (54)$$

For combinations of Bessel functions, limiting ourselves in their expansions to zero and first approximations, we obtain

at $m = 0$

$$\frac{2}{b} K_1(\beta_2 b) + \beta_2 K_0(\beta_2 b) = \frac{2}{b} \left[\left(\ln \frac{\beta_2 b}{2} + c - \frac{1}{2} \right) \frac{\beta_2 b}{2} + \frac{2}{\beta_2} \right] + \beta_2 \left[-\left(\ln \frac{\beta_2 b}{2} + c \right) \right] = -\frac{1}{2}\beta_2 + \frac{4}{\beta_2 b^2} = -\frac{1}{2\beta_2} \left(\frac{8}{b^2} - \beta_2^2 \right);$$

$$K_1(\beta_2 r_2) = \frac{1}{2\beta_2} \left[\frac{4}{r_2} + \frac{r_2}{2} \left(\ln \frac{\beta_2 r_2}{2} + c - \frac{1}{2} \right) \right] \approx$$

$$\approx \frac{1}{2\beta_2} \left[\frac{4}{r_2} + \frac{r_2}{2} \left(c - \frac{1}{2} \right) \beta_2^2 \right],$$

at $m = 1$

$$\frac{2}{a} I_1(\beta_1 a) - \beta_0 I_0(\beta_1 a) = \frac{2}{a} \cdot \left(\frac{\beta_1 a}{4} + \frac{(\beta_1 a)^3}{16} \right) - \beta_1 \left(1 + \frac{\beta_1^2 a^2}{4}\right) = \frac{\beta_1}{2} + \frac{\beta_1^3 a^2}{8} - \beta_1 - \frac{\beta_1^3 a^2}{4} =$$

$$= -\frac{1}{2}\beta_1 - \frac{\beta_1^3 a^2}{8} = -\frac{1}{2}\beta_1 \left(1 + \frac{a^2}{4}\beta_1^2\right);$$

$$I_1(\beta_1 r_1) = \frac{\beta_1 r_1}{4} \left(1 + \frac{r_1^2}{4}\beta_1^2\right).$$

Hence

$$\frac{\frac{2}{a}I_1(\beta_1 a)-\beta_1 I_0(\beta_1 a)}{I_1(\beta_1 r_1)}=-\frac{2}{r_1}\cdot\frac{1+\frac{a^2}{4}\beta_1^2}{1+\frac{r_1^2}{4}\beta_1^2}; \quad (55)$$

$$\frac{\frac{2}{b}K_1(\beta_2 b)+\beta_2 K_0(\beta_2 b)}{K_1(\beta_2 r_2)}=-\frac{2}{r_2}\cdot\frac{\frac{8}{b^2}-\beta_2^2}{\frac{8}{r_2^2}+(c-\frac{1}{2})\beta_2^2}, \quad (56)$$

where $c=\ldots$ is the number

Substituting (55) and (56) into equations (54), we have

$$-\frac{2}{r_1}\left(1+\frac{a^2}{4}\beta_1^2\right)\left\{\sum_{n=0}^{\infty}\beta_0^{2n}\left[2\tilde{u}_{\theta0}^{(0)}+\xi\eta_1(n,r_1)\beta_0^2\tilde{u}_{\theta0}^{(1)}\right]\cdot\right.$$
$$\left.\cdot\frac{(r_1/2)^{2n+1}}{n!(n+1)!}+\frac{\xi}{r_1}\tilde{u}_{\theta0}^{(1)}\right\}=\tilde{\mu}_1^{-1}\left[\left(1+\frac{r_1^2}{4}\beta_1^2\right)\tilde{f}_{r\theta}^{(1)}(k,p)\right];$$
$$-\frac{2}{r_2}\left(\frac{8}{r_2^{*2}}-\beta_2^2\right)\left\{\sum_{n=0}^{\infty}\beta_0^{2n}\left[\left[2\tilde{u}_{\theta0}^{(0)}+\xi\eta_1(n,r_2)\beta_0^2\tilde{u}_{\theta0}^{(1)}\right]\right.\cdot\right.$$
$$\left.\cdot\frac{(r_2/2)^{2n+1}}{n!(n+1)!}+\frac{\xi}{r_2}\tilde{u}_{\theta0}^{(1)}\right\}=\tilde{\mu}_2^{-1}\left[\left(\frac{8}{r_2^2}+\left(c-\frac{1}{2}\right)\beta_2^2\right)\tilde{f}_{r\theta}^{(2)}(k,p)\right] \quad (57)$$

