RELATIVE ORDERING TESTS FOR DRAFT FORCE MODELS IN SOIL TILLAGE
/
TESTE DE ORDONARE RELATIVA PENTRU MODELE ALE FORTEI DE REZISTENTA LA TRACTIUNE IN LUCRARILE SOLULUI

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ABSTRACT

The article presents results of some test sketches - validation and ordering - of the mathematical models proposed for the physical law rank on soil tillage draft force. The results constitute the continuation and partial completion of a method of testing - validation and ordering - the models proposed and published by researchers in the specialized literature of the last seventy years. The material defines and completes the method (initially only a validation method), up to a method of ordering the models according to their accuracy in relation to the experimental results. The proposed tests are intended to increase the coherence of research in the field of searching for a physical law of soil tillage draft force, assuming that it exists. The method can also be applied in case of other physical laws in research, construction or improvement stage.

REZUMAT

Articolul prezintă rezultatele unor scheme de testare - validare și ordonare ale modelelor matematice propuse pentru clasa de legi fizice referitoare la forța de rezistenta la tracțiune în lucrările solului. Rezultatele constituie continuarea și completarea parțială a unei metode de testare - validare și ordonare - a modelelor propuse și publicate de cercetători în literatura de specialitate din ultimii șaptezeci de ani. Materialul definește și completează metoda (inițial doar o metodă de validare), până la o metodă de ordonare a modelelor în funcție de precizia acestora în raport cu rezultatele experimentale. Testele propuse sunt menite să crească coerența cercetărilor în domeniul căutării unei legi fizice a forței rezistenta la tracţiune a solului, presupunând că aceasta există. Metoda poate fi aplicată și în cazul altor legi fizice aflate în stadiul de cercetare, construcție sau perfecționare.

INTRODUCTION

The results presented in this article constitute a continuation and, to a large extent, a completion of those set out in (*Cardei et al., 2020*). Finding and defining a physical law of the soil tillage draft force is a very complex experimental and theoretical research activity, due, first of all, to the large number of parameters involved in the process of interaction of the working bodies and, generally, of the machines with the soil. Also the random character of many parameters and characteristics of the soil constitutes an element of high difficulty. The hope that a physical law of soil tillage draft force exists, involves a large number of researchers in experimental and theoretical research. Most likely, if it exists, the law will materialize in one or more mathematical relationships that will form the mathematical model that will express the sought law. Currently, there are a relatively large number of mathematical relationships that want a place in the hierarchy of claimants to the title of law of the soil tillage draft force. However, despite the large number of such relationships, they can be grouped into several simple categories. An important observation is required here. Validation in the sense of *(Cardei et al., 2020*) refers to the positivity of the coefficients of the terms, but not to the exponents determined by the method of the smallest squares, exponents that, in general, can have a positive or negative sign.

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The observation refers, especially to formulas produced by factors with various exponents, formulas obtained correctly from a dimensional point of view, for example *Moenifar & al., (2014*). From the point of view of the physical sense, the formulas leading to parameters with an uncertain physical dimension are invalidated (*Cardei and Gageanu, 2017*). In *Cardei and Gageanu (2017)* is showed how such situations can be remedied. The mathematical models are comparable by means of precision in report to the experimental data, thus resulting a hierarchy of them. In this article, according to *Cardei et al., (2020)*, by validation is understood first of all the physical consistency of the model expression, in the sense of the two criteria set out by (*Cardei et al., 2020)*. The validation in the classical sense, is made in the ordering stage. The ordering stage also includes what the literature understands by comparing models (*Dadu, 2012*). Often, the authors understand by testing even the experimental validation (comparing the model results with the experimental results (*Marion, 2008*).

Generally, mathematical models are divided into two broad categories: deterministic and stochastic (*Dadu, 2012; Marion, 2008*). In this article only deterministic mathematical models will be considered. Obviously, the usefulness of these tests consists primarily in the validation and ranking of the models in an order according to the accuracy achieved by experimental data. Validation using experimental data causes the ordering to become relative to a batch of experimental data.

