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Composite parameters analysis with boundary element method

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ABSTRACT

In this paper, singular, hyper singular and multi-domain boundary element formulations is developed and applied to the mechanical analysis of two-dimensional isotropic and anisotropic media, respectively. The anisotropy increases the number of elastic constants in the linear stress-strain relationship; hence the construction of fundamental solutions becomes difficult. Subsequently, the fundamental solution of Cruse & Swedlow to deal with anisotropic regions was also incorporated into the formulations. The unique integrations inherent to the method are regularized through the Singularity Subtraction Method allowing the use of polynomial contour elements with high order approximations. Also presented is the multi-region technique for modelling composite structures made up of different materials. The multi-region technique is adopted to couple the interfaces of non-homogeneous multiphase bodies. In the applications, the two plane states, State Plane stress (SPS) and State Plane deformation (SPD) were considered, and the responses obtained with the BEM were compared with finite element responses via the Ansys software. Finally, it is shown how the internal magnitudes, stress and displacement, can be obtained from the integral equations. The results obtained demonstrate good agreement with other reported results and show strong dependence on the material anisotropy.

1 Introduction

The BEM is one of the most well-known numerical methods for solving problems governed by differential equations. This may be an alternative to other numerical methods such as MEF or MDF. The method has a rich mathematical formulation allowing the resolution of several Contour Value Problems (PVC) in the mechanics of solids. The origins of the numerical implementation of integral contour equations, which are the basis of the BEM, have been observed since the early 1960s when computers began to become viable for such analyzes. From that date onwards, the number of published articles on the topic has been increasing.

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Its formulation is based on the transformation of differential equations that govern physical problems into integral equations written on the contour of the domain of analysis. For this, it is necessary to know a fundamental solution for the type of problem to be analyzed.

The first physical problems formulated by integral equations were solved by indirect methods. In this approach, the unknowns are dummy variables associated with the contour, obtained by means of boundary conditions prescribed in a certain number of points. The dummy variables have no physical meaning, and these are used as an aid to obtain the physical quantities of interest. Still in the scope of the methods of resolution of the integral equations, other important works published by Russian authors made that the method of the integral equations became better known in Europe. Among these authors, the work of Kellog [1], which was the pioneer in the use of these integral equations to study the solve problems governed by Laplace's fundamental equation. In his paper, Kupradze [2] proposed a formulation by means of integral equations to determine an approximation for displacements in bodies under linear elastic regime.

Rizzo [3] was the first to solve the integral equations of two-dimensional elasticity problems in a direct way. In their work, the variables resulting from the solution were the displacements and the surface forces, the contour of the problem was discretized by elements of straight geometry. It is also worth mentioning that [4] suggested discretizing domains in subregions to treat non-homogeneous problems. The technique consists basically in imposing conditions of compatibility of displacements and equilibrium of the tensions in the interface of the subregions thus forcing the continuity of the regions. The use of the subregions in the discretization of engineering problems has been adopted in several works of the literature involving different formulations of the BEM [5-8]. As previously mentioned, this technique will be explored in the course of the present study.

A greater generalization of the method came with the contribution of the work of Lachat [9] that parameterized the functions of approximation of the contour elements admitting for the same linear, quadratic and cubic variations. In this work, Lachat solved the integrations of the method numerically through Gauss's quadrature.

Brebbia [10] deduced the integral formulation of elasticity problems from the weighted residuals technique, which supports other numerical techniques. Therefore, from this point, the method based on the integral contour equations has a common root with other numerical methods, such as FEM and [illegible]. Since then, the combination of BEM with other formulations from the weighted residues has been further explored. The coupling of BEM to other numerical methods allows a better utilization of the techniques where each region of the structure can be represented by the method presents the best advantages. Also due to Brebbia is the nomenclature "Method of Contour Elements" [10] which until then was known as the Method of integral equations.

In order to deal with BEM anisotropic materials, [11] proposed two-dimensional elastic fundamental solutions deduced from the analytical formalism of the original complex stress functions of [12-14]. Cruse [11] developed fundamental anisotropic solutions to the three-dimensional elastic problems. Like the fundamental isotropic solution of Kelvin, the anisotropic solution of [11] will also be used in this work to analyze the mechanical response of the structures.

Sahli et al. [15] presented a dynamic formulation of the boundary element method for stress and failure criterion analyses of anisotropic thin plates. The elastostatic fundamental solutions are used in the formulations and inertia terms are treated as body forces. The radial integration method (RIM) is used to obtain a boundary element formulation without any domain integral for general anisotropic plate problems. [16] presents a two-dimensional parametric study of the behavior of uniformly cooled homogeneous linear elastic anisotropic bodies containing cracks using the boundary element method (BEM), investigates the effects of varying material properties, and varying orientation of these material properties, on the magnitude of the stress intensity factors (SIFs) of the cracked bodies.

Azevedo [17] instead of using the formalism of the fundamental solutions of [11] presents an alternative formulation for analyzing anisotropic inclusions in plane problems. For this, the author makes use of the fundamental solution of Kelvin and also of integral representations with field of initial tensions. In the work, the regions of the domain with anisotropic properties are discretized in triangular cells. In these cells, the components of the stress tensor are defined by means of a correction of the elastic stresses of the reference isotropic material proceeding through a penalty matrix.

