

MULTIPLYING AND DIVIDING IN THE FISCAL PROCESS. AN ALGEBRAIC APPROACH

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Abstract: *The paper aims to examine two paired phenomena in the fiscal process (more exactly, in the budgetary obligations collecting), namely multiplying and dividing. The multiplier of the fiscal collecting indicates how and how much the fiscal evasion could increase the budget receipts (if the fiscal administration does work), while the fiscal divisor indicates how and how much the fiscal evasion could decrease the budget receipts (if the fiscal administration doesn't work). The two indicators are algebraically modelled, and some empirical data are used in order to illustrate the mechanisms of multiplying and dividing, respectively, regarding the budgetary receipts when the voluntarily conformation to pay the budgetary obligations leaves to be still desired.*

Keywords: *fiscal evasion, fiscal multiplier, fiscal divisor, budgetary receipts.*

JEL Classification: *B22, B41, C02.*

1. Non-technical introduction

The economic process is one of social inter-actions of goals, means, and actions. Consequently, this process is characterized by causal, structural, and functional relationships. Moreover, many interactions are „endowed” with feedbacks (either negative or positive), feedforwards (anticipations), multiplying and dividing of the economic variables involved (Dinga, 2009). The paper aims to examine the phenomena of multiplying and of dividing assigned to the fiscal-budgetary process, more exactly linked to the fiscal evasion phenomenon. By fiscal evasion is understood that behaviour of an economic actor (consumer, producer, trader, investor, financier etc.) which brings monetary prejudices to the state – more exactly, to the public budget. Since the analysis developed within the paper is focused on the fiscal evasion, it is useful to additionally clarify this concept. To this end, we provide the following considerations in the matter:

- one of the market failure is the incapacity of the free decentralised market to provide public goods (the public goods are those goods which are, inter alia, free to consumption and non-rivalry, that is they are at free disposal of the individuals and their consumption is, principled, unlimited); such incapacity is not a defect of the free market, by the contrary, it is a logical consequence of the free market criterion of behaviour – profit maximizing;
- so, the public goods (called also as public positive externalities – PPE) must be produced and distributed by the state;
- to finance the public goods supply (Dinga, 2018), the state collects, based on the law, mandatory contributions from individuals in the society (either as physical persons or juridical ones) called taxes;
- in the process of such a collecting, occurs the phenomenon of the fiscal evasion, which has as result the diminishing of tax amount collected by the fiscal administration of the state compared with the tax amount due by law (we do not discuss in the present paper more about the fiscal evasion, because it is not the main subject of the study).

In the fiscal evasion process could occurs phenomena of multiplying (for example of the fiscal evasion itself or, by the contrary, of the due tax amount to the public budget) or, after the case, of dividing. In fact, just these phenomena will be further put under our analysis.

2. The concept of multiplier in the economic process

Generally, a multiplier (Cloyne, , et al., 2020) is a coefficient with which a base variable must be multiplied in order to obtain a result conditioned by that base variable. To logically understand the abstract concept of multiplier, some brief clarifications are of usefulness:

- *firstly*, a multiplier must be supra-unitary (being noted a given multiplier with m , then the inequality $m > 1$ must be held);

- *secondly*, between the base variable (V_b), and the result of applying the multiplier (V_m) must subsist a causal relationship, that is, $V_m = f(V_b)$; in fact, the multiplier uses the common multiplication algebraic operation to pass from V_b to V_m , that is

simply

$$V_m = m \cdot V_b;$$

- *thirdly*, the algebraic relationship between base variable and the multiplied variable is reversible, that is, if the tow values are known, the multiplier results algebraically: $m = \frac{V_m}{V_b}$ (the condition of existence, that is, $V_b \neq 0$, is obviously

verified);

- *fourthly*, the function of the multiplied variable can be either linear ($V_m = m \cdot V_b$) or affine ($V_m = n + m \cdot V_b$, where $n \in \mathbb{R}$, and $n \neq 0$); in the most cases, in

economic analyses are used the linear functions of multiplier but, of course, could arise cases in which a constant is held although the multiplier is null;

