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Research Paper

Mechanical and structural reliability based algorithm optimization

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ABSTRACT

The overall aim of this paper is the development of nonlinear formulations of BEM (Boundary Element Method) for the analysis of fracture problems and also the realization of a probabilistic approach to the fatigue problem through the coupling between a mechanical model that deals with the propagation of cracks under fatigue, and structural reliability models. In the BEM/MEF coupling model, the crack propagation model was introduced on nonlinear regime. This model leads to interesting results and allows to analyze some structures where this effect is important. Closing the topic related to the development of BEM formulations, a formulation was also developed for the analysis of fatigue cracking propagation problems. Probabilistic models are applied to the analysis of structures subjected to fatigue. It is known that the integrity of the structures in service essentially depends on its ability to maintain a resistance pattern over time. To this probabilistic model was coupled an optimization algorithm for the determination of parameters such as the geometry dimensions of the structure as well as intervals for the performance of the maintenance and inspection procedures, taking into account objective functions written in terms of structural cost and safety. The investigation shows that direct coupling scheme converged for all problems studied, irrespective of the problem nonlinearity.

1 Introduction

In order to allow the analysis of a larger range of materials, the concept of fracture encompassing the cohesive fracture was extended. The cohesive models have their origins in Dugdale and Baremblatt works [1, 2] and later extensively studied numerically and experimentally in the works of [3, 4].

In this model, the existence of a process zone located in front of the region where there is the proper separation of the fissure surfaces is considered. In this process zone, loss of stiffness occurs with the increase of the micro-cracks and the consequent dissipation of energy. Considering this small zone, a possible approximation would be done to consider that the dissipation takes place in a fictitious fissure placed before the real fissure. In this fictitious region, a fictitious boundary

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aperture in which the transfer of tension occurs between the surfaces of the crack is idealized. The constitutive relation that governs the maximum stress in the process zone and the opening of the faces of the crack reflects the softening behavior.

The simplest representation of this model is to use a linear curve given by two parameters of the material, maximum fictitious aperture and maximum tensile strength. However, in the literature there are several other suggested approximations. Some recent important works on the subject, besides those mentioned, are [5 – 10]. Fracture work using other numerical methods as finite element method and non-knitted methods can be found in [11].

In this way, complex problems can be addressed by making more realistic considerations about the uncertainties present in the structure, based on statistical data, obtaining a more accurate and realistic design, maintenance and inspection design. Models such as those proposed allow the realization of a real risk analysis in relation to the use of the structure or the part in analysis.

With respect to the progress and advances in the formulations of the boundary element method (BEM), the objective of this work is to develop models that deal with the process of crack growth in flat areas composed of fragile, quasi-fragile and ductile materials. Considering these different types of materials, the numerical formulation adopted in the analysis should represent the nonlinear structural behavior due to the process of crack propagation and consequent structural degradation. One of the contributions presented in this paper refers to the employment of the tangent operator in solving the non-linear problems. This operator allows the nonlinear problem to be solved by using a smaller number of iterations, when compared to models that employ constant operator, thus making the formulation more precise and also efficient from a computational point of view. It should be emphasized that the consistent tangent operator depends on the nonlinear law adopted in the problem. Thus for each nonlinear problem, to be solved, one must deduce the terms of this operator.

This work deals with two topics that have been widely discussed by the scientific community: development of the BEM formulations for use in engineering problems as well as reliability and optimization models applied in the analysis of time dependent problems. The resulting model of this triple coupling can be applied to several particular problems. In particular, in this work this model is used to approach problems of design, inspection and maintenance of structures subject to the growth of cracks under fatigue effects, considering a probabilistic analysis of structures.

2 Mechanical fatigue model

The determination of the stress intensity factor using the displacement correlation technique is performed by correlating the displacements determined numerically at the nodal points of the element located at the crack end with analytical solutions.

For 2D structures, the stress intensity factors for modes I (opening) and II (sliding) are given according to the following expressions [12]:

$$K_I = \sqrt{\frac{2.\pi}{r}}.\frac{\mu}{(\kappa+1)}.COD \tag{1}$$

$$K_{II} = \sqrt{\frac{2.\pi}{r}}.\frac{\mu}{(\kappa+1)}.CSD \tag{2}$$

, where: COD "Crack Open Displacement" difference between the displacements perpendicular to the plane of the crack and CSD "Crack Sliding Displacement" difference between the displacements parallel to the plane of the crack. K_I and K_{II} are the stress intensity factors for modes I and II, respectively, r is the distance between the crack tip and the computational point (i.e., mesh node) and μ and κ are the material properties. In order to calculate stress intensity factors at the crack tip, these variables are evaluated for four pairs of mesh nodes near the crack tip. Then, the stress intensity factors at the crack tip are obtained from a local extrapolation process. This process leads to accurate results as presented in [13, 14].

For the determination of the propagation angle the circumferential tension, $\sigma_{\theta\theta}$, must be maximum and consequently a principal stress. According to the concepts of the solid mechanics for this situation to occur, the shear stress must be zero. Thus, to determine the direction of the crack propagation, θ_p, one must take $\tau_{r\theta} = 0$. Through this condition it is possible to obtain:

$$K_I.\sin(\theta_p) + K_{II}.(3.\cos(\theta_p) - 1) = 0 \tag{3}$$

Using trigonometric relationships, it is possible to rewrite the above relationship as:

$$tg\left(\frac{\theta_p}{2}\right)=\frac{1}{4}\left[\frac{K_I}{K_{II}}\pm\sqrt{\left(\frac{K_I}{K_{II}}\right)^2+8}\right] \tag{4}$$

For this purpose, the maximum circumferential stress criterion is used [12]. According to this criterion, the cracks are assumed to grow indirection θ_p, which is perpendicular to the maximum circumferential stress at the crack tip

$$K=K_I\cos^3\left(\frac{\theta_p}{2}\right)-3.K_{II}.\cos^2\left(\frac{\theta_p}{2}\right).\sin\left(\frac{\theta_p}{2}\right) \tag{5}$$

where K: is the effective stress intensity factor.

A widely used criterion is that presented in [15, 16]. This criterion consistently describes only the propagation of fissures in region II and is usually referred to in the literature as the "Paris Law". The relationship between the crack growth rate and the variation of the stress intensity factors is given by:

$$\frac{da}{dN}=C.\Delta K^n \tag{6}$$

where: C and n are the material constants, a represents the crack length, N is the number of loading cycles, and ΔK is the stress intensity factor range.

3 Formulations of the BEM for Cohesive Fracture Analysis

In this section, the nonlinear formulations proposed in this work are presented for the analysis of problems involving nonlinear (cohesive) fracture and also contact between bodies [illegible] (BEM) equations.

First, the model developed for the analysis of nonlinear fracture problems will be discussed. However, it is proposed in this work a formulation that solves the nonlinear problem using a consistent tangent operator. This model also leads to good results; however this formulation requires [illegible] smaller number of iterations to obtain the equilibrium in each increment of load, making it more efficient from a computational point of view.

A model was also developed for the analysis of the cracks propagation in materials that are governed by the concepts of linear elastic fracture mechanics. In this model the stress intensity factors are calculated using the displacement correlation technique. In addition, some theories of modes interaction were also implemented to obtain the crack propagation angle and also the equivalent stress intensity factor. From the formulation point of the BEM, this problem is also treated by considering a consistent tangent operator, which will also be shown in this paper.

As for the contact model, a formulation was developed, employing also a consistent tangent operator, for the analysis of the contact in two different problems. The first one refers to the contact between faces of cracks, that is, in the simulation of the closing of a crack. For the second, this model is applied to the analysis of composite materials which allows the simulation of contact and slip between the various materials that make up the structure. In both applications the Coulomb law is adopted to govern the adhesion behavior of the contact region, that is, the displacements and surface forces in this region.

Several BEM formulations have been proposed in the literature to properly handle crack problems [17–27]. For crack simulation, a finite gap between two crack surfaces has to be considered as well as an accurate integral scheme to evaluate the integrals along the quasi-singular elements.

Initially one can write the general equation of the BEM, in the following way:

$$HU=GP \tag{7}$$

The matrices and vectors described in Eq. (7) can be divided according to their location in the model. The source points may be on the contour, c, or on the faces of fissures, f. Like this:

$$\begin{aligned}H_C^CU_C+H_f^CU_f&=G_C^CP_C+G_f^CP_f\\H_C^fU_C+H_f^fU_f&=G_C^fP_C+G_f^fP_f\end{aligned} \tag{8}$$

, where U_c and U_f are the displacements assigned to the boundary and crack surface nodes, respectively, P_c and P_f are the boundary and the crack surface tractions, respectively, and H_{cc}, H_{fc}, G_{cc} and G_{fc} are the corresponding matrices that account for the displacement and the traction effects. The subscript c indicates that the collocation point is on the boundary and the super scripts specify the boundary (c) or crack surface (f) values.