Ricardela [18] presented BEM formulations for elastoplastic analysis, assuming anisotropic materials from the fundamental [11] solution. Vanalli [19] also made use of this fundamental solution and presented formulations of the BEM and MEF for the viscoplastic and viscoelastic analysis of anisotropic media.

Since the consolidation of the method, several formulations of BEM have been developed to treat different problems such as soil and rock mechanics, soil-structure interaction, fluid mechanics, plasticity, viscoplasticity, anisotropic media, fracture mechanics, contact mechanics , dynamic problems among others [20-26].

In this paper applications of composite and / or anisotropic structures in linear elastic regime are analyzed, using the singular (BEM S) and hyper singular (HS BEM) formulations of the contour elements. In all the applications were considered discontinuous contour elements, thus guaranteeing the hypotheses of continuity of the HS BEM formulation.

2 Boundary element method applied to anisotropic media

The boundary integral equation for linear anisotropic elasticity is derived in the usual manner [27, 28] and is given by

$$C_{ij}(s_k)u_j(s_k)+\int_{\Gamma}T_{ij}(z_k,s_k)u_j(z_k)d\Gamma(z_k)=\int_{\Gamma}U_{ij}(z_k,s_k)t_j(z_k)d\Gamma(z_k) \tag{1}$$

where: i, j = 1,2; C_{ij} is given by $\delta_{ij}/2$ for a smooth boundary; T_{ij} and U_{ij} are the fundamental solution for tractions and displacements respectively; s_k and z_k are the source and field points on the boundary Γ of the characteristic complex plane, defined by $(x_1 + \mu_k x_2)$ and μ_k are the roots of the characteristic equation for the anisotropic material, with $\boldsymbol{k}$ = 1,2. The displacements u_i and the tractions t_j are computed on the boundary Γ. It is worth remembering that this equation is called singular due to the order of singularity $1/r$ of the fundamental solution. Equation (1) is discretized and solved by the usual BEM techniques [29].

The displacements at an internal point p_k from are obtained the boundary values of displacements and tractions as

$$u_i(s_k)=\int_{\Gamma}T_{ij}(z_k,p_k)u_j(z_k)d\Gamma(z_k)-\int_{\Gamma}U_{ij}(z_k,p_k)t_j(z_k)d\Gamma(z_k) \tag{2}$$

The displacements gradients tensor $u_{i,}$, can be evaluated from eq. (2) by differentiation with respect to the coordinates of p_k to give

$$u_{i,l}(s_k)=\int_{\Gamma}T_{ij,l}(z_k,p_k)u_j(z_k)d\Gamma(z_k)-\int_{\Gamma}U_{ij,l}(z_k,p_k)t_j(z_k)d\Gamma(z_k) \tag{3}$$

where

$$T_{ij,l}(z_k,p_k)=-2\,\mathrm{Re}\left[R_{l1}Q_{j1}\left(\mu_1 n_1-n_2\right)A_{i1}/\left(z_1-p_1\right)^2+R_{l2}Q_{j2}\left(\mu_2 n_1-n_2\right)A_{i2}/\left(z_2-p_2\right)^2\right] \tag{4}$$

$$U_{ij,l}(z_k,p_k)=2\,\mathrm{Re}\left[R_{l1}P_{j1}A_{i1}/\left(z_1-p_1\right)+R_{l2}P_{j2}A_{i2}/\left(z_2-p_2\right)\right] \tag{5}$$

n_k are components of the normal vector and R_{ik}, Q_{jk}, P_{jk}, A_{ik} are complex coefficients [11, 27].

Equation (3) is the so called the hyper singular integral equation. This is due to the greater order of singularity of the expression, $1/r^2$.

The BEM formulations can be developed as long as a fundamental solution to the type of material desired in the analysis is known. In this sense, it is intended to present in this section formulations of BEM applied to anisotropic problems deduced from the fundamental solution presented by [11]. From this fundamental solution, the singular and hyper singular contour formulations will be presented to analyze anisotropic elastostatic problems. Furthermore, through the technique of subregions it is possible to analyze composite structures composed of different materials which have general anisotropies up to ideal isotropic.

2.1 multi-region technique

Through the multi-region technique it is possible to analyze composite structures made up of different materials. This technique was initially proposed by [4]. Figure 1 below illustrates the discretization of a domain composed of the multi-region approach:

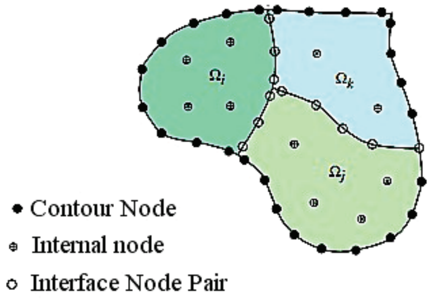

Fig. 1 Discretization of a composite domain using the multi-region technique [30]

For each element that discretizes an interface portion there is another one in the same geometric position with nodal coordinates are equivalent, but their opposite orientation. Each of these coincident elements represents one of the faces of the two regions interconnected by the interface. It is worth noting that the contour elements of the subregions should be oriented counter clockwise so that the normal versor on the surface points out in each region.

