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PREVENTING NEGATIVE VALUES WHEN FORECASTING NON-NEGATIVE TIME SERIES VARIABLES

Ali TFAILY
The Bucharest University of Economic Studies, Bucharest, Romania
tfaily.ali@gmail.com

Abstract
Long run time series variables forecasting is of special importance to academics and professionals alike. In this paper, the disadvantage of the "Natural Logarithm" transformation that prevents generating negative values in the forecast horizon is discussed. An alternative technique that does not suffer from the disadvantage of the "Natural Logarithm" transformation is presented. Both methods have been applied for forecasting the average USD deposits rate offered in the Lebanese retail banking industry.
Keywords: Forecasting, Time series, ARIMA, GARCH, Monte Carlo simulation

1. INTRODUCTION

In this paper, the modeling of the time series variable is based on the ARIMA-GARCH models and the forecast technique used is Monte Carlo Simulation (or MCS). ARIMA-GARCH models proved to be very powerful in capturing the different aspects of a time series variable (i.e. produce good forecasts). This type of forecasting methods, ARIMA-GARCH-MCS, does not prevent negative forecasts when the time series variable does not admit negative values. Given that in economics and finance lots of time series variables cannot admit negative values (e.g. volumes, price, interest rates…), there is a need for a technique that prevents the estimation of negative forecasts. An easy solution is to take the "Natural Logarithm" of the time series to be modeled and apply the ARIMA-GARCH-MCS method on it. This transformation is a simple way for preventing the estimation of negative values over all forecast horizons. But, the problem is that the "Natural Logarithm" transformation technique suffers from a major drawback, a more detailed description of this technique with its alternative, the "No Transformation" technique, will be presented in the methodology section. In the case study section, the average USD deposit rate offered in the Lebanese retail banking industry is forecasted over a period spanning from one month till thirty years. The forecast is done based on the two techniques, the "Natural Logarithm" transformation technique and the “No Transformation” technique. As a base for comparison, the raw forecast estimated using the ARIMA-GARCH-MCS method that allows for negative deposit rates in the forecast horizon is also presented. At the end of this paper, some recommendations and concluding remarks are presented.

2. LITERATURE REVIEW

ARMA models or Autoregressive Moving Average models popularized by Box and Jenkins in the 1970 book “Time Series Analysis: Forecasting and Control” (Box and Jenkins, 1970). ARMA models proved to be very powerful in modeling the expectation of covariance stationary time series variables. A generalization of the ARMA models, the ARIMA or Autoregressive Integrated Moving Average models, proved to be a great tool for modeling the expectation of non-stationary time series variables.

ARCH or Autoregressive Conditional Heteroskedasticity models first proposed by (Engle, 1982), with its extension the GARCH or Generalized Autoregressive Conditional Heteroskedasticity models first proposed by (Bollerslev, 1986) proved to be very powerful in capturing all the variation in the variance of a covariance stationary time series variable. (Christofferson, 2012) states that GARCH models are perfect for modelling the variance of time series variables and that this type of models is flexible enough to account for different aspects of the variance. As an example, to account for the leverage effect (negative shocks have a greater effect on variance than positive shocks), one can use GJR-GARCH (Glosten et al., 1993), NGARCH (Engle and Ng, 1993) or EGARCH (Nelson, 1991).

ARIMA-GARCH models combine well proven models for estimating the expectation (ARIMA models) and the variance (GARCH models) of a time series variable. To insure accurate parameter estimation, the estimation of the ARIMA-GARCH models should be done in one shot (i.e. simultaneously estimate the parameters of the ARIMA and GARCH models). It is not advised to estimate the parameters of the ARIMA model and then estimate the parameters of the GARCH model using the error terms from the ARIMA model. The simultaneous estimation of the parameters can be done using Quasi Maximum Likelihood, this is done automatically by most statistical software.