Let us rewrite (57) equation in a more convenient form for subsequent use

$$\left(1+\frac{a^2}{2}\beta_1^2\right)\left\{\sum_{n=0}^{\infty}\beta_0^{2n}\left[2\tilde{u}_{\theta0}^{(0)}+\xi\eta_1(n,r_1)\beta_0^2\tilde{u}_{\theta0}^{(1)}\right]\cdot\right.$$
$$\left.\cdot\frac{(r_1/2)^{2n+1}}{n!(n+1)!}+\frac{\xi}{r_1}\tilde{u}_{\theta0}^{(1)}\right\}=-\frac{r_1}{2}\tilde{\mu}_1^{-1}\left[\left(1+\frac{r_1^2}{4}\beta_1^2\right)\tilde{f}_{r\theta}^{(1)}(k,p)\right],$$
$$\left(1-\frac{b^2}{8}\beta_2^2\right)\left\{\sum_{n=0}^{\infty}\beta_0^{2n}\left[2\tilde{u}_{\theta0}^{(0)}+\xi\eta_1(n,r_2)\beta_0^2\tilde{u}_{\theta0}^{(1)}\right]\frac{(r_2/2)^{2n+1}}{n!(n+1)!}+\right.$$
$$\left.+\frac{\xi}{r_2}\tilde{u}_{\theta0}^{(1)}\right\}=-\frac{b^2}{2r_2}\tilde{\mu}_2^{-1}\left\{\left[\left(1+\left(c-\frac{1}{2}\right)\frac{r_2^{*2}}{8}\beta_2^2\right)\tilde{f}_{r\theta}^{(2)}(k,p)\right]\right.$$

Let us introduce functions $u_{\theta0}^{(0)}$, $u_{\theta0}^{(1)}$ and operators λ_m^n by the formulas

$$\left[u_{\theta0}^{(1)},u_{\theta0}^{(1)}\right]=\int_0^{\infty}\left.\begin{matrix}\sin kz\\ -\cos kz\end{matrix}\right\}dk\int_{(\ell)}\left(\tilde{u}_{\theta0}^{(0)},\tilde{u}_{\theta0}^{(1)}\right)e^{pt}dp;$$
$$\chi_m^n(\zeta)=\int_0^{\infty}\left.\begin{matrix}\sin kz\\ -\cos kz\end{matrix}\right\}dk\int_{(\ell)}\beta_m^{2n}(\zeta)e^{pt}dp, \quad (58)$$

Inverting conditions (58) over p and k, taking into account (57), we obtain

$$\sum_{n=0}^{\infty}\lambda_0^n\left(1+\frac{a^2}{2}\lambda_1\right)\left[2u_{\theta0}^{(0)}+\xi\eta_1(n,r_1)\lambda_0 u_{\theta0}^{(1)}\right]\frac{(r_1/2)^{2n+1}}{n!(n+1)!}+$$
$$+\frac{\xi}{r_1}\left(1+\frac{a^2}{2}\lambda_1\right)u_{\theta0}^{(1)}=-\frac{r_1}{2}\mu_1^{-1}\left[\left(1+\frac{r_1^2}{4}\lambda_1\right)f_{r\theta}^{(1)}(z,t)\right]; \quad (59)$$
$$\sum_{n=0}^{\infty}\lambda_0^n\left(1-\frac{b^2}{8}\lambda_2\right)\left[2u_{\theta0}^{(0)}+\xi\eta_1(n,r_2)\lambda_0 u_{\theta0}^{(1)}\right]\frac{(r_2/2)^{2n+1}}{n!(n+1)!}+$$
$$+\frac{\xi}{r_2}\left(1+\frac{a^2}{2}\lambda_2\right)u_{\theta0}^{(1)}=-\frac{b^2}{2r_2}\mu_2^{-1}\left[\left(1+(c-\frac{1}{2})\frac{r_2^2}{8}\lambda_2\right)f_{r\theta}^{(2)}(z,t)\right] \quad (60)$$