The crack presents two faces, one of them located to the right and the other to the left of its average geometric line. In Eq. (8) the fountain points located on the fissure can be separated into fountain points belonging to the left face and the right face. Like this:

$$
\begin{aligned}
H_C^C U_C + H_f^{CR} U_f^R + H_f^{CL} U_f^L &= G_C^C P_C + G_f^{CR} P_f^R + G_f^{CL} P_f^L \\
H_C^f U_C + H_f^{fR} U_f^R + H_f^{fL} U_f^L &= G_C^f P_C + G_f^{fR} P_f^R + G_f^{fL} P_f^L
\end{aligned} \tag{9}
$$

, where indices R and L, distinguish the source points located on the right and left faces of the fissure respectively. The system of equations presented in Eq. (9) can be solved for the known in the contour.

4 Structural Reliability model

The design, sizing and prediction of good structural functioning, lead to checks of the structural elements with respect to requirements resulting from physical and mechanical knowledge and also from the experience of their analysts and builders. These requirements translate, in more or less complex forms, into criteria whose objective is to define threshold values for structural variables such as stresses arising from requests, displacements, deformations, among others. In a structural reliability analysis, each criterion can be understood as a potential event and its consequences as failure scenarios. The verification of each criterion, therefore, results in verification of each potential failure mode. To do so, one must describe and formulate the problem considering its variables with due uncertainties. The objective is then to evaluate a probability, that of finding a fault situation, considering the statistical knowledge of each variable and its influence on the structural behaviour [28].

In structural engineering, reliability problems can be formulated by means of their capacity or resistance, R, and demand or effect of the actions, S. The analysis is usually based on the calculation of the complement of reliability, ie, the propensity to fail P_f. If resistance and request are statistically independent random variables, with known probability distributions and time stationary, the probability of failure P_f, can be evaluated by the solution of the following equation [29]:

$$P_f = \text{Prob}[(R-S) \le 0] = \int_0^\infty F_R(x).f_S(x)dx \tag{10}$$

In Eq. (10) $F_R(x)$ is the cumulative probability function of the resistance and $f_S(x)$ is the probability density function of the request. Eq. (10) is known as convolution integral with respect to "x", corresponding to the sum of all the request cases for which the resistance is smaller than the request.

This equation can also be written in terms of the cumulative probability function of the request, $F_S(x)$, and the probability density function of the resistance $f_R(x)$. Like this:

$$P_f = \text{Prob}[(R-S) \le 0] = \int_0^\infty [1 - F_S(x)].f_R(x)dx \tag{11}$$

The first order reliability or FORM method provides an estimate of the probability of failure of the structure by linearization of the limit state function at the design point in the standard normal space. Linearization is done through a hyper plane tangent to the fault surface at the design point. The FORM approximation is sufficiently accurate for cases where the curvature of the fault surface is small and the probability of failure has a small value. In addition, the error in this type of approach depends on the concavity of the fault surface, ie for concave surfaces, the approach is in favour of safety, whereas for convex surfaces, the FORM is against safety.

The second order reliability method SORM is an attempt to improve the approximation of the failure probability based on more information about the failure surface of the structure. The principle is exactly the same as the FORM approach, but requires a better understanding of the geometry of the boundary state function in the neighbourhood of the design point. In this type of approach, the boundary state function is treated as a hyper-surface of the second degree instead of the hyper plane tangent. Additional information about the limit state function is its main curvatures, in addition to the reliability index. The method requires that at the point of design, the quadratic approach surface is continuous and that it is twice differentiable, besides having the same tangent plane and the same main curvature as the real limit state function. There are several quadratic approximations available in the technical literature for the shape of the hyper surface used in SORM. The choice depends on the required accuracy as well as the available processing time. Among the various options for SORM are: Approximation by a centred hyper-sphere, eccentric hyper-sphere and asymptotic approximations [30, 31].

Finally, as seen in this topic we tried to present some important concepts of reliability theory. Other definitions on probability, structural life and reliability can be found in the literature, but the definitions presented are perfectly compatible with the use of reliability theory in this work.

5 5. Optimization model

Three models developed in this work will be presented in this section for the analysis of the maintenance and design of structures submitted to fatigue. The first model deals with the determination of the optimal inspection moment for a given reliability index. In this case the model determines the number of cycles in which maintenance is to be performed to maintain the desired level of safety. In this model are considered perfect and imperfect maintenance. In the first case, the structural part is replaced, once the number of critical loading cycles is reached, by another equal and in good condition. With imperfect maintenance, it is accepted that the faces of the cracks are closed with some type of filling material before the replacement of the structural element. It should be emphasized that in this model only the inspection moment is determined without, however worrying about the cost of the inspection. The cost variable is considered in the other models of optimization and reliability built in this work.

The second model refers to a Reliability Based Design Optimization (RBDO) model where the geometry dimensions of the structural element are obtained from the reliability and optimization analyses. The objective of this model is to obtain the geometric dimensions of the structural element in order to minimize the volume of the structure, considering a given level of structural safety desired, in order to obtain the minimum cost of the structure production. Thus, the equation to be minimized must relate the dimensions of the structure to its volume while the constraint equation of the optimization problem relates the structural dimensions to the reliability index being constructed by means of response surfaces of the structure geometry variables.

The third developed model aims to obtain the minimum dimensions of the structure as well as the maintenance and inspection intervals that lead to the minimum cost of the structure taking into account the costs of design, inspection, maintenance and failure. Thus, the function to be minimized is a cost function that covers design, inspection, maintenance, and failure costs. The restriction function of the analysis is constructed based on the evolution of the reliability index over time, being defined by means of response surfaces of the variables.

5.1 Sequential Quadratic Programming (SQP)

According to [32, 33] SQP is one of the most efficient methods for solving nonlinear programming problems. The main idea of this class of methods is to transform a constrained optimization problem into an unrestricted optimization problem by generating quadratic sub problems, which are more easily solved at each step.

This set of methods was popularized mainly from the mid-70s with the emergence of the Quasi-Newton versions and their generalizations. The works of [34-36] stand out at the beginning of the development of the method. SQP research deals with the efficient use of second derivatives of objective function, particularly in difficult-to-solve problems. Thus the SQP methods are generalizations of the Newton method for the general optimization problem where a problem with objective function and non-linear constraints is currently addressed. The main idea of the method is to linearize the optimality conditions of the problem, expressing the equations resulting from this process in a solvable system. Linearization allows the adoption of algorithms with fast local convergence making the method efficient. In this way the SQP works by replacing, at each iteration, the objective function by a quadratic approximation of the Lagrangian function of the original problem at a point x_k and the constraints by linear approximations also at the point x_k. This process even justifies the name of the method. This approximation can be done by expanding the Taylor series Lagrangian function and taking the first three terms for the objective function, the first two terms, for the constraints. In this way the sub problem to be solved at each iteration k is a quadratic problem with linear constraints, which, compared to the original problem, can be considered to be easier to solve.

These methods can be considered primitive-dual methods, in the sense that they work simultaneously in the space of the primary variables and in the space of the multipliers of Karush-Kuhn-Tucker (KKT), dual variables.

In general, nonlinear programming algorithms solve problems of obtaining extremes by calculating, at each iteration, two main parameters: direction of descent (or rise if the problem is maximization) and distance to travel in the calculated direction (unidirectional end). Through the SQP, the descent (or climb) directions are obtained for each variable considered in the problem. The problem of obtaining the unidirectional end is solved using another type of optimization algorithm, in this case applied to unidirectional problems. There are several algorithms to treat this last problem, being possible to emphasize the methods of polynomial approximations and also dichotomy. However in this work we chose to use the Golden Section method to solve the one-way problem.

For a more in-depth discussion of this method and also of other non-linear programming methods it is suggested to refer to the following references [32, 37-39].