3 Applications in elastic regime

In this paper applications of composite and / or anisotropic structures in linear elastic regime are analyzed, using the singular (BEM S) and hyper singular (HS BEM) formulations of the contour elements. In all the applications were considered discontinuous contour elements, thus guaranteeing the hypotheses of continuity of the HS BEM formulation. In the first four applications, the two plane states, State Plane stress (SPS) and State Plane Deformation (SPD) were considered, and the responses obtained with the BEM were compared with finite element responses via the Ansys software. Already in the last application, the problem was dealt with in SPS and the results were compared with reference in the literature. All plane structures were considered in unit thicknesses. The first application concerns a bending plate composed of isotropic regions. The second example refers to a composite beam consisting of an isotropic region and an anisotropic region. The third example shows an anisotropic plate with rigid isotropic inclusions. In examples 2 and 3 structures are evaluated whose algebraic systems were obtained by BEM with different fundamental solutions. The results of this last application were compared with numerical responses obtained in other works of the literature.

3.1 Inhomogeneous plate

In the first example, consider a rectangular plate whose domain consists of three different isotropic materials. Also consider that in the plate are imposed zero displacement boundary conditions on one of its faces, and boundary conditions of forces prescribed on two other faces. Figure 2 illustrates the problem by presenting its geometry, the elastic properties of the three materials and also the contour conditions.

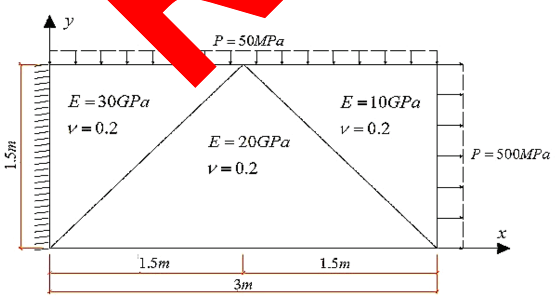

Fig. 2 inhomogeneous cantilever plate

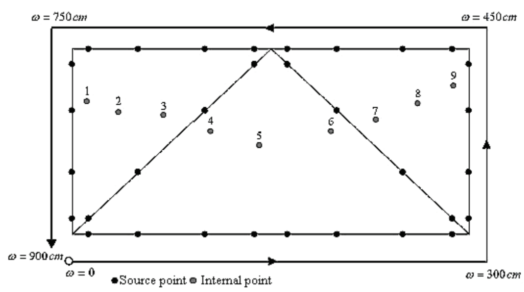

Fig. 3 Mesh with Cubic Contour Elements

In order to solve the problem with the multi-region technique, the contour and the material interfaces of the sheet were discretized with a mesh of 40 nodes and 10 cubic approximation contour elements as shown in Figure 3.

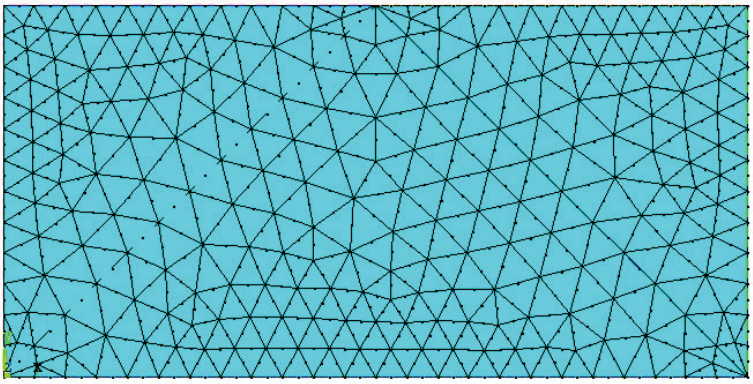

Fig. 4 Discretization in triangular finite element quadratic approximation

The example was processed with both the BEM S formulation and the HS BEM for further comparison of results with finite element analysis. The structure was evaluated in State Plane Stress and State Plane Deformation. The finite element discretization adopted in Ansys consists of 1001 nodes and 464 elements of the triangular type "PLANE 183" of quadratic approximation. In addition, the discretization was planned in such a way that some of the nodes belonging to the contour coincide in coordinates with the source points of the BEM mesh, thus facilitating the comparison of the result. Figure 4 below illustrates the discretization adopted in Ansys.

In order to compare the results of the BEM, BEM S and BEM HS, the displacement values were first analyzed in the 24 source points of the contour mesh. For the comparison, a coordinate ω was adopted which circumvents the plate with origin in the lower left corner, $\omega = 0$, and end at the same point, $\omega = 900$ *cm*. Figure 3 schematizes this coordinate. The results were presented in the graphs of Figure 5 to compare the displacement fields in the contour.

It is possible to verify the similarities of the displacement fields obtained with both numerical techniques. However, to better assure the validity of the results, the stresses σx, σy, and τxy were also evaluated in 9 internal points of the structure. The internal points were numbered as shown in Figure 3 and each sub-region contains 3 of these points. The stress response is illustrated in Figures 6 and 7.

As for the displacement field, similar behavior between the stresses fields found via Ansys and via BEM is also verified. Therefore, it can be concluded that the BEM S and BEM HS formulations have resulted in satisfactory solutions to the respective mechanical problem.