Proceeding in this way there will be orders of the batch of proposed formulas, but relative to certain experimental data. Therefore, it is very likely that the order of accuracy of the formulas will differ for different experimental data sets. This is normal, primarily because both mathematical models (formulas) and experimental data neglect many parameters that influence the soil tilling processes. Another usefulness of these tests is the elimination of those formulas that have no physical meaning (due to the physical dimension of their terms) or present negative terms in an additive representation of the soil tillage draft force. *Cardei et al., (2020*) explains why this requirement of theoretical validation is included in tests. For example, we sought to avoid classical formulas as they appear in *Ion and Ion, (2019*).

MATERIALS AND METHODS

The working material of this article consists of three basic mathematical models for the draft force, of which the first two, each together with three variants, altogether nine models or nine formulas for testing. The first two basic models, together with their variants, are among the most used in the literature, for example: *Letosnev, (1959); Krasnicenco, (1964); Scripnic and Babiciu, (1979); ASAE, (2003); Ranjbarian et al., (2017); Askari and Khalifahamzehghasem, (2006); Ormenisan, (2014); Cardei et al., (2019)*.

In the paper of *Cardei et al., (2019)*, it was showed that there is a formula for the draft resistance force to tillage that generally includes the most commonly used formulas today. A generalization was given in *Cardei et al., (2020)*. The third basic model tested is taken from *Moenifar et al., (2014)*, which is very interesting, especially for dynamic reasons, but it can be a very good competitor to the best coverage of all the considered experimental cases.

Table 1 lists the parameters that appear in the formulas (mathematical models) analysed in this paper.

Table 1

Parameters of the process: notations, significance and units of measurement

Notation	Name	Unit
F	Draft force	N
F_i	The values of the interpolated force in the experimental points	N
A	The static coefficient of the draft force term	N
B	The coefficient of the draft force term that depends on the working speed	kg/s
C	The coefficient of the draft force term that depends on the square of the working speed	kg/m
φ	Dimensionless factor describing the influence of soil texture: φ = 1 fine, φ = 2 average, and φ = 3 coarse	-
v	Working speed	m/s
b	Working body width	m
a	Working depth	m
k	Coefficient that characterizes specific soil deformation resistance	MPa
ε	Coefficient which depends on the shape of the active surface of the body and the soil properties (*Letosnev, 1959*)	kg/m^3
f	Coefficient analogous to friction coefficient	-

Table 1

(continuation)

Notation	Name	Unit
G	Plough weight	N
g	Gravitational acceleration	m/s^2
ρ	Soil mass density	kg/m^3
α	Horizontal blade angle (rake angle)	rad
n	Number of working bodies	-
φ_s	The lateral displacement angle of the furrow	rad
δ	Coulter sharpening angle	rad
G_A	The degree of dislocation of the soil	-
a_i	Working depth, experimental data	m
b_i	Working body, width experimental data	m
v_i	Working speed, experimental data	m/s
$\mathcal{F}_i$	Draft force, experimental data	N
$\bar{\mathcal{F}}$	The average value of the forces determined experimentally	N
i	Index of the experimental data	-
$\mathbf{e_x}$	The set of experimental data	-
N	The number of experiences in the experimental data set	-
ε_{rel}	global relative error	%
ε_{max}	maximum relative error	%
R^2	Coefficient of determination	-
c	Soil cohesion	Pa
c_a	Soil – working body adhesion	Pa
x	Model parameter - coefficient	-
y, z, t	Model parameters -exponents	-

For working body definition see paper of *Singh, (2017*). If there are n working bodies, each one with working width $b\,'$, is considered the relation for the total working width: $b = nb'$.