- *sixthly*, the way of multiplier action is based on mechanisms of impulse transmission (see, for example, the well-known multiplier of the autonomous, i.e., public, investment: $m_{ai} = \frac{1}{s}$, where with s has been noted the marginal

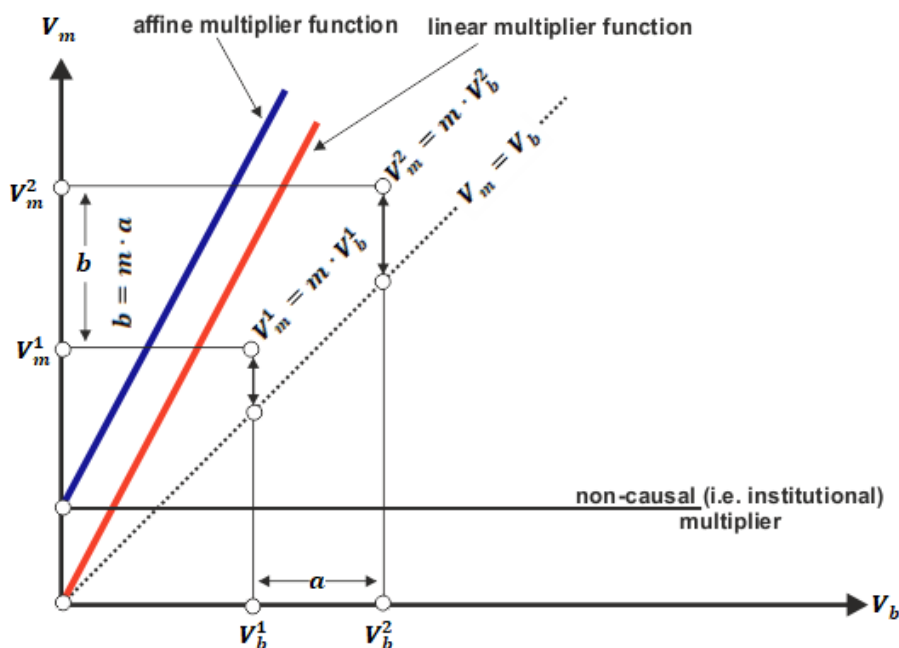
propensity to saving;

- *seventhly*, the multiplier coefficient can transform into a divisor one, when the economic process suffers a radical change in its propensity; to be mentioned such a reversibility is indefinite possible (if there is just a cyclicity in the reversibility in case should, of course, examined separately from the present research);

- *eighthly*, the multiplier question (Dinga., 2018) can be studied from the point of view of its stabilization (or not); for example, in such a case could be studied a so called echo function, which describe the trajectory of an impulse introduced in the economic process – such an impulse, either fiscal or monetary or of any other kind, will lose its potential to multiply the base variable concerned after a while, and this amortization can be algebraically described and analysed.

Figure 1 gives a synoptic image of the concept of economic multiplier (in fact, of any multiplier) (Favero & Karamysheva, 2017).

Figure 1. The abstract mechanism of a multiplier



Source: author

3. The concept of divisor in the economic process

The concept of divisor is the mirrored image of the multiplier. Algebraically, there is no difference between multiplier and divisor, except the numerical value of the coefficient through which the result variable is obtained from the base one. So, if is noted with d the divisor coefficient, the we can write: $V_d = d \cdot V_b$, under the condition $d < 1$. To be mentioned all the considerations done for the multiplier are valid also for the divisor, mutated mutandis (for example, in Figure 1, all the results given by the dividing process will be situated under the prime bisecting. In the economic field there are many cases in which the divisor works, so we'll consider some cases:

- any tax rate divides the base variable (tax base), so giving a net value of it;
- inflation divides the purchasing power of any nominal income;
- the rate of minimal mandatory reserves, enacted by the central bank and assigned to commercial bank deposits, divides those deposits.