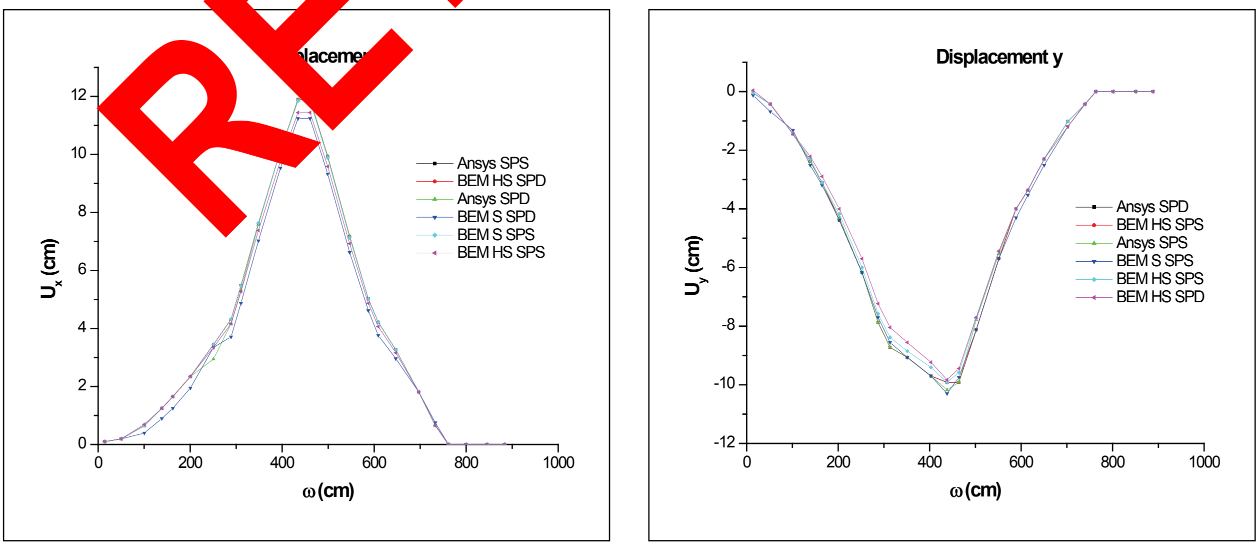

Fig. 5 Contour displacements response: BEM versus Ansys

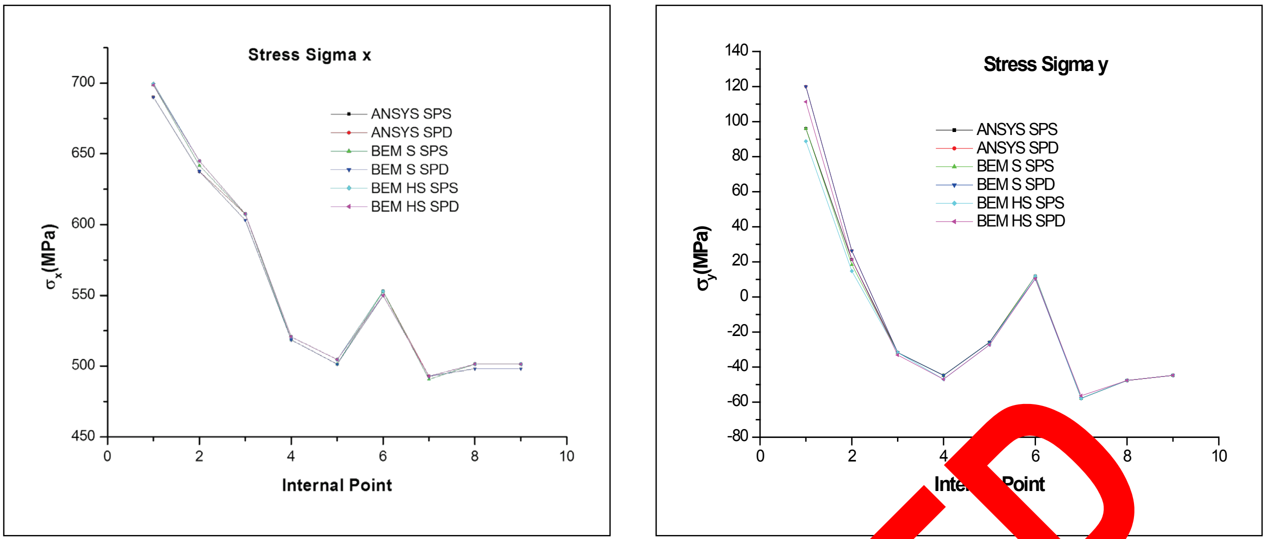

Fig. 6 Response of independent normal stresses: BEM versus Ansys

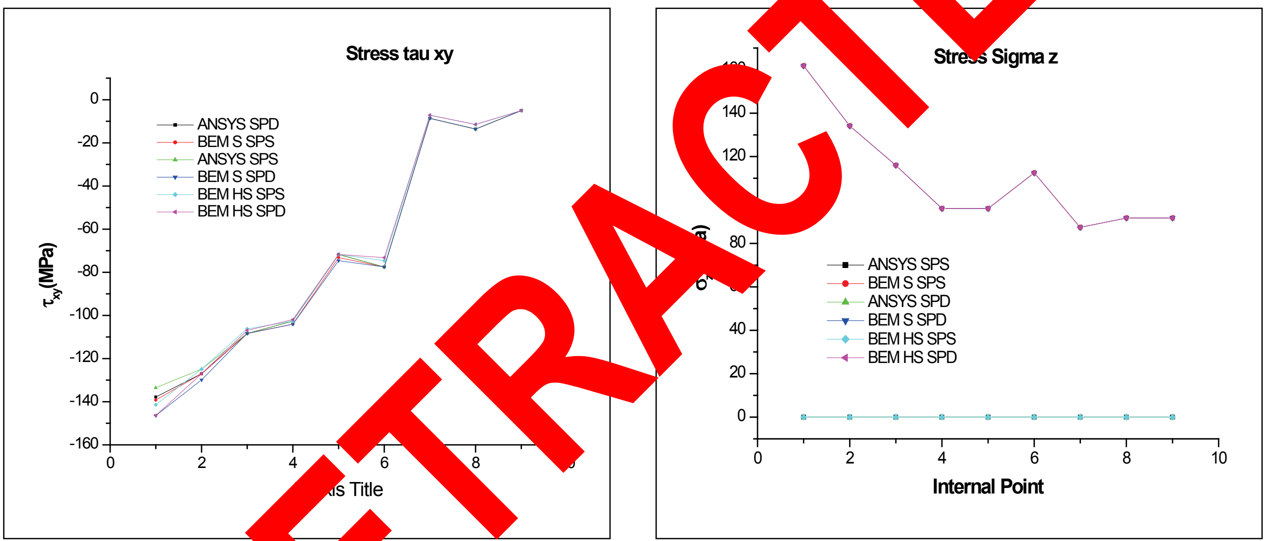

Fig. 7 Shear stress and normal stress σ_z response: BEM versus Ansys

3.2 Non-homogeneous anisotropic beam

For the second example we consider a linear elastic behavior of a beam composed of isotropic and anisotropic sub-regions is simulated using the BEMS and BEMHP formulations. For this, the contour of the upper region was evaluated from the fundamental solution of Kelvin, and the contour of the inferior region from the fundamental solution of [11]. The displacement compatibility imposed by the multi-region technique guarantees the continuity of the displacements at the materials interface. The problem was evaluated considering the SPS and SPD plans states. Figure 8 shows the geometry and the solicitation of the composite beam as well as the elastic properties of the upper isotropic region.

The lower region was considered to consist of a laminated material presented by [19] whose layers are assumed homogeneous orthotropic and are arranged inclined 30 degrees counter clockwise with respect to the axis x. Under these conditions, the lower beam region presents