Let us introduce the following notation [......]

$$A_{1i}=2\sum_{n=0}^{\infty}\lambda_0^n\frac{(r_i/2)^{2n+1}}{n!(n+1)!};$$
$$A_{2i}=\frac{1}{r_i}+\sum_{n=0}^{\infty}\eta_1(n,r_i)\lambda_0^{n+1}\frac{(r_i/2)^{2n+1}}{n!(n+1)!}; \quad (61)$$

Taking into account (61), Eqs. (60) can be rewritten in a more convenient form for what follows:

$$\left(1+\frac{a^2}{2}\lambda_1\right)\left[A_{11}u_{\theta0}^{(0)}+\xi A_{12}u_{\theta0}^{(1)}\right]=$$
$$=-\frac{r_1}{2}\mu_1^{-1}\left[\left(1+\frac{r_1^2}{4}\lambda_1\right)f_{r\theta}^{(1)}(z,t)\right];$$
$$\left(1-\frac{b^2}{8}\lambda_2\right)\left[A_{12}u_{\theta0}^{(0)}+\xi A_{22}u_{\theta0}^{(1)}\right]=$$
$$=-\frac{b^2}{2r_2}\mu_2^{*2}\left\{\left[1+\left(c-\frac{1}{2}\right)\frac{r_2^2}{8}\lambda_2\right]f_{r\theta}^{(2)}(z,t)\right\} \quad (62)$$

Based on the expressions for β_m (44)*, it is easy to obtain that the operators λ_m^n, introduced by formulas (58), with the reverse transition according to Fourier and Laplace, in variables z,t have the following forms

$$\lambda_m^n(\zeta)=\left[\rho_m\mu_m^{-1}\left(\frac{\partial^2\zeta}{\partial t^2}\right)-\left(\frac{\partial^2\zeta}{\partial z^2}\right)\right]^n, \quad (63)$$

where μ_m – elastic operators of layer materials.

Equations (62) in accordance with formulas (63) for operators λ_m^n are integro-differential equations of unbounded order. These equations contain the main parts $u_{\theta0}^{(0)}$ and $u_{\theta0}^{(1)}$ of the torsional displacement of $u_{\theta0}$ points of a certain “intermediate” surface of the middle layer of a three-layer conical shell. The specified "intermediate" surface has a radius, the values of which are included in the interval $r_1\le\xi\le r_2$. In accordance with the numerical value of the radius ξ, this “intermediate” surface can pass into the middle one at $\xi=\frac{r_1+r_2}{2}$ and the contact between the layers of the shell surface at $\xi=r_1$ and $\xi=r_2$. Consequently, equations (62), depending on the values of the radius ξ, can be the equations of oscillation of a three-layer cylindrical shell relative to the main parts of the torsional displacement of the points of the middle or contact surfaces of the middle layer.

The obtained equation in particular cases transforms into the equations of vibration of a two-layer elastic conical shell, into the equations of vibration of a single-layer elastic conical shell, and others.

Results and Discussions

Let's consider the problem of longitudinal-radial vibrations of a shield clamped in the longitudinal direction, at $z=0$ and $z=l$, where l - length conical shell in the direction of the axis Oz. As vibration equations, we take the system (62). The boundary conditions of the problem have the form

$$u_{\theta 0}^{(0)}=0;\ \frac{\partial^2 u_{\theta 0}^{(0)}}{\partial z^2}=0;\ \frac{\partial u_{\theta 0}^{(1)}}{\partial z}=0;\ \frac{\partial^3 u_{\theta 0}^{(1)}}{\partial z^3}=0\cdot$$

The initial conditions are assumed to be zero.