After estimating an ARIMA-GARCH model that accurately represents all the movements in a certain time series variable, MCS technique can then be used in order to estimate the forecasts based on this model. A simulation technique (like MCS or other) is a must given the complexity of the ARIMA-GARCH models (a closed form solution for the forecasts is sometimes difficult to formulate even if one assumes that the error terms of the ARIMA-GARCH model are Normally distributed; it is worth noting that one

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can use a t-distribution, a skewed t-distribution, an asymmetric t-distribution or other to model the error terms for MCS).

Although very powerful, the ARIMA-GARCH-MCS method does not prevent negative forecasts when the time series variable does not admit negative values (e.g. volumes, price, interest rates…). This concern is of special importance when using this method to estimate forecasts over the long run (e.g. 2, 5, 10 or 30 years). As stated previously, one solution is to take the "Natural Logarithm" of the time series to be modeled and apply the ARIMA-GARCH-MCS method on it. But, the "Natural Logarithm" transformation technique suffers from a major drawback. In what follows, in the methodology section, a more detailed description of the "Natural Logarithm" transformation technique with its alternative, the "No Transformation" technique, will be presented. In the case study section, the average USD deposit rate offered in the Lebanese retail banking industry is forecasted over a period spanning from one month till thirty years using the "Natural Logarithm" transformation technique, the "No Transformation" technique and as a reference, the raw ARIMA-GARCH-MCS method that allows for negative deposit rates in the forecast horizon. At the end, some recommendations and concluding remarks are presented.

3. METHODOLOGY

In this section, a detailed description of the "Natural Logarithm" transformation technique with its alternative, the "No Transformation" technique, will be presented.

Let X_t be the time series to be model. The "Natural Logarithm" transformation technique consist of the following steps:

- Instead of modeling X_t directly, $ln(X_t)$, the "Natural Logarithm" of X_t is estimated and then modeled using an ARIMA-GARCH model.
- Using MCS, the different simulated values of $ln(X_{T+Z})$ are generated (X_T is the last observation of the time series variable X_t).
- The simulated values of X_{T+Z} are then calculated as $X_{T+Z} = e^{ln(X_{T+Z})}$ which are ensured to be positive since the Exponential function e^X is positive for all X.

- The forecasted value $E(X_{T+z})$ is then estimated as the average of the different X_{T+z} that have been generated.
- The lower and upper bounds of the $(100\% - \alpha)$ confidence interval for X_{T+z} are estimated as the $\frac{\alpha}{2}$ percentile of the simulated X_{T+z} (for the lower bound) and as the $100\% - \frac{\alpha}{2}$ percentile of the simulated X_{T+z} (for the upper bound).

The “Natural Logarithm” transformation technique is simple to implement and it does prevent the forecasted values of a time series variable from going negative. But, the major drawback of this technique is that the patterns that are present in X_t are not preserved when the transformation $ln(X_t)$ is applied. Since, $\frac{dln(X_t)}{dX_t} = \frac{1}{X_t} \neq constant$, the amount of change $\Delta X_t = x$ is not being transformed into the same change in $\Delta \ln(X_t)$ for different values of X_t. As an example, some values are presented in the table below:

TABLE 1. TABLE SHOWING DIFFERENT VALUES FOR X_t WITH THE CORRESPONDING VALUES FOR ΔX_t AND $\Delta \ln(X_t)$

Case	X_1	X_2	ΔX_t	$\Delta \ln(X_t)$
1	0.01	0.015	0.005	0.405
2	0.03	0.035	0.005	0.154

The above problem is especially important when the values of X_t are close to zero. Also, another important problem is that the “Natural Logarithm” transformation treats negative and positive changes in X_t asymmetrically, this is again due to the fact that $\frac{dln(X_t)}{dX_t} = \frac{1}{X_t} \neq constant$.

To solve the problem of not equally treating the changes in X_t introduced by the “Natural Logarithm” transformation technique, a “No Transformation” technique must be formulated. Below are the different steps of the “No Transformation” technique:

- X_t is modeled directly using an ARIMA-GARCH model.