a general anisotropy with the following elastic constants: $E_x = 19.681\ GPa$, $E_y = E_z = 11.248\ GPa$, $G_{xy} = G_{yz} = G_{xz} = 7.933\ GPa$, $\nu_{yx} = 0.529$, $\eta_{xy,} = -1.242$ and $\eta_{xy,} = -0.042$. In the case of SPD, we also considered the elastic constants in the direction z: $\nu_{zy} = 0.30$, $\nu_{zx} = 0.15$ and $\eta_{xy,} = 0.75$.

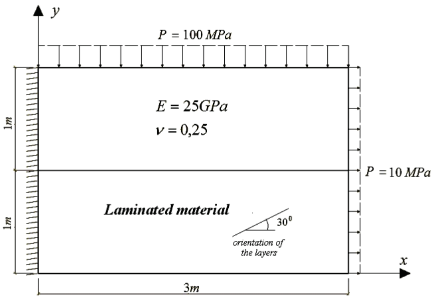

Fig. 8 Composite beam subjected to bending.

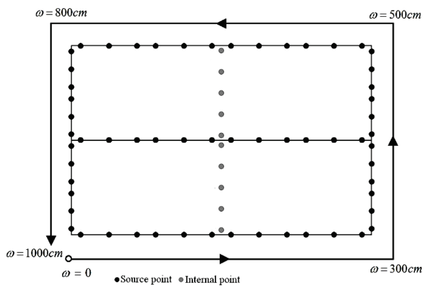

Fig. 9 Contour elements mesh and open contour coordinates

Each of the two subregions was discretized with a contour mesh composed of 12 quadratic approximation elements. Therefore, the discretization of the entire contour and also the interface of the problem totaled 72 nodes. Figure 9 shows the adopted mesh showing the open contour coordinate scheme and also 10 internal points along the height of the section where the stress fields will be evaluated.

The problem was modelled on finite elements using Ansys to allow a comparison of results. The Ansys discretization, shown in Figure 10, adopts 19521 nodes and 6400 triangular "PLANE 1… elements of quadratic approximation. Once again the discretization was planned in such a way that the coordinates of some of the contour nodes coincided with the source points of the BEM mesh.