The solution of the system of equations (62), which includs the conditions for fixing the ends, and also the functions of external actions, is represented in the form.

$$u_{\theta 0}^{(0)}=\sum_{m=1}^{\infty}\tilde{u}_{\theta 0}^{(0)}(t)\sin\frac{m\pi z}{l};\ u_{\theta 0}^{(1)}=\sum_{m=1}^{\infty}\tilde{u}_{\theta 0}^{(1)}(t)\cos\frac{m\pi x}{l};$$

$$f_{r\theta}=\sum_{m=1}^{\infty}f_{r\theta m}(t)\sin\frac{m\pi z}{l};\qquad (64)$$

The substitution of (64) into (62) leads to a system of two fourth-order differential equations with respect to the functions $\tilde{u}_{\theta 0}^{(0)}(t)$ and $\tilde{u}_{\theta 0}^{(1)}(t)$. The problem was solved numerically at the following values of the physico-mechanical and geometric parameters of the three-layer conical shell:
$\xi=0.9h_0$; $l=0.4m$; $r_0=0.04m$; $d=0.005m$; $r_2=0.08m$; $\rho_0=30kg/m^3$; $\rho_1=2700kg/m^3$; $\rho_2=2700kg/m^3$; $E_0=0.165\cdot 10^9\ Pa$; $E_1=69\cdot 10^9\ Pa$; $E_2=69\cdot 10^9\ Pa$; $\nu_0=0.03125$; $\nu_1=0.33$; $\nu_2=0.33$; $f_{zm}(t)=3t^2$. The results are shown in Fig. 2-5 in the form of graphs of the longitudinal and transverse displacements of the points of the middle layer and normal stresses in its various sections.

Conclusion

From the presented graphs in Fig. 2-3 it follows that the longitudinal displacement of the points of different sections reach their maximum at values between 0.6 and 0.8 of the dimensionless time. Negative values of longitudinal displacement indicate that the shield for weight the period of action of the external load undergoes compression.

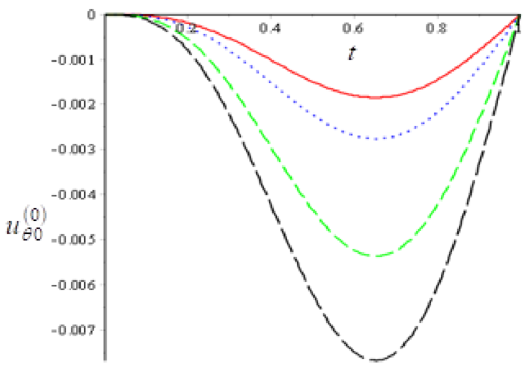

Fig. 2. Dependence of displacement $u_{\theta 0}^{(0)}$ on time at $z=0.2$ (—); 0.3 (.....); 0.4(– –); 0.6(—.).

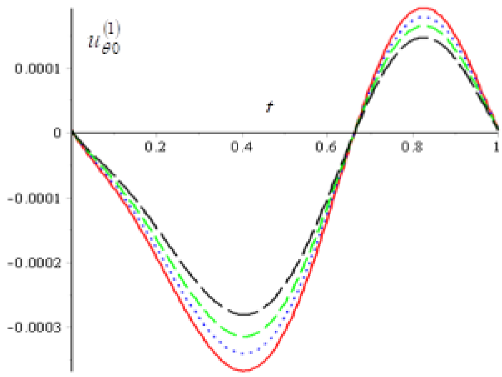

Fig. 3. Dependence of displacement $u_{\theta 0}^{(1)}$ on time at $z=0.2$ (—); 0.3 (.....); 0.4 (– –); 0.6(—.).

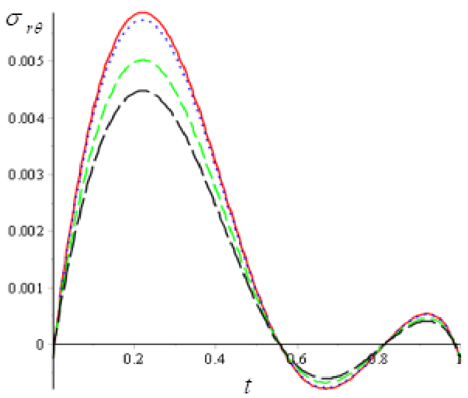

Fig. 4. Dependence of displacement $\sigma_{r\theta}$ on time $z=0.2$ (—); 0.3 (.....); 0.4 (– –); 0.6(—.).