5.2 Equations of the SQP Method

In this item will be presented the equations used by the SQP and also the philosophy of the method. Initially, it is observed that problems that only contain equality restrictions are not very common in engineering practice, but the initial discussion to be presented here will be restricted to this case. Thus the following problem will be addressed which one wishes to solve:

$$\text{experience plan (EP)} \quad \begin{array}{lll} \textit{Minimize} & f(x) & \\ \textit{Subject to} & h_i(x) = 0 & i = 1.....m \\ x \in R^n & & \end{array} \tag{12}$$

, where: $f : R^n \to R^n$ *and* $h : R^n \to R^n$ are functions that are continuously differentiable functions and h a vector of m functions h_i. The Lagrangian function for this problem is given by:

$$L(x,\lambda) = f(x) + \sum_{i=1}^{m} \lambda_i h_i(x) \tag{13}$$

The main idea of the SQP for the EP problem is, from x_k, make an approximation that generates a quadratic sub problem and, after solving this sub problem, obtain the new point x_{k+1}. One way to find the optimal solution to this sub problem is to find the KKT point. This search is done by solving the system with $n + m$ variables x and λ and $n + m$ equations.

$$F(x,\lambda) = \begin{bmatrix} \nabla f(x) + \sum_{i=1}^{m} \lambda_i \nabla h_i(x) \\ h_i(x) \end{bmatrix} = 0 \tag{14}$$

We will use A^k to denote the Jacobian matrix of constraints h at point x_k, that is:

$$A^{k^t} = \left[\nabla h_1(x^k), \nabla h_2(x^k), ...\nabla h_m(x^k) \right] \tag{15}$$

and the hessian matrix in x of the Lagrangian function associated with the problem EP at the point (x_k, λ_k) will be denoted by:

$$W^k = \nabla^2_{xx} L(x^k, \lambda^k) = \nabla^2 f(x^k) + \sum_{i=1}^{m} \lambda_i^k \nabla^2 h_i(x^k) \tag{16}$$

Since A, at the optimal point, has a complete rank, the optimal solution (x_k^*, λ_k^*), of the EP problem satisfies Eq. (14). The Newton pitch of the iteration k is given by:

$$\begin{bmatrix} x^{k+1} \\ \lambda^{k+1} \end{bmatrix} = \begin{bmatrix} p^k \\ \nu^k \end{bmatrix} + \begin{bmatrix} x^k \\ \lambda^k \end{bmatrix} \tag{17}$$

Where p_k and v_k is the solution of the following KKT system:

$$\begin{bmatrix} W^k & A^{k^t} \\ A^k & 0 \end{bmatrix}\begin{bmatrix} p^k \\ v^k \end{bmatrix} = \begin{bmatrix} -\nabla_x L(x^k,\lambda^k) \\ -h(x^k) \end{bmatrix} \tag{18}$$

It should be noted that the Jacobian matrix of Eq. (14) at the point (x_k, λ_k) is

$$J^k = J(x^k,\lambda^k) = \begin{bmatrix} W^k & A^{k^t} \\ A^k & 0 \end{bmatrix} \tag{19}$$

Newton's pitch is well defined when the matrix J^k is non-singular. As this condition is met, Newton's algorithm for nonlinear systems converges quadratically to the solution. However, Newton's method has some drawbacks:

1) The system shown in Eq. (18) can determine not only the possible local minimizers, but also the maximizers as well as saddle points.

2) The sequence (x_k, λ_k) may not converge if the choice of the starting point is not sufficiently close to the optimal solution.

The choice of a starting point close to the optimal solution of the problem is the main drawback for the construction of the general and real algorithm based on Newton's method. In order to remedy these drawbacks and still make use of the quadratic convergence of the Newton pitch, when the starting point is close to the optimal solution, the Newton method associated with other methods is used.

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5.3 Structure of the Method

There is another way of looking at Eq. (18) system. Suppose that in the iteration k the quadratic sub problem defined is:

$$(QS)\quad \begin{aligned} &\text{Minimize} && \frac{1}{2}p^t W^k p + \nabla^t f^k p \\ &\text{Subject to} && A^k p + h(x^k) = 0 \end{aligned} \tag{20}$$

Where W^k is given by Eq. ([illegible]). It should be noted that the objective function of the QS problem differs only by a constant of the quadratic approximation of the Lagrangian function associated to the QS problem, defined as follows:

$$L(p,\lambda) = \frac{1}{2}p^t W^k p + \nabla^t f^k p + \lambda^t (A^k p + h(x^k)) \tag{21}$$

In fact, given (x_k, λ_k) the quadratic model for the Lagrangian function is:

$$M_L(p) = L(x^k,\lambda^k) + \nabla^t L(x^k,\lambda^k)p + \frac{1}{2}p^t W^k p \tag{22}$$

But we have that:

$$L(x^k,\lambda^k) = f(x^k) + \lambda^{k^t} h(x^k) \tag{23}$$

$$\nabla^t L(x^k,\lambda^k)p = \nabla^t f(x^k) + \lambda^{k^t} A^k p \tag{24}$$

Thus Eq (22) can be rewritten as:

$$M_L(p) = \frac{1}{2}p^t W^k p + \nabla^t f(x^k)p + v \tag{25}$$

Since the constant v can be defined as:

$$v = L(x^k,\lambda^k) + \lambda^{k^t} A^k p = f(x^k) + \lambda^{k^t}\left(A^k p + h(x^k)\right) \tag{26}$$

Considering the restriction of the sub problem QS, we have $\nu = f(x^k)$. Thus, each iteration of the SQP consists of minimizing the quadratic model of the Lagrangian function, subject to the linearization of the constraints.

If the conditions used to show the uniqueness of J^k occur, the sub problem QS has a unique solution (p_k, μ_k) satisfying:

$$\begin{bmatrix} W^k p + \nabla f^k + A^{kt} \mu^k \\ A^k p + h^k \end{bmatrix} = 0 \tag{27}$$

It should be noted that p_k and ν_k can be identified as the solution of the Newton Eq. (18). Subtracting $A^{k^t} \lambda^k$ on both sides of Eq. (18) yields:

$$\begin{bmatrix} W^k & A^{kt} \\ A^k & 0 \end{bmatrix} \begin{bmatrix} p \\ \lambda^{k+1} \end{bmatrix} = \begin{bmatrix} -\nabla f(x^k) \\ -h(x^k) \end{bmatrix} \tag{28}$$

By the non-singularity of the coefficients of the matrix we have that $p = p^k$ and k $\lambda^{k+1} = \nu$[illegible]

In conclusion, the KKT system shown in Eq. (18) for the EP problem is equiv[illegible]t to th[illegible] opti[illegible]onditions for the QS problem.

The interpretation in terms of Newton's method facilitates the conver[illegible]e analys[illegible] whil[illegible]e structure of Quadratic Sequential Programming allows developing practical algorithms to solve [illegible]oble[illegible] like QS.

The structure of the SQP for nonlinear problems with equality [illegible]int is eas[illegible]xtended to problems with inequality constraints of the type:

$$A^k p + g(x) \; [illegible] \tag{29}$$

At each step the quadratic problem must be solved:

$$(QS[illegible]) \quad [illegible]ize \quad \frac{1}{2} p W^k p + \nabla^t f^k p \qquad S[illegible] \quad A^k p + g(x) \le 0 \tag{30}$$

The problem can be solved simil[illegible] to the ma[illegible]r described for the equality problem. The existing modification relates to the fact that in step p^k and the ne[illegible] esti[illegible]e of the m[illegible]iplier λ^{k+1} are defined by the solution and the Lagrange multipliers corresponding to the problem [illegible].

5.4 SQP Method Algorith[illegible]

Consider that [illegible] wa[illegible]o sol[illegible] [illegible]llowing optimization problem:

$$\begin{array}{lll} \textit{Minimize} & f(x) & \\ \textit{Subject to} & g_j(x) \le 0 & ,j=1,\ldots n_g \end{array} \tag{31}$$

Assuming that in the i^{th} iteration the iterative process lies on a given point x_i, the direction of descent (rise) of the objective function must first be determined for the determination of the desired end. This direction, s, is obtained from the resolution of the following quadratic programming problem:

$$\begin{array}{lll} \textit{Minimize} & \varphi(s) = f(x_i) + s^T.g(x_i) + \frac{1}{2} s^T.A(x_i, \lambda_i).s & \\ \textit{Subject to} & g_j(x_i) + s^T.\nabla g(x_i) \le 0 & ,j=1,\ldots n_g \end{array} \tag{32}$$

Where g is the gradient of the objective function f, A is a definite positive approximation for the Hessian of the Lagrangian function. After solving this quadratic programming problem, the values for the Lagrange multipliers will be obtained, as well as the slope directions, variables (s_i, λ_i). Thus the next point of the iterative process is obtained by means of the following expression:

$$x_{i+1} = x_i + \alpha.s \tag{33}$$

Being that it is found by minimizing the following one-dimensional function:

$$\Psi(\alpha) = f(x) + \sum_{j=1}^{n_g} \mu_j \left|\min\left[0, g_i(x)\right]\right| \tag{34}$$

In this equation μ_j is equal to the absolute value of the Lagrange multipliers for the first iteration, as shown in Eq. (35):

$$\mu_j = \max\left[\left|\lambda_j^{(i)}\right|, \frac{1}{2}\left(\mu_j^{(i-1)} + \left|\lambda_j^{(i-1)}\right|\right)\right] \tag{35}$$

, where the index i denotes the number of the iteration. In this work, the unidirectional problem is solved using the Golden Section method. The matrix A is a definite positive matrix that approximates the Hessian of the objective function. In the first iteration this matrix is defined as an identity matrix being updated as the iterative process progresses. This update is done using the equation proposed by BROYDON- FLETCHER-SHANNO-GOLDFARB, equat[illegible]S, and recommended in [39]. Like this:

$$A_{new} = A - \frac{A.\Delta x.\Delta x^T.A}{\Delta x^T.A.\Delta x} + \frac{\Delta l.\Delta l^T}{\Delta x^T.\Delta\text{[illegible]}} \tag{36}$$

Given that in this equation Δx and Δl are defined as:

$$\Delta x = x_{i+1} - x_i \tag{37}$$

$$\Delta l = \nabla_x L(x_{i+1}, \lambda\text{[illegible]}\nabla_x L(x_i, \lambda_i) \tag{38}$$

, where L is the Lagrangean function and ∇_x represents th[illegible]agrangi[illegible]function gradient with respect to the variables x. To ensure that A is positive definite, Δl is modified[illegible]followi[illegible]n is true.

$$\Delta x^T.\Delta\text{[illegible]} < 0\text{[illegible]}.\Delta x \tag{39}$$

If Eq. (39) is true, Δl must be recalcu[illegible]d by [illegible] follo[illegible]g equation:

$$\Delta l\text{[illegible]} + (1-\theta)A.\Delta x \tag{40}$$

where:

$$\theta = \frac{0,8.\Delta x^T.A.\Delta x}{\Delta x^T.A.\Delta x - \Delta x^T.\Delta l} \tag{41}$$

Given the equ[illegible]ions a[illegible]e SQP [illegible]orithm, it will be shown the applications resulting from the coupling between this optimization m[illegible]od a[illegible] the r[illegible]bi[illegible] mechanistic model.