- Using MCS, the different simulated values of X_{T+Z} are generated while imposing the following restriction on choosing the value of the error term of MCS: only a shock ε_{T+Z} that will preserve $0 \leq X_{T+Z}$ will be accepted (i.e. if a certain shock ε_{T+Z} that has been drawn does not produce a value for X_{T+Z} such that $0 \leq X_{T+Z}$, another value for ε_{T+Z} will be drawn, this process is repeated till the drawn value for ε_{T+Z} produces a value for X_{T+Z} such that $0 \leq X_{T+Z}$.
- The forecasted value $E(X_{T+Z})$ is then estimated as the average of the different X_{T+Z} that have been generated.
- The lower and upper bounds of the $(100\% - \alpha)$ confidence interval for X_{T+Z} are estimated as the $\frac{\alpha}{2}$ percentile of the simulated X_{T+Z} (for the lower bound) and as the $100\% - \frac{\alpha}{2}$ percentile of the simulated X_{T+Z} (for the upper bound).

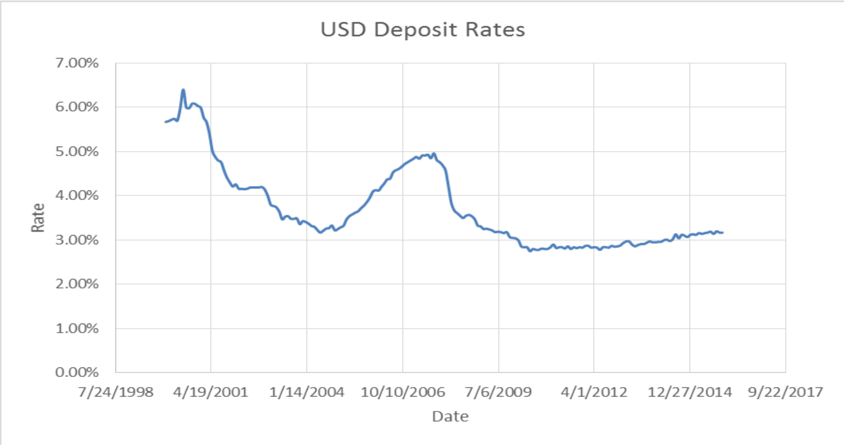
The "No Transformation" technique can produce non-negative forecasts for a time series variable X_t without the major drawback of the "Natural Logarithm" transformation technique (i.e. without messing the patterns of the time series variable X_t). The only disadvantage of the "No Transformation" technique is that it is more computationally intensive than the "Natural Logarithm" transformation technique but this is a minor issue given today's computational power.

In the next section, the average USD deposit rate offered in the Lebanese retail banking industry is forecasted over a period spanning from one month till thirty years using the "Natural Logarithm" transformation technique, the "No Transformation" technique and as a reference, the raw ARIMA-GARCH-MCS method that allows for negative deposit rates in the forecast horizon.

4. CASE STUDY

The case study will focus on forecasting the average USD deposit rate offered in the Lebanese retail banking industry (or R_t). The data for the average monthly deposit rates are downloaded from "Banque du Liban" website (the Lebanese central bank website). The data that is used span from January 2000 till December 2015, below is a graph showing the downloaded data:

FIGURE 1. THE AVERAGE MONTHLY USD DEPOSIT RATES OFFERED IN THE LEBANESE RETAIL BANKING INDUSTRY FROM JANUARY 2000 UNTIL DECEMBER 2015



First, the raw ARIMA-GARCH-MCS method that allows for negative deposit rates in the forecast horizon is estimated followed by the "Natural Logarithm" transformation technique and then by the "No Transformation" technique. For all of the three techniques, the error terms that will be used for MCS are assumed to follow a standard normal distribution.