The method of evaluating the mathematical models proposed in the specialized literature, used to obtain the results presented in this article, has been set out and exemplified in *Cardei et al., (2019)*. In short, this method consisted in identifying model parameters using the least squares method. Validation, $\mathbf{V}$, was done demanding the fulfilment of the criteria:

C1 - formulas must have a physical meaning, in the sense of dimensional correctness;

C2 - in the additive composition of the formulas, generally negative terms will not be accepted because the negative components would have the meaning of some components that lead to the decrease of the draft tillage resistance.

Criterion C1 is reflected in the physical dimension of the model parameters and in their physical unit (see table 1). Criterion C2 will be generally supplemented with the indication that the values of the model parameters should be included in the intervals specified by the literature, if any. In addition to the model validation, a proposal for precision test of approximation was added to this paper, $\mathbf{P}$. This test proposes an ordering of the tested formulas, according to the accuracy that they perform in relation to the experimental data relative to which the validation is performed (identifying the parameters model). In order to estimate the accuracy of a model against a batch of experimental data, $\mathbf{e_x}$, two variants are proposed in this article.

The first precision estimator is based on the average value of the sum of the squares of the data errors calculated by interpolation, relative to the experimental data, relative to the average value of the experimental data:

$$\varepsilon_{rel} = \frac{\sqrt{\sum_{i=0}^{N-1}(F_i - \mathcal{F}_i)^2}}{N\bar{\mathcal{F}}} \cdot 100 \tag{1}$$

and it will be called *global relative error*.

Another precision evaluator starts from the definition of the infinite or maximum norm (*Trench, 2013; En.wikipedia.org; Colojoara, 1983*):

$$\mathcal{E}_{max} = \frac{\max\limits_{i=0\ldots N-1} |F_i - \mathcal{F}_i|}{\bar{\mathcal{F}}} \cdot 100 \qquad (2)$$

$\mathcal{E}_{\max}$ is the maximum relative error (it receives relative character by dividing by the average value of the force). Both operators are given as a percentage.

With these definitions, the testing process can be synthetically presented as a process:

$$\mathbf{T}=\mathbf{T}(\mathbf{V},\mathbf{P},\mathbf{e}_{\mathbf{x}},\mathbf{m}) \qquad (3)$$

where $\mathbf{m}$ is a model set, and the result of this process is:

$$\mathbf{R}_{\mathrm{T}}=(\mathbf{m}_{\mathbf{v}},\mathbf{t}_{\mathbf{m}}) \qquad (4)$$

A couple of results, the first consisting of validated mathematical models (formulas), $\mathbf{m}_{\mathbf{v}}$, and the second, $\mathbf{t}_{\mathbf{m}}$, in a ranking of the validated models depending on the accuracy achieved for the batch of experimental data that was worked on.

The experimental data for which, the candidate formulas for the title of draft force law are tested, come from six papers that have been freely accessed: *Akbarnia et al., (2014)*, *Ranjbar et al., (2013)*, *Naderloo et al., (2009), Fechete-Tutunaru et al., (2018), and Moenifar et al., (2014)*. This set of experimental data forms in our case the ex-set of experimental data, from the definition (3). In general, the data from the sources above contain working depth and width, working speed and draft force. Some also contain soil moisture, the type of plough or different angles characteristic of the working bodies. For the latter, separate tests were performed on each additional process characteristic compared to the basic ones (depth, width and working speed and draft force). In some reasonable cases a global validation can be done over a number of additional parameters.

The mathematical models of the tillage draft force, whose results of the qualitative and quantitative evaluation are presented in this paper, are the following:

$$F = fG + kabn + \varepsilon abv^2 \qquad (5)$$

$$F = fG + kab \qquad (6)$$

$$F = kab \qquad (7)$$

$$F = kab + \varepsilon abv^2 \qquad (8)$$

$$F = \varphi(A + Bv + Cv^2)ab \qquad (9)$$

$$F = \varphi(A + Bv)ab \qquad (10)$$

$$F = \varphi(A + Cv^2)ab \qquad (11)$$

$$F = \varphi(Bv + Cv^2)ab \qquad (12)$$

The set of nine formulas (5) - (13) forms the set of mathematical models, $\mathbf{m}$, from the definition of the test process (3). For the moment, the models of the soil tillage draft force that depend on the second power of the working depth, as in *Okoko, et al., (2018); Kushwaha, et al., (1993)*, have not been considered.