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6 Optimiza[illegible] mo[illegible] for determining the inspection instance and maintenance

In this item we di[illegible]ss a model developed in this work for the determination of the ideal moment for performing the structural inspection and maintenance for a given desired level of security. This model, resulting from the triple coupling between mechanical fatigue, reliability and optimization, considers two types of maintenance: perfect maintenance and imperfect one. In the first type of maintenance, it is considered that the structure is replaced by a similar one in perfect condition after reaching the number of critical loading cycles, which is determined according to the desired structural safety. In this model we aim to determine the number of load cycles limit for which the structure should be replaced.

In the imperfect maintenance model, the number of inspections and maintenance planned before the replacement of the structural element by another should be initially defined. Maintenance is said to be imperfect because the structure is not replaced when it reaches the limit state, but is repaired. In the studied case, growth of cracks under fatigue, maintenance is carried out by inserting a type of bonding material between the faces of the cracks. The limit state considered for determining the time of inspection for imperfect maintenance is the length of the cracks. Thus, the number of cycles of critical loading is determined considering a certain level of safety and the maximum length of cracks.

The level of security to be considered in each case depends on the importance of the structural element considered in the structural system to which it belongs. Onoufriou [40] presents a table, which is reproduced below, containing the values for the target reliability index according to the importance of the structural element studied. These values will be adopted in the analyzes developed in this work.

Table1 Target Reliability Indices according to [40]

Consequence of Failure	Target Reliability Index	Probability of Failure
Very serious	4.2	$1{,}4.\ 10^{-5}$
serious	3.7	$1{,}1.\ 10^{-4}$
Not serious	3.1	$9{,}7.\ 10^{-4}$
Local Effects	2.3	[illegible]
Does not affect	1.0	[illegible]

In both models, the optimization algorithm chosen is the Golden Section since the optimization problem depends on a single variable that is the number of active loading cycles. It should be emphasized that in this model the cost of maintenance as well as the materials involved in structural repair is not discussed. This variable will be considered in the other models constructed in this work.

6.1 Perfect Maintenance Model

In this model, the objective is to determine the ideal time for the inspection and maintenance, for a given level of desired safety, in order to replace the structural element considered by another in good condition. In this model, the optimization problem to be solved is that presented in Eq. (42):

Determine the number of loading cycles in order to:

$$\text{Minimize} \qquad f(x,y,z)=\left|\beta_{Structural}(x,y,z)-\beta_{Target}\right| \tag{42}$$

in this equation, x represents the random variables of the reliability problem, y the variables to be optimized and z the parameters of the mechanical fatigue model. β_{Target} indicates the target reliability index chosen for the moment of maintenance, and $\beta_{Structural}$ is the reliability index calculated with the parameters of the reliability model and the number of load cycles determined by the optimization model.

In this model the boundary states are: stress intensity factor greater than the threshold stress intensity factor, a crack propagation rate greater than the limit propagation rate, and finally a connection of the crack with some side of the structure.

6.2 Perfect Maintenance Model Applied to a Beam under Three-Point Bending

In this item, the perfect maintenance model will be applied to the study of the structure shown in Fig (1). It is a beam of S_v of length and W_v of height with a central notch of depth equal to a_0, which was requested to flexion in three points under concentrated load F. In this analysis, the propagation of the crack under fatigue will be carried out considering the oscillatory loading regime composed of a complete loading and unloading cycle.

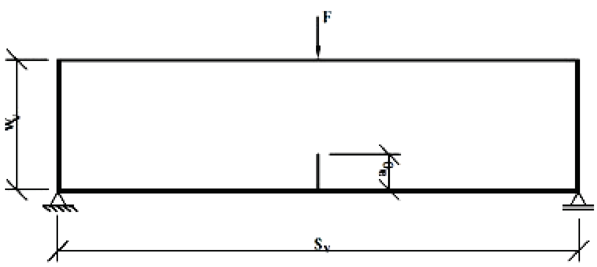

Figure 1 Structure considered.

The following properties were adopted for the constituent material of the structure: longitudinal modulus of elasticity $E=2.1 \times 10^8\ kN/m^2$, Poisson coefficient $\nu = 0.20$, tensile strength factor $K_c = 1.04 \times 10^5\ kN/m^{3/2}$, tensile strength factor of the Paris law $\Delta K_{th} = 1.0\ kN/m^{3/2}$ and exponent n parameter of the Paris law $n = 2.70$. The Paris law was integrated considering the increase in crack length equal to $\Delta a = 0.05$ m.

The reliability model was constructed considering 3 random variables: the active load, F, the parameter C of the Paris law and the initial length of the crack, a_0. The following statistical properties were adopted for these random variables: $F \sim N(5.0;\ 0.80)\ kN$, parameter C of the Paris *law* $C \sim LN\ (3,0.10^{-10}\ ;1,8.10^{-10}\ )\ m/cycles\ (kN/m^{3/2})^n$ and initial crack length $a_0 \sim N(\ 0,01;\ 0,003)\ m$. The other variables of the analysis are considered as deterministic. The span of the beam was considered equal to $s_v = 5.0\ m$ and the height of the beam was allowed equal to $W_v = 1.25\ m$. The tolerance adopted for the convergence of the analysis was considered equal to $1x10^{-4}$. To complete the analysis data, the target reliability index for the optimization model was considered equal to β_{Target} = 3,10. This value is recommended in [40], being shown in Table 1, for ruptures that do not seriously affect the behavior of the structural system to which it belongs.

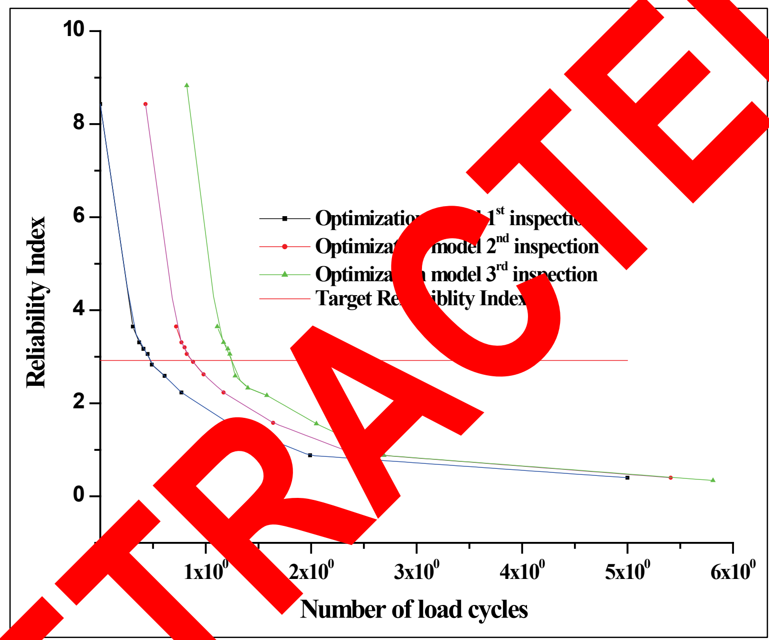

Figure 2- Diagram β x Number of Cycles.

In Fig. (2), the convergence diagram for the Golden Section optimization model is presented. The analysis was developed considering three interventions in the considered beam. As shown in this figure, the convergence of the reliability / optimization model for each of these curves was obtained using 26 calls from the reliability model. In addition, these curves show a good performance of the optimization algorithm which converges smoothly to the solution.

In Fig. (3), the maintenance curves obtained by the used model are shown. It is verified that the maintenances should be carried out when the number of load cycles acting on the structure is equal to 3,942 × 10^5 or multiples of that value. At that moment the structure will meet the reliability index equal to the desired one in the analysis.