Raw technique:

First, one must run an Augmented Dickey-Fuller test (Dickey and Fuller, 1979) on R_t, in order to check if R_t is stationary. Below is figure showing the Augmented Dickey-Fuller test performed using the software "Gretl":

FIGURE 2. THE AUGMENTED DICKEY-FULLER TEST PERFORMED ON R_t

```
Augmented Dickey-Fuller test for Rate
including 3 lags of (1-L)Rate
(max was 14, criterion AIC)
sample size 188
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.0170077
  test statistic: tau_c(1) = -2.45059
  asymptotic p-value 0.1279
  1st-order autocorrelation coeff. for e: -0.031
  lagged differences: F(3, 183) = 13.697 [0.0000]

  with constant and trend
  model: (1-L)y = b0 + b1*t + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.0240224
  test statistic: tau_ct(1) = -2.36797
  asymptotic p-value 0.3965
  1st-order autocorrelation coeff. for e: -0.035
  lagged differences: F(3, 182) = 13.906 [0.0000]
```

From the above figure, the p-values of both tests, the one with trend and the one without trend, suggest the presence of a unit root. Below is figure showing the Augmented Dickey-Fuller test performed on ΔR_t using the software "Gretl".

FIGURE 3. THE AUGMENTED DICKEY-FULLER TEST PERFORMED ON ΔR_t

```
Augmented Dickey-Fuller test for dRate
including 2 lags of (1-L)dRate
(max was 14, criterion AIC)
sample size 188
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.437361
  test statistic: tau_c(1) = -4.79184
  asymptotic p-value 5.338e-005
  1st-order autocorrelation coeff. for e: -0.026
  lagged differences: F(2, 184) = 7.516 [0.0007]

  with constant and trend
  model: (1-L)y = b0 + b1*t + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.457937
  test statistic: tau_ct(1) = -4.92259
  asymptotic p-value 0.0002548
  1st-order autocorrelation coeff. for e: -0.024
  lagged differences: F(2, 183) = 6.942 [0.0012]
```

From the above figure, the p-values of both tests, one with trend and one without trend, suggest the absence of a unit root (i.e. R_t is integrated of order one). Given that R_t is integrated of order one, an ARIMA(p,1,q)-GARCH model will be used or an ARMA(p,q)-GARCH model applied on ΔR_t will be used. After estimating several ARMA(p,q)-GARCH models using "Gretl", the below model has been selected given the value of the Akaike Information Criterion (Akaike, 1973):

FIGURE 4. THE CHOSEN ARMA-GARCH MODEL FOR ΔR_t ESTIMATED USING "GRETL"

```
Function evaluations: 29
Evaluations of gradient: 9

Model 5: GARCH, using observations 2000:02-2015:12 (T = 191)
Dependent variable: dRate
QML standard errors

             coefficient   std. error      z      p-value
  -------------------------------------------------------
  alpha(0)   2.40776e-07   6.76683e-08    3.558   0.0004   ***
  alpha(1)   0.936891      0.339780       2.757   0.0058   ***

Mean dependent var   -0.000131   S.D. dependent var    0.000930
Log-likelihood        1109.089   Akaike criterion     -2212.179
Schwarz criterion    -2202.422   Hannan-Quinn         -2208.227

Unconditional error variance = 3.81524e-006
```

From the above figure, the estimated ARMA-GARCH model for ΔR_t is:

$$\begin{cases} \Delta R_t = C + \theta \times \Delta R_{t-1} + \sigma_t \times Z_t \\ \sigma_t^2 = \omega + \alpha \times [\Delta R_{t-1} - \theta \times \Delta R_{t-2}]^2 + \beta \times \sigma_{t-1}^2 \end{cases}$$