In addition to models (5) - (12), for the soil tillage draft force, an example of a model which is established using the dimensional analysis is considered in *Moenifar et al., (2014).*

Dimensional analysis is used for the same purpose by *Larson, (1964)*. In order to test the model in *Moenifar et al., (2014)*, it is necessary to know the cohesion, adhesion and density of the soil, in addition to the experimental data used in the presented tests.

The model (5) is known as Goriacikin's formula, and (9) as ASABE Universal Draft Equation (*Tewari, 2018*). With this all the working data are presented, that is, all the components of the test process $\mathbf{T}$, (3) are known.

Before proceeding with the presentation of the results, it should be specified that:
- we do not consider before or after testing, that any model is true or false;
- we consider that all experimental results are correct.

With these two working principles, the conclusions of the tests can state whether a model is valid for an experimental data set or not and, from the valid models, a model will be a model will be selected in the desired order according to their accuracy. Thus, the validation and precision of a model will become a notion relative to a certain set of experimental data. Under such conditions, there is the possibility that certain models may be statistically significant.

These will be the models that must be followed and studied in order to reach a hypothetical convergence to an existing hypothesis of a physical law of soil tillage draft force. Of course, there is the possibility that that law will depend on the particular conditions of experimentation. Then, fundamental problems of human thought arise in relation to the laws of nature and their perception by man.

RESULTS AND DISCUSSION

We now proceed to the presentation of the results of the T test, respectively the system of test results, (4), R_T, that is, to specify the validated models and to order them in the descending order of accuracy (the option of the descending order is not mandatory). In order to select valid models, and order them, one can proceed by eliminating the models with negative model parameters (terms), respectively by manual or automatic ordering, using a norm composed of one or both errors (1) and (2), and correlation, and the index of determination, which forms the group of quality parameters.

In order to obtain a mixed, qualitative and quantitative measure of each model, it is recommended the ratio between quality factors (correlation and / or determination index) and errors (one or both). This ratio must be maximized. A typical result for this analysis shows that in table 2. By applying the least squares method to the models (5) - (12), we obtain values of the model parameters as in table (2), columns 2-7.

The quality and quantitative estimators of the interpolations performed are grouped in columns 8-11 of table 2. A table of the type of table 2 results by processing each set of experimental data, relative to the batch of formulas or models tested and specified on the first column of the table 2.

The results from the table 2 are obtained by applying the least squares method for formulas (5) - (12) and experimental data from *Ranjbarian et al., (2017),* for soil moisture with a value of 22%.

Table 2

The values of the models parameters and estimators of quality and precision

Formula	fG	k	ε	φA	φB	φC	correlation	R^2	ε_{rel}	ε_{max}
(5)	2413.484	14576.06	9547.043				0.944	0.891	2.414	15.17
(6)	2413.484	22997.6	0				0.818	0.669	4.201	38.861
(7)	0	36952.22	0				0.818	0.387	5.715	40.779
(8)	0	28530.67	9547.043				0.929	0.609	4.565	33.369
(9)				39636.86	-27465.7	24936.03	0.927	0.599	4.626	33.755
(10)				21488.63	17208.32	0	0.929	0.609	4.563	33.881
(11)				36821.081	0.000	148.66	0.82	0.394	5.683	40.664
(12)				0.000	793440.95	-718326	-0.29	-226.67	110.155	711.706

The values of the parameters for the formula (13), in the case of the experiments (*Ranjbar et al., 2013*) at 22% soil moisture, are found in *Cardei et al., (2020)*.