6.3 Imperfect Maintenance Model Applied to a Beam under Three-Point Bending

It will be discussed in this item the application of the imperfect maintenance model to the study of the structure shown in Fig. 1. In this analysis the propagation of the fatigue crack will be carried out under oscillatory loading composed of a complete loading and unloading cycle.

The reliability model was constructed considering 2 random variables: the active load, F and the parameter C of the Paris

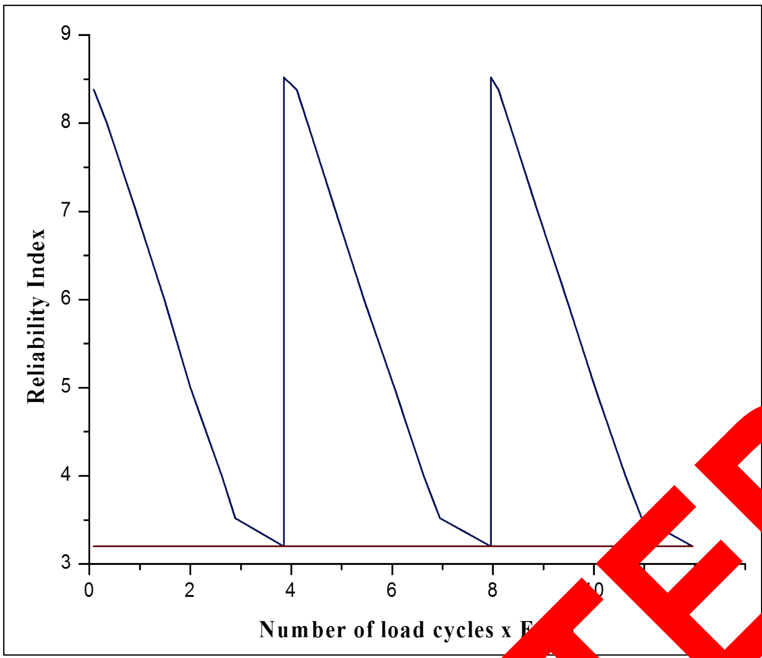

Figure 3 Maintenance Cur[illegible]edictive)

law. The following statistical properties were adopted for the [illegible] random v[illegible]iables: $F \sim N(5.0;\ 0.80)$ kN, parameter C of the Paris *law* $C \sim LN\ (3{,}0.10^{-10}\ ;1{,}8.10^{-10}\ )$ *m/cycles* $(kN/m^{3/2})^n$. [illegible] other [illegible]riables of the analysis are considered as deterministic. The span of the beam was considered equal [illegible] 5.0 m, th[illegible] of the beam was allowed equal to $W_v=1.25m$ and the initial crack length from $a_0 = 0.01$ m. The tolerance [illegible]opte[illegible] the convergence of the analysis was considered equal to $1x10^{-4}$. To complete the analysis data, the ta[illegible]eliabi[illegible]ex for the optimization model was considered equal to $\beta_{Target} = 2.30$, according to the criteria of T[illegible] 1. Th[illegible]tructu[illegible] was analysed considering that the maintenance process will consist of an imperfect maintenance and [illegible]per[illegible]t m[illegible]ce.

The number of loading cycles, t[illegible]btained [illegible]he optimization model, to perform the imperfect maintenance will be calculated considering as a limit sta[illegible] the [illegible]th of the [illegible]ck equal to 10 cm. In this way the optimization model will calculate the number of load cycles in w[illegible]n maintena[illegible] is to be performed so that the reliability index of the structure is equal to the desired target. The imperf[illegible]ainte[illegible]nce will b[illegible]carried out considering that the bonding material placed on the faces of the crack has a rigidity equal to [illegible] = $1{,}0.\ 10^9$ kN / m.

In Fig. (4) the[illegible]ve[illegible]nce d[illegible]ra[illegible]or the Golden Section optimization model is presented. In a comparative way, the analysis of this [illegible]cture[illegible]as carri[illegible] out considering also the perfect maintenance model. As this figure shows for the construction of eac[illegible]e, tha[illegible]or the convergence of the model, 21 calls of the reliability model are necessary.

In Fig. (5) the maint[illegible]nce curves for the analysis of the considered structure are presented. The green curve shows the evolution of the reliability index in relation to the number of loading cycles applied until the first inspection, when the crack reaches a length of 10 cm. It is noted that for imperfect maintenance the first inspection should be done when the number of loading cycles is equal to 6.873×10^5. At that time the adhesive material is applied to the faces of the crack and the structure is exposed again to the loading of fatigue. From this point on we must verify the curve in red which represents the evolution of the reliability index in relation to the number of cycles of load acting after performing the maintenance in the structural element. It is observed that after the maintenance the structure recovers part of its capacity resistant to fatigue and consequently part of its structural safety. According to the optimization analysis performed, the structural element studied should be replaced when the number of cycles of loading acting equals 1.01×10^6. Only by comparison the structure was also analysed considering perfect maintenance. The evolution of the reliability index in relation to the number of load cycles acting on the perfect maintenance model is shown by the blue curve. It is observed that by means of this model the structure must be replaced when exposed to $7{,}845 \times 10^5$ charge cycles. Thus, by performing only imperfect maintenance on the structural element, it is possible to increase the useful life of the structure by approximately 23%, which is a significant gain. This is further increased by conducting more inspections over time.

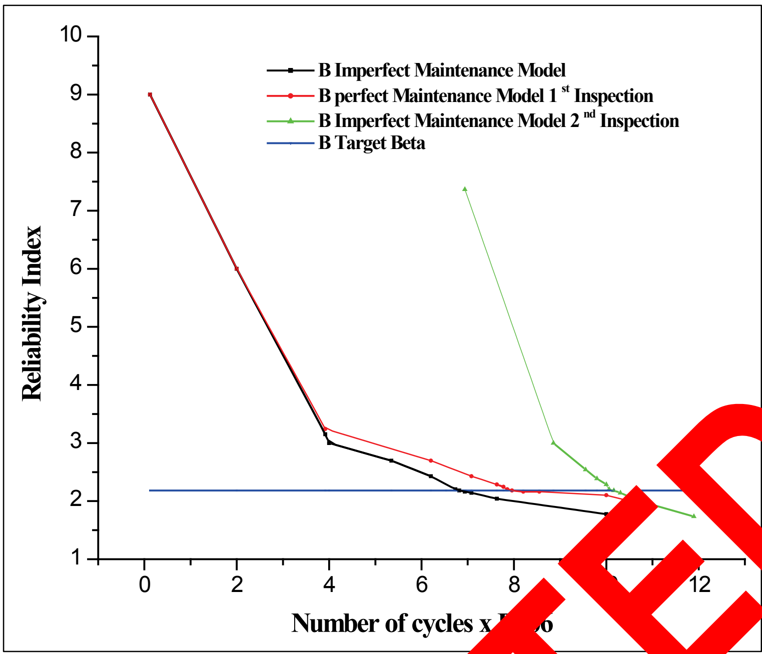

Figure 4- Diagram b vs Number of Cycles

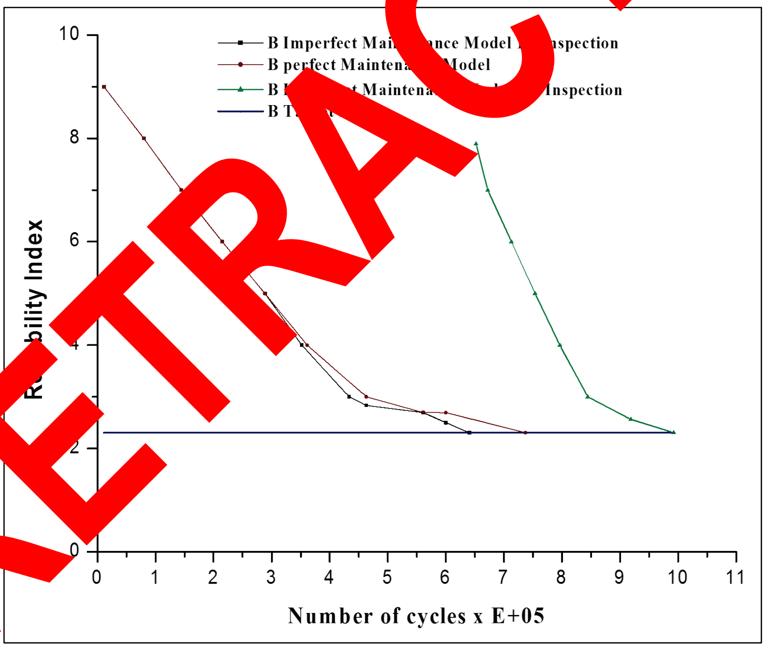

Figure 5 Evolution of b with the number of load cycles applied. Maintenance Curves.

6.4 The reliability analysis

The reliability analysis of the structure shown in Figure (5) will be performed by considering two different scenarios that differ in the number of random variables considered in the analysis. In the first scenario, 3 random variables will be considered, while in the second scenario, 5 random variables will be considered.