where,

$\hat{C} = 0$

$\hat{\theta} = 0$

$\hat{\omega} = 0.000000240776$

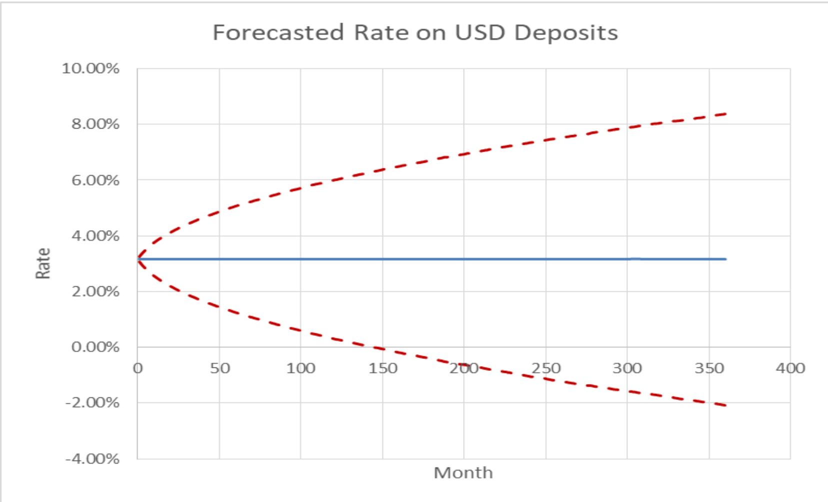
$\hat{\alpha} = 0.936891$

$\hat{\beta} = 0$

For the GARCH model on ΔR_t to converge to a long run variance, one must have $\{\hat{\alpha} + \hat{\beta} < 1\}$ (Christoffersen, 2012), this is true for the estimated model. This condition on the GARCH coefficients insures that the model is mean reverting and can be used for simulation (this is especially important for long run forecasting).

After modeling R_t as an ARIMA(0,1,0)-GARCH(1,0) model, one can use MCS to estimate the forecasts for R_t. The simulation was done using the software "Matlab". Below is a graph showing the forecasted R_t values from one month till thirty years (the error terms of the ARIMA(0,1,0)-GARCH(1,0) model are assumed to follow a standard normal distribution):

FIGURE 5. THE FORECASTED AVERAGE MONTHLY USD DEPOSIT RATE WITH ITS 95% CONFIDENCE INTERVAL



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"Natural Logarithm" transformation technique:

First, one must run an Augmented Dickey-Fuller test on $ln(R_t)$, in order to check if $ln(R_t)$ is stationary. Below is figure showing the Augmented Dickey-Fuller test performed using the software "Gretl":

FIGURE 6. THE AUGMENTED DICKEY-FULLER TEST PERFORMED ON $\boldsymbol{ln(R_t)}$

```
Augmented Dickey-Fuller test for LnRate
including 3 lags of (1-L)LnRate
(max was 14, criterion AIC)
sample size 188
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.0143342
  test statistic: tau_c(1) = -2.23812
  asymptotic p-value 0.1928
  1st-order autocorrelation coeff. for e: -0.019
  lagged differences: F(3, 183) = 13.420 [0.0000]

  with constant and trend
  model: (1-L)y = b0 + b1*t + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.0210989
  test statistic: tau_ct(1) = -2.16406
  asymptotic p-value 0.5092
  1st-order autocorrelation coeff. for e: -0.022
  lagged differences: F(3, 182) = 13.564 [0.0000]
```

From the above figure, the p-values of both tests, the one with trend and the one without trend, suggest the presence of a unit root. Below is figure showing the Augmented Dickey-Fuller test performed on $\Delta ln(R_t)$ using the software "Gretl":

FIGURE 7. THE AUGMENTED DICKEY-FULLER TEST PERFORMED ON $\boldsymbol{\Delta ln(R_t)}$

```
Augmented Dickey-Fuller test for dLnRate
including 2 lags of (1-L)dLnRate
(max was 14, criterion AIC)
sample size 188
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.429439
  test statistic: tau_c(1) = -4.72379
  asymptotic p-value 7.244e-005
  1st-order autocorrelation coeff. for e: -0.016
  lagged differences: F(2, 184) = 7.781 [0.0006]

  with constant and trend
  model: (1-L)y = b0 + b1*t + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.44851
  test statistic: tau_ct(1) = -4.84335
  asymptotic p-value 0.0003591
  1st-order autocorrelation coeff. for e: -0.014
  lagged differences: F(2, 183) = 7.177 [0.0010]
```