4. Multiplying and dividing in the fiscal field

4.1. The concept of public money

This concept is not outside confusions and misunderstanding, so we'll provide some considerations synthesized from the online specialized literature (Dinga., 2018).

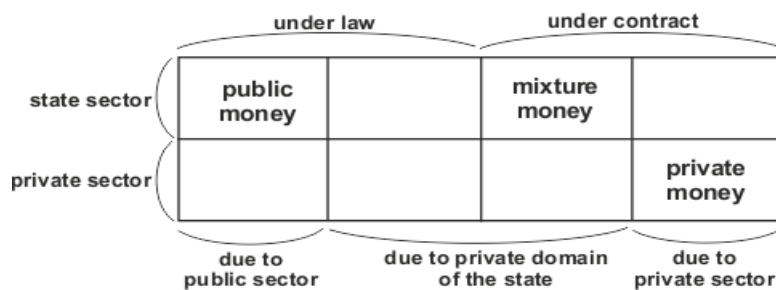
- money must be classified by its nature;
- the nature of money is identifiable based on the criterion: property right;
- the property right must be viewed from the perspective of the final due destination of money in case;
- there are, based on the above, three kinds of money:
 - public money: that money, no matter who is its owner, which is due, by law, to the state as government, without a correspondingly counterpart (for example: the tax included in the gross wage is public money, as well as the

VAT included in the price charged by the seller, although that money is owned by the worker or by the seller);

- private money: that money which is due, by contract, to someone, except the state, no matter the particular reason of such an obligation to transfer money;
- mixture money: that money due to the state, based on contract, not on the law.

In the present paper the public money is of interest only (see Figure 2).

Figure 2. The typology of money



(with the permission of publication *The Market for Ideas*).

4.2. The concept of fiscal field

By fiscal field is understood that field of economic activity in which are collected and spent the public money. Not all public institutions and authorities are included into the fiscal field – for example, National Bank of Romania, although is completely publicly capitalized, has private money; the same, Financial Supervisory Authority works under the aegis of private money). Typically, the fiscal field is associated with the Government, more specifically, with Ministry of Public Finance, and much more specifically, with the functioning of the consolidated general budget (CGB). So, the fiscal field can be defined as the *set of activities and transactions linked to collecting and spending the public money, under the law*.

4.3. The concept of fiscal evasion

Generally, the fiscal evasion is a behaviour of someone who, with or without intention, do not pay, voluntarily, fully and on time, his/her/its obligations to the state (another term here is the budgetary obligations). In present terminology, it is about of avoiding (or about of evading from) to pay the public money – for example, the tax on personal income, the VAT, the customs taxes and so on. Beyond the civil component (the amount due to the state) the fiscal evasion behaviour constitutes also a criminal offence and is criminally punished.

5. Algebraic modelling of the fiscal evasion multiplier

In the fiscal evasion phenomenon occurs processes of multiplying. Let's put some notations:

- *FE*: the monetary amount of fiscal evasion, as it is discovered by the control of fiscal administration;
- *PP*: the monetary amount of pecuniary penalty (civil punishment);

- k : the coefficient of penalization for the lag time of delaying to pay the budgetary obligations;
- d : the duration of delaying to pay the budgetary obligations (according to the measurement units used to define the coefficient d);
- DP : the monetary amount of delaying penalization.

So, the multiplier of fiscal evasion could be formalized as:

$$m_{FE} = \frac{FE+PP+DP}{FE} = 1 + \frac{PP+DP}{FE} = 1 + \frac{PP+k \cdot d \cdot FE}{FE} = 1 + \frac{PP}{FE} + k \cdot d = 1 + \varphi \quad (1)$$

where with φ is noted the expression $\frac{PP}{FE} + k \cdot d$, which could be called as the *value added of the discovered fiscal evasion* (indeed, much of fiscal evasion remains undiscovered either due to the incompetency of the fiscal administration or due to the corruption of fiscal administrations civil servants. As PP and k are done by law (that is, they are constant), we can note PP with α , so the fiscal evasion multiplier becomes:

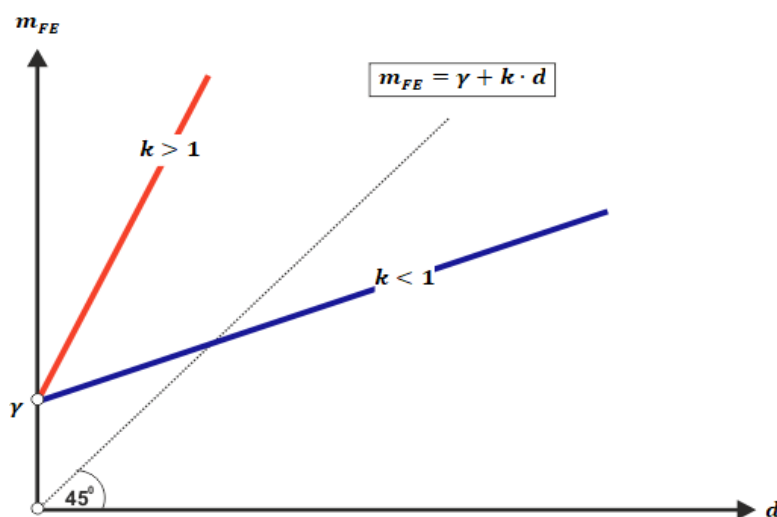
$$m_{FE} = 1 + \frac{\alpha}{x} + k \cdot d \quad (2)$$

where $FE = x$, because it is the main economic variable in the formula (although d is also a variable, it not of economic kind, but of calendar kind. In fact, the algebraic formula of the fiscal evasion multiplier is three-variate polynomial (it is depending on two independent variables). A didactical case of the three-dimensional graph of the fiscal evasion multiplier can be viewed in Figure 4.

From a qualitative point of view, if would be considered the fiscal evasion multiplier as depending on a single independent variable, we have the following analytical conclusions:

- (a) for x given: $m_{FE} = \gamma + k \cdot d$, where $\gamma = 1 + \frac{\alpha}{x} = \text{constant}$ (Figure 3)

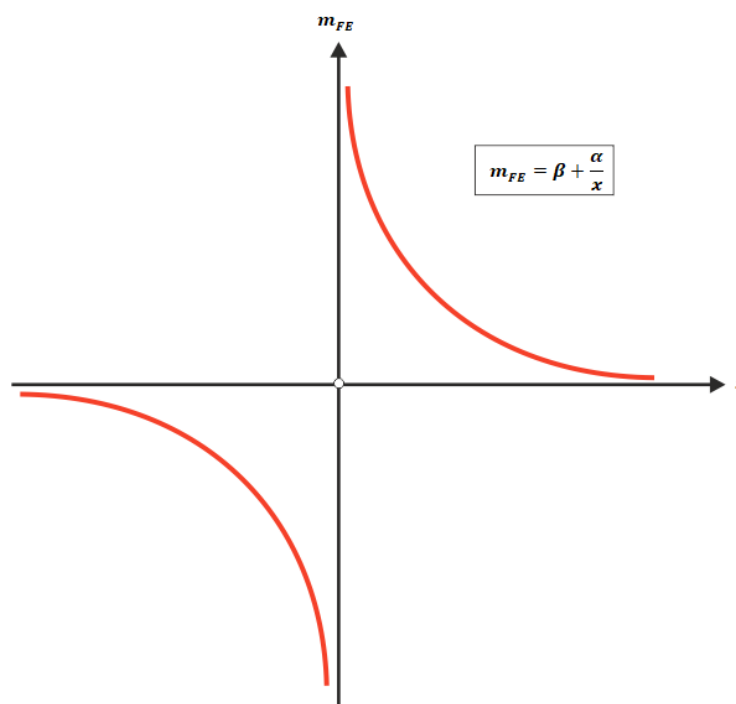
Figure 3. The fiscal evasion multiplier for the affine case



Source: author.