The quantitative estimators reduced to the unit are calculated below, that is, each of the columns (9) and (10) are reduced to unit by dividing by the maximum value. Also, the correlation and the index of determination are reported to the absolute maximum value, for a more compact graphical representation. We also add a validation estimator that has a positive value if the model parameters are strictly positive and a negative one otherwise. The absolute value of the validation estimator is chosen so that it can be represented graphically together with the values of the other estimators (we have chosen 0.1). This last group of estimators is given in table 3.

Table 3

Selection parameters obtained from the parameters and estimators calculated in table 2
All estimators are reduced to one unit

Formula	ε_{rel}	ε_{max}	correlation	R^2	validation
(5)	1.000000	1.000000	0.422397	0.372005	0.100000
(6)	0.866525	0.750842	0.735083	0.952966	0.100000
(7)	0.866525	0.434343	1.000000	1.000000	0.100000
(8)	0.984110	0.683502	0.798775	0.818289	0.100000

Table 3
(continuation)

Formula	ε_{rel}	ε_{max}	correlation	R^2	validation
(9)	0.981992	0.672278	0.809449	0.827754	-0.100000
(10)	0.984110	0.683502	0.798425	0.830844	0.100000
(11)	0.868644	0.44220	0.994401	0.997180	0.100000
(13)	0.958686	0.815937	0.667892	0.720665	0.100000

The material presented in the Results chapter is produced for each set of experimental data.

For the purpose of qualitative and quantitative selection will be presented validations and classifications for an example of the norm considered.

The validation operator will be considered compulsory for ordering. In the example, it was used only the maximum error (the product between the validation estimator value and the inverse of the maximum error).

The results given in the Table 4 are obtained for the batch (1)-(8) of mathematical models of the tillage draft force and for the data the experiences provided in *Akbarnia et al., (2014).*

Table 4

Synthesis of validation and of the modelling performances

Model **Experimental data**	**(5)**	**(6)**	**(7)**	**(8)**	**(9)**	**(10)**	**(11)**	**(12)**	**(13)**
Akbarnia et al., (2014)	0.000	0.000	0.271	0.349	0.000	0.354	0.273	0.000	0.152
Ranjbar et al., (2013), soil moisture 16.1%	0.464	0.257	0.176	0.215	0.000	0.215	0.177	0.000	0.268
Ranjbar et al., (2013), soil moisture 22.0%	0.414	0.238	0.175	0.219	0.000	0.219	0.176	0.000	0.262
Ranjbar, Rashidi, Najjarzadeh, & Niyazadeh, (2013), soil moisture 25.4%	0.427	0.245	0.177	0.22	0.000	0.221	0.178	0.000	0.257
Ranjbar et al., (2013), all soil moisture	**0.732**	**0.435**	**0.315**	**0.387**	**0.000**	**0.388**	**0.315**	**0.000**	**0.463**
Naderloo et al., (2009), mouldboard plough	0.284	0.094	0.09	0.213	0.000	0.192	0.093	0.000	0.145
Naderloo et al., (2009), disk plough	0.000	0.000	0.058	0.085	0.000	0.084	0.059	0.000	0.072
Naderloo et al., (2009), chisel plough	0.515	0.141	0.122	0.223	0.000	0.229	0.125	0.000	0.089
Naderloo et al., (2009), all ploughs	**0.8**	**0.235**	**0.27**	**0.521**	**0.000**	**0.504**	**0.277**	**0.000**	**0.305**
Fechete-Tutunaru et al., (2018), angle of cutting 30, rake 25	0.000	0.000	0.076	0.076	0.000	0.076	0.076	0.000	0.605
Fechete-Tutunaru et al., (2018), angle of cutting 30, rake 35	0.000	0.000	0.087	0.087	0.000	0.087	0.087	0.000	0.618
Fechete-Tutunaru et al., (2018), angle of cutting 30, rake 50	0.000	0.000	0.094	0.094	0.000	0.094	0.094	0.000	0.592
Fechete-Tutunaru et al., (2018), angle of cutting 45, rake 25	0.000	0.000	0.076	0.076	0.000	0.076	0.076	0.000	0.597
Fechete-Tutunaru et al., (2018), angle of cutting 45, rake 35	0.000	0.000	0.083	0.083	0.000	0.083	0.083	0.000	0.603
Fechete-Tutunaru et al., (2018), angle of cutting 45, rake 50	0.000	0.000	0.101	0.101	0.000	0.101	0.101	0.000	0.612