From the above figure, the p-values of both tests, the one with trend and the one without trend, suggest the absence of a unit root (i.e. $ln(R_t)$ is integrated of order one). Given that $ln(R_t)$ is integrated of order one, an ARIMA(p,1,q)-GARCH model will be used or an ARMA(p,q)-GARCH model applied on $\Delta ln(R_t)$ will be used. After estimating several ARMA(p,q)-GARCH models using "Gretl", the below model has been selected given the value of the Akaike Information Criterion:

FIGURE 8. THE CHOSEN ARMA-GARCH MODEL FOR $\Delta ln(R_t)$ ESTIMATED USING "GRETL"

```
Function evaluations: 55
Evaluations of gradient: 21

Model 1: GARCH, using observations 00-03-31:15-12-31 (T = 190)
Dependent variable: dLnRate
QML standard errors

             coefficient    std. error      z      p-value
  ---------------------------------------------------------
  dLnRate_1   0.262399      0.113307      2.316    0.0206   **

  alpha(0)    0.000228975   4.21392e-05   5.434    5.52e-08 ***
  alpha(1)    0.452582      0.171900      2.633    0.0085   ***

Mean dependent var   -0.003079   S.D. dependent var    0.021340
Log-likelihood        488.1425   Akaike criterion    -968.2850
Schwarz criterion    -955.2969   Hannan-Quinn        -963.0237

Unconditional error variance = 0.000418282
Likelihood ratio test for (G)ARCH terms:
  Chi-square(1) = 31.8995 [1.62362e-008]
```

From the above figure, the estimated ARMA-GARCH model for $\Delta ln(R_t)$ is:

$$\begin{cases} \Delta\ln(R_t) = C + \theta \times \Delta\ln(R_{t-1}) + \sigma_t \times Z_t \\ \sigma_t^2 = \omega + \alpha \times [\Delta\ln(R_{t-1}) - \theta \times \Delta\ln(R_{t-2})]^2 + \beta \times \sigma_{t-1}^2 \end{cases}$$