6.4.1 1st Scenario

In the reliability analysis performed were considered as random variables the force applied at the right end of the structure, F_x, the coefficient C of the Paris law and the distance between the holes, D_f. The following statistical properties were adopted for these random variables: acting load $F_x \sim$ LN (6.0; 1.0) kN/m, distance between holes $D_f \sim$ N (0,025; 0,001) *m* and parameter *C* of the Paris law, C ~ LN ($1,63.10^{-13}$; $4,0.10^{-14}$) *m/*cycles ($kN/m^{3/2}$)^n. The other variables involved in the problem were considered as deterministic. The initial crack length was allowed to equal a = 0.50 mm, the diameter of the holes of the plate *D = 5.0* mm and the number of cycles of applied load equal to N^{acting}_{cycles} = 4.0 10^{13} Cycles. The tolerance adopted for the convergence of the analysis was considered equal to1.10^{-4}.

The results were obtained in terms of the coordinates of the design point and the reliability index. In Fig. (6), Fig. (7), Fig. (8) and Fig. (9) we present the results for the convergence of the random variables of the analysis and also for the reliability index, *β*.

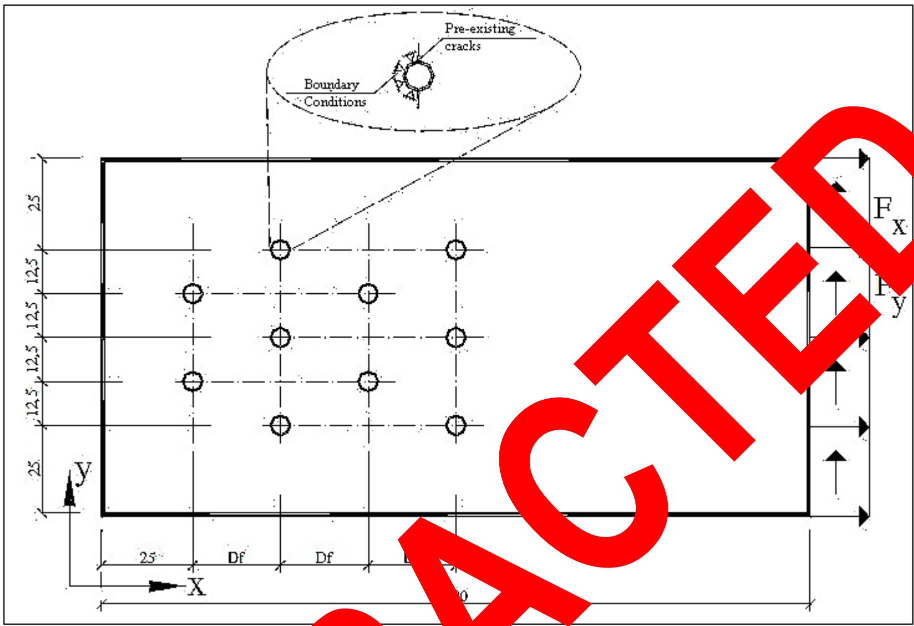

Figure 5 Structure analysed. Dimensions in millimetres.

These diagrams show good direct coupling performance. In this analysis, 19 iterations were performed, resulting in 76 calls from the mechanical model to convergence. During the analysis, some points were observed where the algorithm faced difficulties of convergence. However the method was able to overcome these points and achieve convergence. This indicates that, in addition to being efficient, this method is also robust.

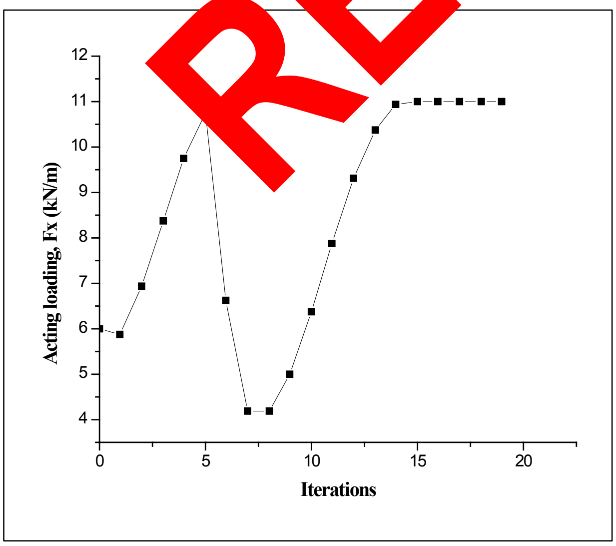

Figure 6- Convergence for loading

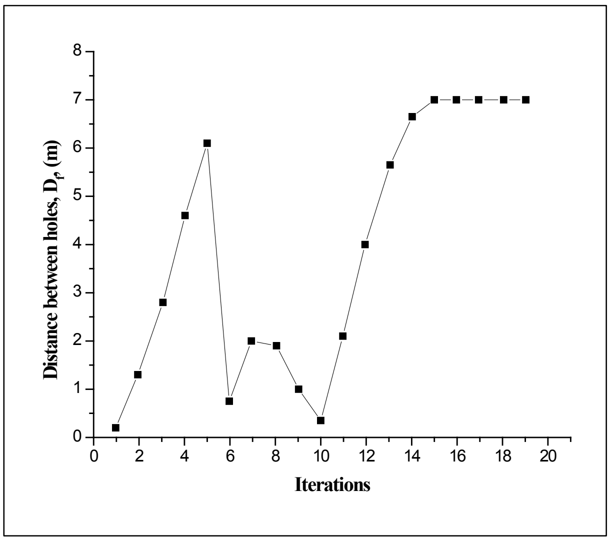

Figure 7- Convergence parameter C (Law Paris)

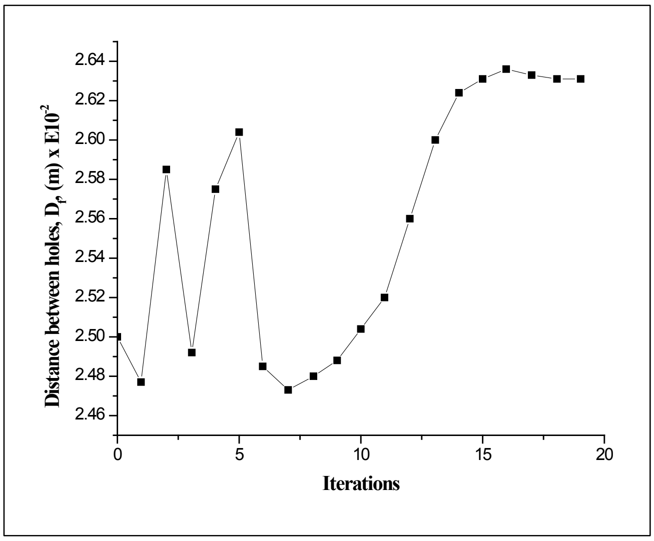

Figure 8- Convergence for Df

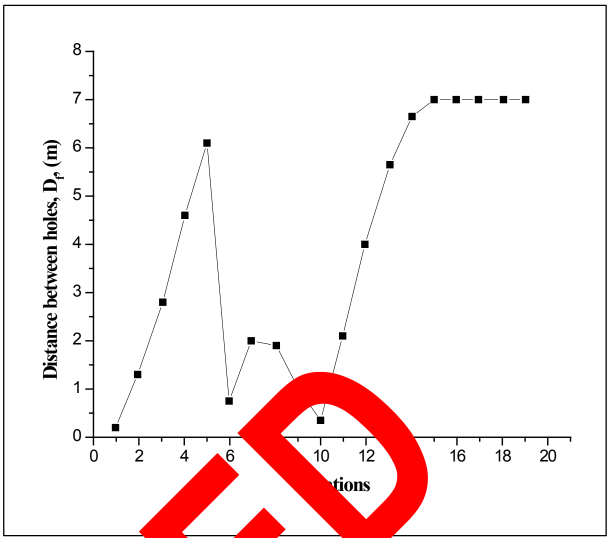

Figure 9- Convergence for β

6.4.2 2nd Scenario

In the second scenario of the reliability analysis of this problem are considered 5 random variables. The analysis will be carried out assuming that the working load, *Fx*, the parameter C of the Paris law, the diameter of the holes, *D*, the distance between the holes, D_f, and the initial length of the crack, a_0 are random variables. The following statistical properties were adopted for these random variables: acting load F_x ~ LN (6.0; [illegible]) kN/m, distance between holes D_f ~ N (0,025; 0,001) *m* and parameter *C* of the Paris law, C ~ LN (1,63.10^{-13}; 4,0.10^{-14}) m/cycles(kN/m$^{3/2}$)^n, holes diameter *D* ~ *N* (0.005; 0.0001) and initial crack length a_0 ~ *LN* (0, 0005; 0.0002[illegible]). The number of active charging cycles was considered as deterministic being equal to N^{acting}_{cycles} = 4.0 10^{13} Cycles. The other variables involved in the problem were considered as deterministic. The tolerance adopted for the convergence of the analysis was considered equal to1.10^{-4}.