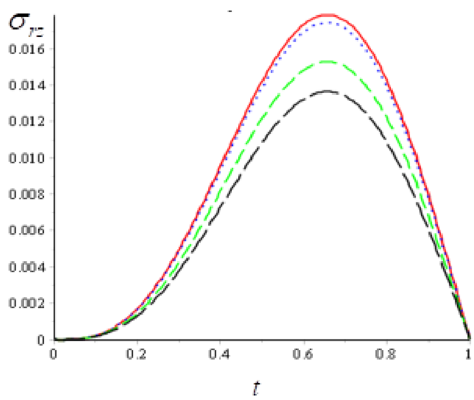

Fig. 5. Dependence of displacement σ_{rz} on time at $z=0.2$ (—); 0.3 (.....); 0.4 (– –); 0.6(—.).

The transverse displacement of the point of the cross sections has a sinusoidal character as a function of time. At the same time, it reaches its maximum at a value of the dimensionless time close to 0.4. The maximum value of the longitudinal displacement corresponds to the zero value of the lateral displacement. In addition, at the beginning of the process and further to the time value of 0.63-0.66, the transverse displacement is negative, and at 0.65 <t <0.7. Further, it remains positive with a relative maximum at 0.82.

Following graphs (Figs. 4-5) are in good agreement with the dependencies of displacements, having relative maxima at the points where the displacements are minimal. At the points of maximum displacement values, it should be noted that corresponding stresses are minimal.

References:

1. Hudojnazarov, H.H. (2003). *Nestacionarnoe vzaimodejstvie krugovyh cilindricheskih uprugih i vjazkouprugih obolochek i sterzhnej s deformiruemoj sredoj*. (p.325). Tashkent: izd-vo imeni Abu Ali ibn Sina.
2. Filippov, I.G., & Filippov, S.I. (2007). *Kolebatel`nye i volnovye processy v sploshnyh szhimaemyh sredah*. (p.430). Moscow: Izd-vo MGSU.
3. Markus, S. (1988). *The mechanics of cylindrical shells*. (p.195). Amsterdam: Elsevier".
4. Netrebko, A.V., & Pshenichnov, S.G. (2015). Nekotorye zadachi dinamiki linejno- vjazkoup-rugih cilindricheskih obolochek konechnoj dliny. *Problemy prochnosti i plastichnosti*, t.77, №1, pp.14-22.
5. Francesco, P. (2007). Vibrations of circular cylindrical shells: Theory and experiments. *Journal of Sound and Vibration*, 303, 154-170.
6. Khalmuradov, R.I., & Khudoynazarov, Kh.Kh. (2002). *Theory of axisymmetrical vibrations of circular cylindrical shells.* The 7th Conference "Shell Structures, Theory and Applications", Gdansk-Jurata (Poland), October 9-11, 2002. (pp.131-132). Gdansk: Gdansk University of Technology.
7. Hudojnazarov, H.H., Skripnjak, V.A., & Jahshiboev, Sh. (2018). Nestacionarnye poperechnye kolebanija trehslojnoj vjazkouprugoj plastinki. *Uzbekskij zhurnal Problemy Mehaniki*, №2, pp.27-32.
8. Jalgashev, B.F., & Berdiev, Sh.D. (2020). Uravnenija krutil`nyh kolebanij trehslojnoj krugovoj cilindricheskoj uprugoj obolochki. *Problemy arhitektury i stroitel`stva*, №2, pp.120-125.
9. Filippov, I. G., & Kudainazarov, K. (1990). Refinement of equations describing longitudinal-radial vibrations of a circular cylindrical viscoelastic shell. *Soviet Applied Mechanics*, 26(2), 161-168. doi:10.1007/bf00887110.
10. Hudojnazarov, H.H., & Burkutboev, Sh.M. (2017). Matematicheskaja model` krutil`nyh kolebanij cilindricheskogo sloja s uchetom protekaushhej zhidkosti i vrashhenija. *Matematicheskoe modelirovanie i chislennye metody*, № 4, pp.31-56. DOI: Retrieved from https://doi.org/10.18698/2309-3684-2017-4-3147
11. Hudojnazarov, H.H., & Jalgashev, B.Ja. (2017). *Vzaimodejstvie cilindricheskih sloev i obolochek s vjazkoj zhidkost`u*, (p.138). LAMBERT Academic Publishing.