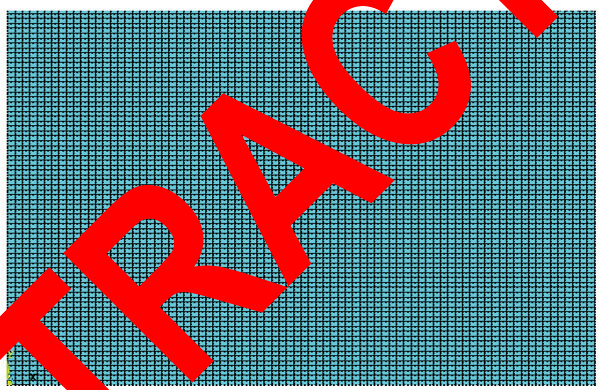

Fig. 10 Discretization in triangular finite elements of quadratic approximation

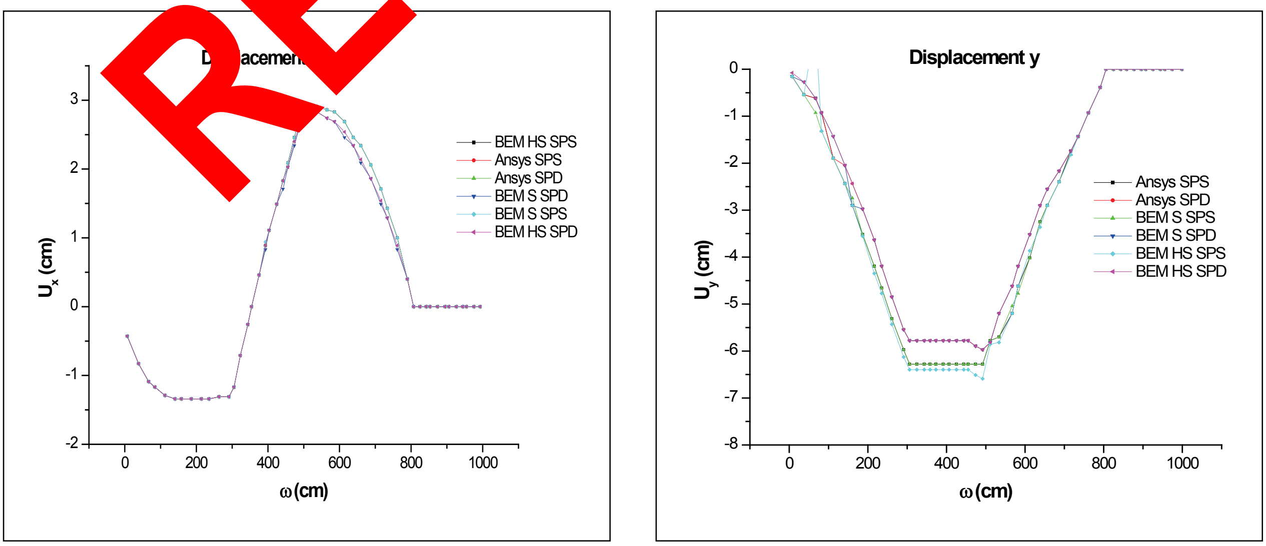

Fig. 11 Contour displacements response: BEM versus Ansys

The solutions of displacement in the contour and stresses along the height y in the internal points are presented in Figures 11, 12 and 13. The responses obtained with Ansys and with the BEM formulations were then compared for the validation of the results.

The graphs show the accuracy of the BEM and the multi-region technique to treat compound elastic problems. Even with a slightly refined contour mesh, the discontinuity of tensions along the height was precisely captured, as shown in the above results. In addition, both BEM S and BEM HS formulations were able to reproduce the fields obtained via Ansys for the contour displacements and the stresses at the internal points in both the SPS and SPD plane states. In Figure 14 the undeformed configuration, deformed SPS and deformed SPD composite beam displacements are presented considering enlarged 10 times.