Table 4
(continuation)

Model *Experimental data*	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Fechete-Tutunaru, Gaspar, & Gyorgy, (2018), angle of cutting 60, rake 25	0.000	0.000	0.072	0.072	0.000	0.072	0.072	0.000	0.571
Fechete-Tutunaru, Gaspar, & Gyorgy, (2018), angle of cutting 60, rake 35	0.000	0.000	0.083	0.083	0.000	0.083	0.083	0.000	0.599
Fechete-Tutunaru, Gaspar, & Gyorgy, (2018), angle of cutting 60, rake 50	0.000	0.000	0.094	0.095	0.000	0.095	0.094	0.000	0.676
Fechete-Tutunaru et al., (2018), all bodies type	**0.000**	**0.000**	**0.765**	**0.767**	**0.000**	**0.767**	**0.765**	**0.000**	**5.474**
Cardei et al., (2017) Optimum working conditions for variable width ploughs,	1.357	1.203	0.166	0.170	0.000	0.17	0.166	0.000	0.214
Total	**4.193**	**2.613**	**2.317**	**2.849**	**0.000**	**2.838**	**2.329**	**0.000**	**7.395**
Total percent	**17.092**	**10.651**	**9.444**	**11.611**	**0.000**	**11.569**	**9.492**	**0.000**	**30.141**

The following observations resulting from table 4 are:

O1) - for the experimental data from *Akbarnia et al., (2014),* variant (10), which is a linear velocity dependence in the set of formulas suggested by ASAE, (2003), is the most appropriate, followed by formula (8), of Goriacikin inspiration, variant which excludes the term of friction given by the weight of the tillage machine;

O2) - the three experimental datasets offered by *Ranjbar et al., (2013),* are best modelled, all, regardless of soil moisture, even the Goriacikin variant, (5), followed by (13), then by variants (6) and (8) of the formula (5);

O3) - the results of the processing of the experimental data from *Naderloo et al., (2009)*, show that the Goriacikin model (5) better models the soil tillage draft force in the case of the mouldboard plough and chisel plough, while the variant (8) of the same model is better for the plough disk;

O4) - the experimental data series presented in *Fechete-Tutunaru et al., (2018)*, by processing according to the test presented in this article, shows that, for the experimental device used in *Fechete-Tutunaru et al., (2018),* the best theoretical model, without exception, is given by the formula (13);

O5) - the experimental data are most efficiently modelled by the original Goriacikin model, (5), together with all its variants, in particular (6);

O6) - an observation of structure is found by comparing the Modified ASABE Universal Draft Equation formula, (*Tewari, 2018*), with formula (5), namely that both contain two constant terms in relation to the speed of movement.

The observations **O1) - O5)** can be used for the selection of the most exacting mathematical model claimant to the title of law of soil tillage draft force, for each set of experimental data.

If instead of the maximum error, relation (2), the relative global error, relation (1), is used for selection, the situation does not change. Moreover, the estimators are well correlated with each other. Also in table 4, invalid formulas or models are those that have in their cell the number 0.000, according to the validation operator value, conventionally chosen value.

For an example of applying the method in the case of a mathematical model of the soil tillage draft force as a product, the formula proposed in *Moenifar et al., (2014)*, is taken:

$$F = x\rho b^2 v^2 \left(\frac{c + c_a}{\rho v^2}\right)^y \left(\frac{a}{b}\right)^z (\sin\alpha) \tag{13}$$

Formula (13) is deduced using dimensional analysis and considering ten parameters that influence the process. Finally, only seven of the parameters appear in the soil tillage draft force formula.