As in the first scenario the results were obtained in terms of the coordinates of the design point and the reliability index. In Fig. (10), (11), Fig. (12) and Fig. (13) are presented the results for the convergence of the random variables of the analysis and also for the reliability index, *β*.

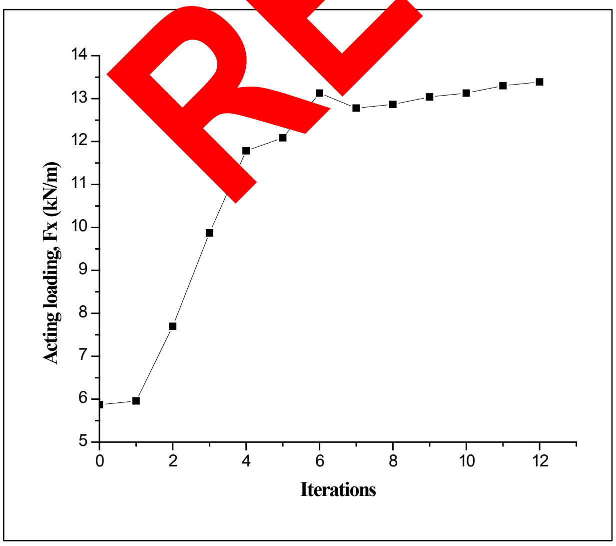

Figure 10- Convergence for loading

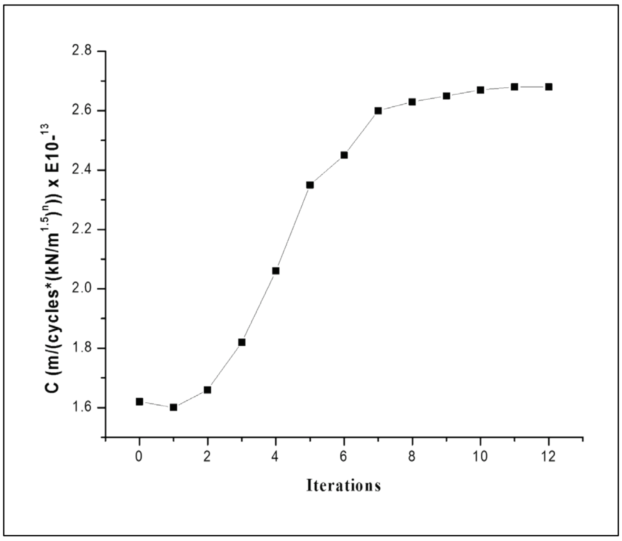

Figure 11- Convergence parameter C (Law Paris)

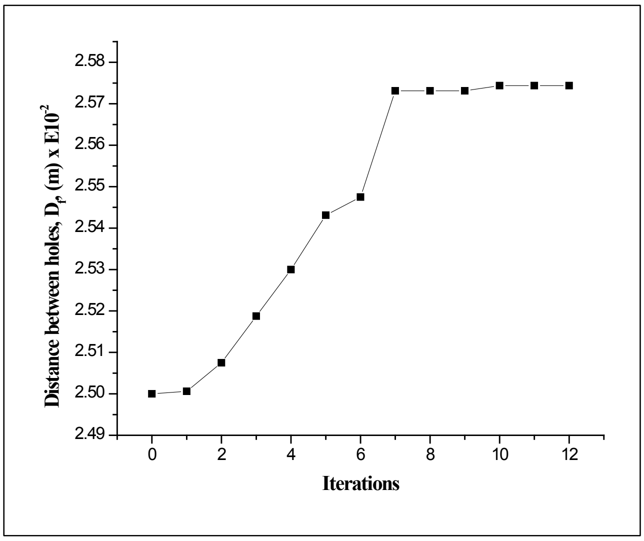

Figure 12- Convergence for D_f

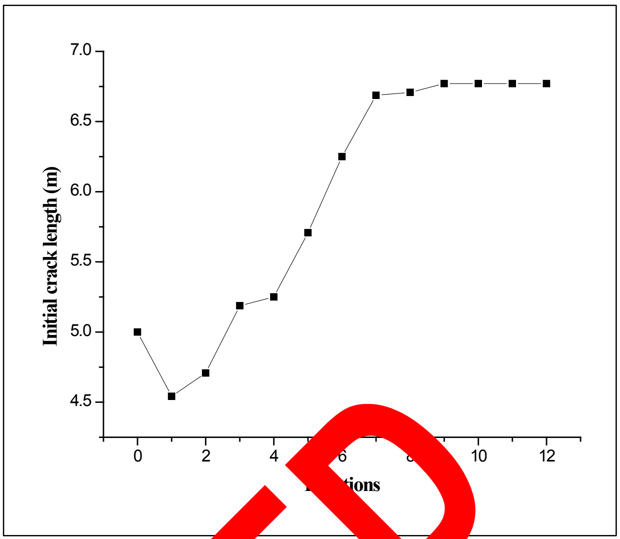

Figure 13- Convergence for a_0

These diagrams show good direct coupling performance. In this analysis 12 iterations were performed, resulting in 72 calls from the mechanical model to convergence. In this analysis there were no excerpts with the occurrence of instabilities and convergence difficulties, which led to the convergence of the problem with only 12 iterations.

Based on the results of this section, it is verified that the model using Direct coupling is a robust and efficient method of reliability.

7 Conclusion

In this paper, problems involving the coupling between a mechano-reliability model and an optimization algorithm were discussed. The parameters that result from this analysis are the dimensions of the geometry of the structural element and also the intervals for carrying out the maintenance and inspection procedures.

A BEM formulation for contact analysis at the interface between bodies and also cracks was developed. Adhesion and contact problems are of great interest in engineering, since in many situations the loads are transmitted through the friction between the components of the structural elements. This formulation also employs a consistent tangent operator for the solution of the nonlinear system of equations, the surface forces in the contact region being governed by Coulomb's law. This model leads to results comparable with those obtained by the ANSYS program. Despite providing good results, the formulation was applied to simple problems. This formulation may be in the future coupled with other non-linear models for the simulation of complex problems such as structural analysis of the superstructure / foundation / soil.

Another interesting contribution of this work refers to the models that treat stiffened domains. This formulation derives from the BEM/MEF coupling where the BEM equations discrete the domain under analysis and those of the MEF the stiffeners. However, the models discussed here consider two degrees of freedom per node in each rigger, thus enabling approach to problems where riggers form a lattice configuration within the body. The nonlinear effects of plastification of the riggers and adhesion of these to the domain were also treated, being presented in this work the equations for their consideration.

On the BEM/MEF coupling model, the crack propagation model was introduced on a linear elastic regime. This model leads to interesting results and allows us to analyse some structures where this effect is important. This latter model can be improved in the future by considering the nonlinear effects on crack propagation, where the consistent tangent operator model applied to quasi-fragile materials could be inserted.

Closing the topic related to the development of BEM formulations, a formulation was also developed for the analysis of fatigue cracking propagation problems. This model is applicable to materials of fragile and ductile behaviour and has great application in several structures, especially those inserted in the context of mechanical structures (aeronautics, naval, automobile, etc). This model uses the law of Paris and the results obtained through this formulation were interesting. It was possible to approach structures with multiple cracks, thus making it possible to study structures where damages grow in different positions inside the body. In the future, this model could be extended to the consideration of fatigue and corrosion in structures formed by quasi-fragile materials such as concrete.

Probabilistic models were also considered in this study, being applied to the analysis of structures submitted to fatigue. It is known that the integrity of the structures in service essentially depends on its ability to maintain a resistance pattern over time. The formation and growth of cracks due to fatigue processes are among the main causes of ruptures of real structural systems, such as those present in aeronautical structures, naval structures, automobile and marine structures in general. Thus, precise analysis of the growth of pre-existing cracks during the use of a structure or equipment plays a fundamental role in the design of a project, which aims to guarantee its functionality during a useful life. The algorithms FORM, SORM, MSR and the direct coupling between FORM and mechanical model (direct coupling) were implemented. According to the results obtained, it can be seen that the direct coupling provided better responses when compared to the MSR. This is because, in this method, no approximation is made on the form of the limit state equation, the gradients of this equation being obtained directly through queries to the mechanical model. Although the MSR also provides accurate results, it has been found that this method is more computationally costly, and does not offer convergence for reliability analysis with certain experimental plans.

To this probabilistic model was coupled an optimization algorithm for the determination of parameters such as the geometry dimensions of the structure and also intervals for the maintenance performance and inspection procedures, taking into account objective functions written in terms of structural cost and safety. The optimization algorithms SQP and Golden Section were implemented. It should be emphasized that this model was applied in the analysis of simple examples, only with the purpose of showing its great potentiality. Thus, work dealing with the problem with precise and general mechanical models will always provide contributions. Thus, these triple coupling models are good suggestion for future research.