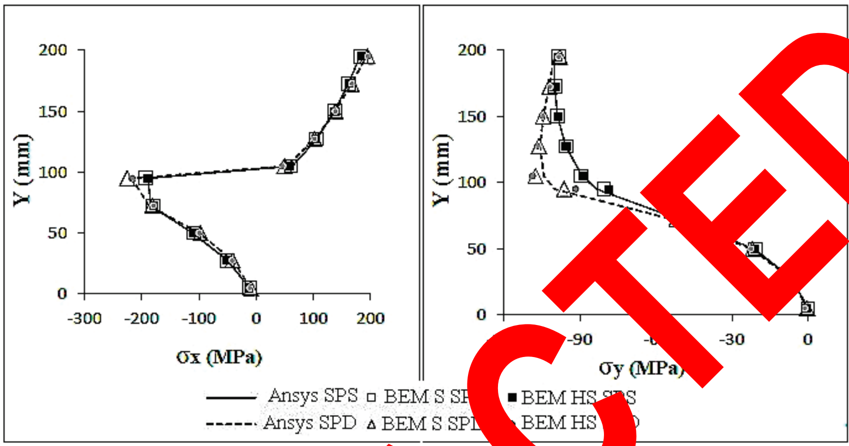

Fig. 12 Response of independent normal stresses: BEM versus Ansys

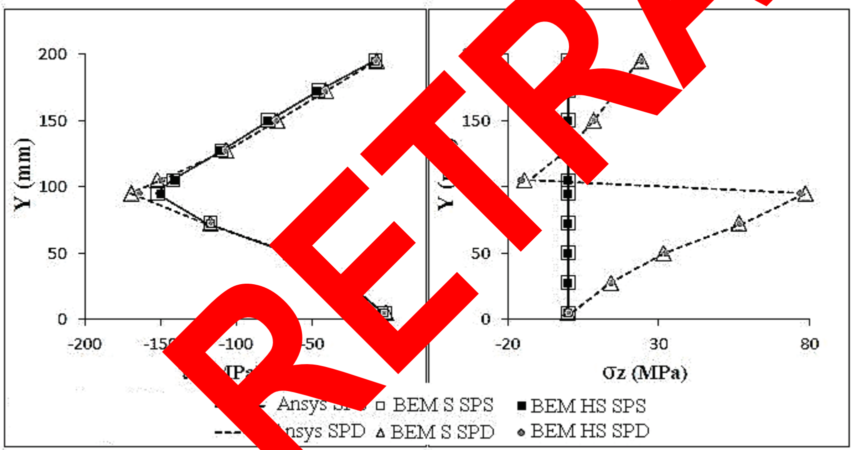

Fig. 13 Shear stress and normal stress σ_z response: BEM versus Ansys

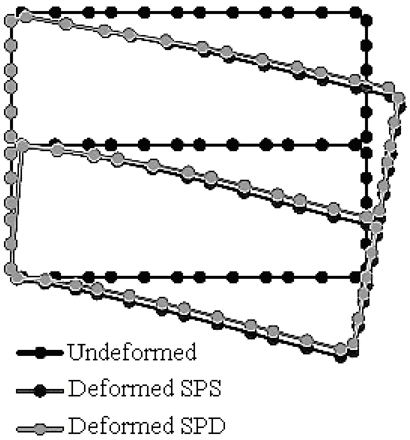

Fig. 14 Undeformed configuration and 10 times deformed mesh: SPD and SPD

3.3 Anisotropic plate with rigid inclusions

In the third application the accuracy of the BEM is evaluated in order to obtain voltage fields with high gradients resulting from sudden changes in material rigidity. For this, a plate containing nine rigid isotropic inclusions was evaluated. The plate consists of the same anisotropic material as the second application of this paper. The included inclusions have circular geometries of radius equal to 2 cm and are located in the axis of the structure spaced of 8 in 8 cm. The material constants adopted for the inclusions were $E = 300$ *GPa and* $v = 0.2$. Therefore, such inclusions are almost three times more rigid than the greater rigidity of the plate in the x direction, $E_x = 124.04$ *GPa*. The composite structure is embedded in one end and drawn in the other as shown in Figure 15 which also brings more geometric data of the problem.

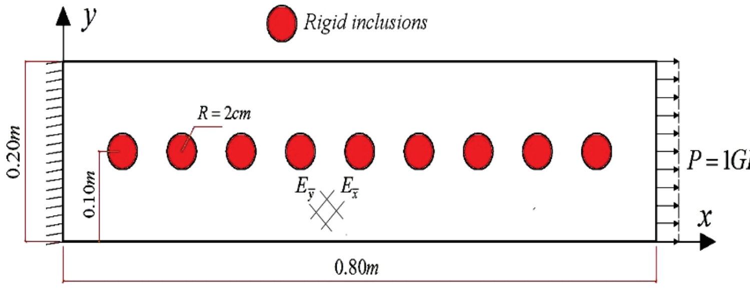

Fig. 15 Anisotropic plate with rigid inclusions.

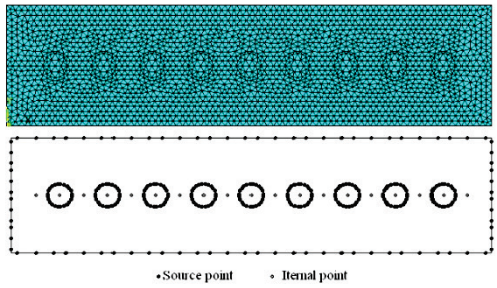

Fig. 16 Comparison between the contour and domain discretizations

In the discretization of the contour of the plate 28 elements of quadratic approximation was adopted. In the case of the inclusions, each one was discretized with 8 curved contour elements also of quadratic approximation. For insertion of the inclusions it was also necessary to discretize the contour of holes in the plate. Thus, the contour displacements of these holes can be made compatible with those of the contour of the inclusions by means of the multi-region technique. Each hole was also discretized with 8 quadratic contour elements equal to the contour of the corresponding inclusion, but with a counter-orientation, ie, clockwise. With the opposite orientation, the normal version to the contour of the holes points to the center of the holes, indicating that in that region there would be no material if there was no inclusion.

It is worth mentioning that the BEM formulation naturally captures holes by adopting contrary orientation for the elements since, with this, the contributions of the holes are of signs contrary to those of inclusions. Therefore, the rigidity of the hole domains is subtracted from the H and G matrices of the algebraic system.

Considering the elements adopted in the contour of the plate, in the nine holes and in the nine corresponding inclusions, the mesh has 172 elements and 516 nodes. Already in the finite element domain discretization, via Ansys, a mesh with 8673 nodes and 4236 triangular approximation "PLANE183" elements was adopted. Such refinement was necessary since, using a mesh with approximately half the number of nodes, the result of the stresses was not as close to the result of the BEM as expected. Therefore, the advantage in terms of the computational cost of BEM in relation to MEF for elastic problems containing inclusions, complex geometries or even cracks is evident. Figure 16 shows the domain and boundary meshes as well as 19 points on the axis of the problem where the stress fields will be evaluated.

The analyzes were carried out again considering the two SPS and SPD plane states and the two formulations BEM S and BEM HS. The solutions of stresses along the 19 internal points of the workpiece axis, obtained with the developed formulations are presented in Figures 17 and 18 where they are compared with the numerical Ansys response.