For formulas of type (13), the dimensionality condition of the parameters x, y, z, t is not required because they are dimensionless by the correct construction mode (dimensional analysis).

However, the condition of positivity is valid for the coefficient x, considering the positive values of the other factors (the angle of position within the constructive and technological limits, has the positive sine). The exponents y, z and t can be negative.

An important mention to be made in the case of model (13) of the draft force is that, if the model parameter, y is positive, then the traction force is zero as long as the speed is not strictly positive.

This behaviour makes it easy to use this model in dynamic calculations, which does not happen with the models that contain term independent of speed, constant, zero (possible in models (5) - (11)).

For testing this formula, the value of soil cohesion and density from *Moenifar et al., (2014)* was used, also there where the experimental data did not specify them, and for estimating the adhesion we used the relation from (*https://www.finesoftware.eu*). This may explain any errors that occur in the results.

Results of the tests of formula (13) on the experimental data from *Akbarnia et al., (2014), Ranjbar et al., (2013), Naderloo et al., (2009), Fechete-Tutunaru et al., (2018)*, are given in the article of *Cardei et al., (2020).*

CONCLUSIONS

The validation and ordering test, of a mathematical model applying for the title of physical law of the draft force generated by the soil working machines is useful for selecting the best model, from a collection of tested models and relative to a lot of experimental data. The validation is conventional and can be ignored, obviously with the price of increasing the risk of some theoretical disadvantages.

The validation and ordering test presented in this article shows that, for now, if we give priority to the accuracy of the mathematical models in relation to the experimental data, then the physical law of the soil tillage draft force has a relative character.

Different models are performing for different experimental data. Acceptance of this situation is a common attitude in the practice of designing and operating machinery for soil works. But, from a theoretical point of view, the situation can be uncomfortable.

Theoretically, one can accept the explanation that if the above law exists, then we are in an intermediate stage of construction. This explanation is likely to be accepted, considering that in the formulas used in current practice, less than one-fifth of the physical parameters involved in the process (known) appear explicitly.

On the other hand, even if we were to build formulas with a large number of parameters, experiences that would sweep all these parameters would be very expensive, almost impossible to achieve.

Under these conditions, it is possible that we will never reach a formula (mathematical model) that bears the name of physical law of the soil tillage draft force, which is not a problem, at least from a practical point of view. Theoretically, however, this situation shows the inability of science to solve complex multi-parametric phenomena. The situation is not new, since such problems arise for well-known mathematical models such as the force of gravitational attraction. Modern physics knows that the same natural phenomenon can be described by different models, with the same precision, but it is difficult to answer for current science when we discover that phenomena apparently belonging to the same category develop or can be developed according to different laws.

From a formal point of view, the notion of physical law can be enforced in the sense of giving the classic form of physical law expressible through an elementary functional relation, a non-elementary expression, practically admitting more relations for a notion of physical law, that becomes so, a collection of relationships, each valid in well-defined fields or intervals of some of the parameters that influence the phenomenon.

For future attempts to formulate a law of draft force, remember that most of the current forms have the following characteristics:

- are additive formulas of polynomial form in relation to the speed of advancement (with certain exceptions), containing also one or two constant terms;
- the model parameters that appear as coefficients of the terms in the formulas, are dependent on: soil moisture and texture, soil density and degree of compaction, the geometrical characteristics of the working bodies and the interaction characteristics between the working parts and soil;
- the basic parameters retained by almost all formulas, are: working width and depth, working speed;
- there can also be considered formulas produced by parameters (possibly deductible by dimensional analysis) and which can be extended by summing with model parameters;
- soil characteristics (moisture, texture, density, compaction etc.) are very numerous and have a random character in space and time, so we must expect the "law" of variation of the draft force to change from place to place and in time.

The only certainties, for the time being, that the research can give to the design and manufacture of agricultural machines for soil tillage, are the upper limits of the draft force, sufficient information for the machines to work safely.

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