Finally, it should be noted that this paper addressed issues that may be considered innovative in some fields as formulations of the BEM and also in structural reliability.

REFERENCES

[1]- D.S. Dugdale, Yielding of steel sheets containing slits. J. Mech. Phys. Solid. 8(1960) 100-104. doi:10.1016/0022-5096(60)90013-2

[2]- G.I. Barenblatt, The mathematical theory of equilibrium cracks in brittle fracture. Adv. Appl. Mech. **7**(1962) 55-129. doi:10.1016/S0065-2156(08)70121-2

[3]- A. Hillerborg, M. Modéer, P.E. Petersson, Analysis of crack formation and crack growth in concrete by means of fracture mechanics and finite elements. Cement Concrete Res. (6)(1976) 773-782. doi:10.1016/0008-8846(76)90007-7

[4]- A. Carpinteri, Mechanics of Materials and Fracture. Pitagora Editrice, Bologna, 1992.

[5]- F. Barpi, S. Valente, Size-effects induced bifurcation phenomena during multiple cohesive crack propagation. Int. J. Solids Struct. 35(16) (1998) 1851-1861. doi:10.1016/S0020-7683(97)00158-3

[6]- J. Planas, M. Elices, G.V. Guinea, F.J. Gómez, D.A. Cendón, I. Arbilla, Generalizations and specializations of cohesive crack models. Eng. Fract. Mech. 70(14) (2003) 1759-1776. doi:10.1016/S0013-7944(03)00123-1

[7]- A. Carpinteri, P. Cornetti, F. Barpi, S. Valente, Cohesive crack model description of ductile to brittle size-scale transition: dimensional analysis vs. renormalization group theory. Eng. Fract. Mech. 70(14) (2003) 1809-1839. doi:10.1016/S0013-7944(03)00126-7

[8]- V. Tvergaard, Cohesive zone representations of failure between elastic or rigid solids and ductile solids. Eng. Fract. Mech. 70(14) (2003) 1859-1868. doi:10.1016/S0013-7944(03)00128-0

[9]- M. Elices, G.V. Guinea, J. Planas, J. Gomez, The cohesive zone model: Advantages, Limitations and Challenges. Eng. Fract. Mech. 69(2) (2002) 137-163. doi:10.1016/S0013-7944(01)00083-2

[10]- Y.W. Mai, Cohesive zone and crack–resistance (R)-curve of cementitious materials and their fibre-reinforced composites. Eng. Fract. Mech. 69(2) (2002) 219-234. doi:10.1016/S0013-7944(01)00086-8

[11]- E.D. Leonel, O.B.R. Lovon,W.S. Venturini, Boundary element formulation applied to multi-fractured bodies. In: Proceedings of the Bound elem method XXVIII Computational Mechanics Publications, Sounthampton, 2006.

[12]- Y. Mi, Three-dimensional analysis of crack growth. Topics in engineering, vol. 28. Computational Mechanics Publications; 1996.

[13]- E.D. Leonel, W.S. Venturini. Dual boundary element formulation applied to analysis of multi-fractured domains. Eng. Anal. Bound. Elem. 34(12) (2010) 1092–99. doi:10.1016/j.enganabound.2010.06.014

[14]- E.D. Leonel, W.S.Venturini, Multiple random crack propagation using a boundary element formulation. Eng. Fract. Mech. 78(6) (2011) 1077–1090. doi:10.1016/j.engfracmech.2010.11.012

[15]- P.C Paris, M.P. Gomez, W.P. Anderson, A rational analytic theory of fatigue. Trend Eng. 13(1961) 9-14.

[16]- P.C. Paris, F. Erdogan, A critical analysis of crack propagation laws. J. Basic. Eng.-T ASME 85(1963) 528-534. doi:10.1115/1.3656900

[17]- A.P. Cisilino, M.H. Aliabadi, Three-dimensional boundary element analysis of fatigue crack growth in linear and non-linear fracture problems. Eng. Fract. Mech. 63(6) (1999) 713–733. doi:10.1016/S0013-7944(99)00047-8

[18]- D.N. Dell'Erba, M.H. Aliabadi, Three-dimensional thermo-mechanical fatigue crack growth using BEM. Int. J. Fatigue 22(4) (2000), 261–273. doi:10.1016/S0142-1123(00)00011-6

[19]- H.L. Oliveira, E.D. Leonel, Dual BEM Formulation Applied to Analysis of Multiple Crack Propagation. Key Eng. Mat. 560(2013) 99-106. doi:10.4028/www.scientific.net/KEM.560.99

[20]- E.D. Leonel, W.S. Venturini, A.A. Chateauneuf, BEM model applied to failure analysis of multi-fractured structures. Eng. Failure Anal. 18(6) (2011) 1538–49. doi:10.1016/j.engfailanal.2011.[illegible].014

[21]- X. Yan, A finite main crack interaction with an arbitrarily oriented micro- defect. Eng. Failure Anal. 13(6) (2006) 971–980. doi:10.1016/j.engfailanal.2005.04.004

[22]- P.O. Bouchard, F. Bay, Y. Chastel, Numerical modelling of crack propagation: automatic remeshing and comparison of different criteria. Comput. Method. Appl. M. 192(35-36) (2003) 3887–3908. doi:10.1016/S0045-7825(03)00391-8

[23]- X. Yan, Automated simulation of fatigue crack propagation for two-dimensional linear elastic fracture mechanics problems by boundary element method. Eng. Fract. Mech. 74(14) (2007) 2225–2246. doi:10.1016/j.engfracmech.2006.10.020

[24]- E.D. Leonel, W.S. Venturini, Non-linear boundary element formulation with tangent operator to analyse crack propagationin quasi-brittle materials. Eng. Anal. Bound. Elem. 34(2) (2010) 122–129. doi:10.1016/j.enganabound.2009.08.005

[25]- H. Kebir, J. Roelandt, L. Chambon, Dual boundary element method modelling of air craft structural joints with multiple site damage. Eng. Fract. Mech. 73(4) (2006) 418–434. doi:10.1016/j.engfracmech.2005.08.006

[26]- F.R.M. Romlay, H. Ouyang, A.K. Ariffin, A.N. Mohamed, Modeling of fatigue crack propagation using dual boundary element method and Gaussian Monte Carlo method. Eng. Anal. Bound. Elem. 34(3) (2010) 297–305. doi:10.1016/j.enganabound.2009.09.[illegible]

[27]- C.G. Nogueira, E.D. Leonel, Probabilistic models applied to safety assessment of reinforced concrete structures subjected to chloride ingress. Eng. Failure Anal. 31(2013) 76-89. doi:10.1016/j.engfailanal.2013.01.023

[28]- B. M. Ayyub, R.H. McCuen, Probability, statistics, and reliability for engineers and scientists. CRC press, 2016.

[29]- R.E. Melchers, A.T. Beck, Structural reliability analysis and prediction. John Wiley & Sons, 2018.

[30]- M. Modarres, M.P. Kaminskiy, V. Krivtsov, Reliability engineering and risk analysis: a practical guide. CRC press, 2016.

[31]- O. Larsson, Reliability analysis. Lecture notes, *Lund University*, 2015.

[32]- J. Nocedal, S.J. WRIGHT, Numerical Optimization. Springer Series in Operations Research. Springer Verlag, 1999

[33]- M. Lukic, C. Cremona, C. Probabilistic optimization of welded joints maintenance versus fatigue and fracture. Reliab. Eng. Syst. Safe. 72(3) (2001) 253-264. doi:10.1016/S0951-8320(01)00019-9

[34]- M.J.D. Powell, A fast algorithm for nonlinearly constrained optimization calculations. In: G.A. Watson editor, Numerical Analysis Dundee, Number 630 in Lecture notes in Mathematics, p. 144-157. Springer-Verlag. 1977.

[35]- M.J.D. Powell, Algorithms for nonlinear constraints that use Lagrangian functions. Math. Program. 14 (1) (1978) 224-248. doi:10.1007/BF01588967

[36]- S.P. Han, A globally convergent methods for nonlinear programming. J. Optimiz. Theory. App. 22(3) (1977) 297-309. doi:10.1007/BF00932858

[37]- J.F. Bonnans, J.C. Gilbert, C. Lemarechal, C.A. Sagastizabal, Numerical Optimization: Theoretical and Practical Aspects. Springer Verlag, Berlin, 2002.

[38]- R.T. Haftka, M.P. Kamat, Elements of Structural Optimization. Dordrecht: Martinus Nijhoff Publishers. 1985.

[39]- G.N. Vanderplaats, Numerical Optimization Techniques for Engineering Design, Colorado Srings : Vanderplaats Research & Development. 2001.

[40]- T. Onoufriou, Reliability based inspection planning of offshore structures. Mar. Struct. 12(7-8) (1999) 521-539. doi:10.1016/S0951-8339(99)00030-1.