(b) for d given: $m_{FE} = \beta + \frac{\alpha}{x}$, where $\beta = 1 + k \cdot d = \text{constant}$ (Figure 4)

Figure 4. The fiscal evasion multiplier for the hyperbolic case



Source: author.

6. Modelling of the fiscal evasion divisor

To deal with the fiscal evasion divisor it must firstly to describe the economic process by which the phenomenon of dividing occurs. Further such a description is provided.

- when fiscal evasion happens, a some amount of public money (that is, of private money due, under the law, to the state) is not received by the state (the public budget);
- the decrease of the fiscal revenues leads to an increase of the fiscal (or budgetary) deficit, since the expenditures committed by the government must be done no matter what happens with the fiscal programmed revenues;
- as consequence, such increasing of the budgetary deficit must be financed by increasing the public debt;
- increasing of the public debt means an increase with the interest which must be paid to the new holders of the bonds;
- such payments with the interests assigned to the increased public debt constitutes the economic content of the concept of fiscal evasion divisor.

Let's make some notations:

- BD_{FE} : the budget deficit increasing due to the fiscal evasion FE (obviously,

$$BD_{FE} = FE);$$

- PD_{FE} : the public debt caused by fiscal evasion FE (obviously, $PD_{FE} = BD_{FE}$);

- r_{PD} : the rate of interest (yield rate) paid by the government for the public debt (assigned to the bonds issued);
- t : the maturity horizon of the bonds issued in financing the public debt caused by fiscal evasion (typically t could be 10; so, these bonds accounts for the Maastricht criterion of real economic convergence with EU, namely the nominal interest rate on long term);
- ω : the value lost by the undiscovered fiscal evasion.

So, it can be written:

$$\omega = FE + r_{PD} \cdot t \cdot FE = FE \cdot (1 + r_{PD} \cdot t) \quad (3)$$

$$d = \frac{FE}{FE \cdot (1 + r_{PD} \cdot t)} = \frac{1}{1 + r_{PD} \cdot t} \quad (4)$$

Since $r_{PD} > 0$, and $t > 0$, it results that $r_{PD} \cdot t > 0$, so $1 + r_{PD} \cdot t > 1$, so $d < 1$, so d is a divisor of fiscal evasion.

Some qualitative conclusions can be drawn here:

- like m , d is a three variate function, that is, we have two independent variables: r_{PD} and t ;
- unlike m , d function is symmetric between r_{PD} and t ; so, no matter which of the two variables is presupposed constant, the d function depending on the another has the same graphical image.

7. On the total multiplier-divisor effect of the fiscal evasion

It is of theoretical and fiscal policy (Flynn, et al., 2020) interest to put together the fiscal evasion multiplier and the fiscal evasion divisor in order to find a total monetary influence of the two processes linked to the fiscal evasion. In is noted with ω the total influences of the two mentioned processes, then, from the algebraical formalizations it can be written:

$$\omega = 1 + \frac{PP}{FE} + k \cdot d + \frac{1}{1 + r_{PD} \cdot t} \quad (5)$$

8. Conclusions

The paper has examined the phenomena of multiplying and of dividing assigned to the fiscal-budgetary process, more exactly linked to the fiscal evasion phenomenon. From the logical standpoint was defined the concept of economic multiplier like been a coefficient with which a base variable must be multiplied in order to obtain a result conditioned by that base variable and were been brought some clarifications on this matter.

For the concept of divisor, which is the mirrored image of the multiplier, arguments of an algebraic nature were brought in support of the statement that there is no difference between multiplier and divisor, except for the numerical value of the coefficient by which the resulting variable is obtained from the first. Continuing the paper, the concepts of fiscal

field and tax evasion were defined and the algebraic modeling of the tax evasion multiplier was carried out.

The author's scientific concerns in this field will be concretized with another paper, in which we will examine qualitatively and quantitatively (including empirical data) the total influences on the public budget of the phenomena of multiplication and division in the tax evasion process.

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