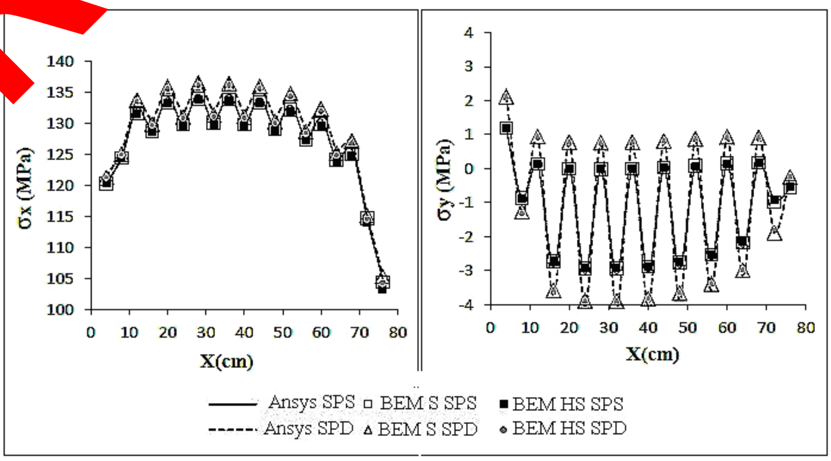

Fig. 17 Independent normal stress response: BEM versus Ansys

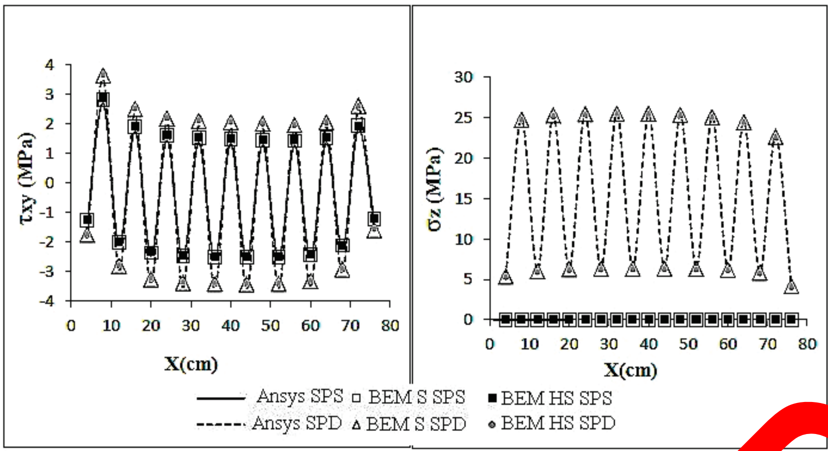

Fig. 18 Shear stress and normal stress σ_z response: BEM versus Ansys

Observing the results, it is verified that even with the considerable voltage oscillations along the axis of the plate, again the BEM response was as precise as that of the Ansys, but with much less degrees of freedom to be analyzed. Thus, in problems where stress field precision is required for specific regions of a structure, such as in regions close to crack, BEM can be a very valuable alternative. Although the propagation of crack in the present work is carried out along pre-established interfaces, it is worth noting that the accuracy of the BEM to capture the stress response is of extreme value in analyzes where the cracking path is not known. This is because, with the accuracy of the voltage field, it is possible to determine good approximations for the Voltage Intensity Factors, by means of which many propagation criteria are formulated.

4 Conclusion

The fundamental anisotropic solution, first presented by Cruse & Swedlow, was developed using the formalism of Lekhnitsky (1963) and the theory of complex functions. From the fundamental anisotropic solution of Cruse & Swedlow, it is possible to obtain the Somiglian identity for anisotropic elastostatic problems. For this, we start from the equilibrium equation of stresses now weighted by the new fundamental solution and through the Weighted Residues Method and relations of the theory of elasticity; identity is obtained by suitable mathematical manipulations.

Both the singular and hypersingular formulations were proposed for the solution of two-dimensional elasticity problems involving anisotropic materials. The formulations treated in the present work were able to reproduce good results for some interesting mechanical applications suggested in the paper. This method is conceptually simple because only the fundamental solution for a potential problem is needed. Three example problems have been presented to illustrate the veracity of the numerical implementation. They include those with stress concentrations for which the BEM is well known to be very well suited to treat. Where possible, the numerical results obtained from the BEM analysis have been compared with known solutions in the literature or with those obtained by the finite element method, and very good agreement between them have been obtained. The performance of the proposed implementations turned out to be highly accurate.

Future developments will, on the one hand, aim at the three-dimensional case that increases the possibility of representations of the most varied structures, the possibility to introduce the BEM that simulate particles and voids, culminating in an increasingly real simulation of the heterogeneous materials, and the implementation of a temporal integrator allowing analysis of problems in which such effects are significant to the results. With an eventual use of a temporal integrator, it becomes interesting to parallelize the obtained code, since the computational cost spent in these analyzes with consideration of the dynamic behavior would be much larger, when compared to the static cases. On the other hand, the implementation of a damage model would be a matter of extreme importance, one of the main functions of fiber use is the control of cracking, in addition to this phenomenon being able to occur simultaneously with plasticity.

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