where,

$\hat{C} = 0$

$\hat{\theta} = 0.262399$

$\hat{\omega} = 0.000228975$

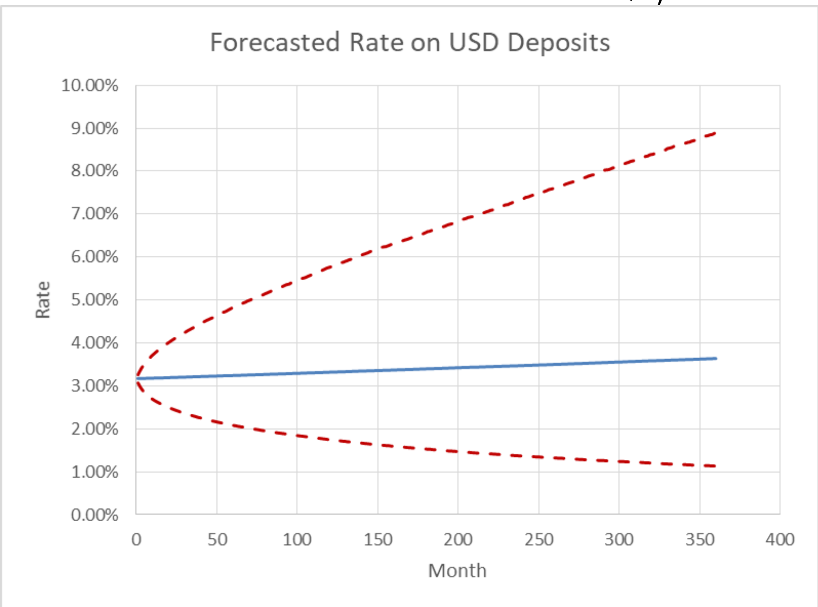
$\hat{\alpha} = 0.452582$

$\hat{\beta} = 0$

For the GARCH model on $\Delta ln(R_t)$ to converge to a long run variance, one must have $\{\hat{\alpha} + \hat{\beta} < 1\}$, this is true for the estimated model. For the AR(1) model on $\Delta ln(R_t)$ to be stationary, $\hat{\theta}$ the estimated coefficient of the AR(1) model must be between $\{-1 < \hat{\theta} < 1\}$ (Asteriou and G.Hall, 2007) this is true for the estimated model. These two condition on the AR and GARCH coefficients will insure that the model is mean reverting and can be used for simulation (this is especially important for long run forecasting).

After modeling $ln(R_t)$ as an ARIMA(1,1,0)-GARCH(1,0) model, one can use MCS to estimate the forecasts for R_t. The simulation was done using the software "Matlab". Below is a graph showing the forecasted R_t values from one month till thirty years (the error terms of the ARIMA(1,1,0)-GARCH(1,0) model are assumed to follow a standard normal distribution):

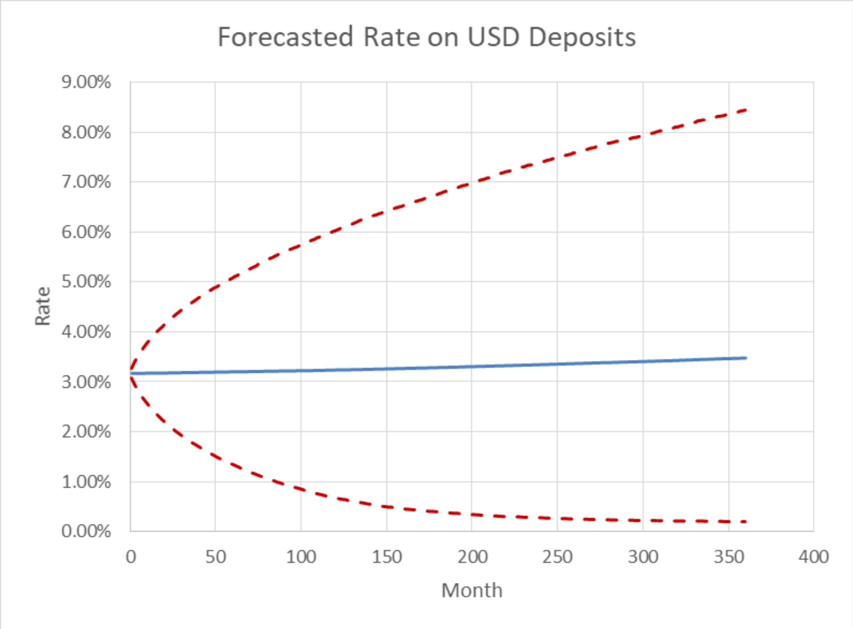
FIGURE 9. THE FORECASTED AVERAGE MONTHLY USD DEPOSIT RATE WITH ITS 95% CONFIDENCE INTERVAL (USING THE "NATURAL LOGARITHM" TRANSFORMATION TECHNIQUE)



"No Transformation" technique:

Using the same model that was estimated previously for R_t (i.e. modeling R_t as an ARIMA(0,1,0)-GARCH(1,0) model), one can use MCS to estimate the forecasts for R_t while imposing the restriction of the "No Transformation" technique on drawing the error terms for MCS. The simulation was done using the software "Matlab". Below is a graph showing the forecasted R_t values from one month till thirty years (the error terms of the ARIMA(0,1,0)-GARCH(1,0) model are assumed to follow a standard normal distribution):

FIGURE 10. THE FORECASTED AVERAGE MONTHLY USD DEPOSIT RATE WITH ITS 95% CONFIDENCE INTERVAL (USING THE "NO TRANSFORMATION" TECHNIQUE)



5. RESULTS AND DISCUSSION

In the previous section, the average one month USD deposit rate offered in the Lebanese retail banking industry was forecasted over a period spanning from one month till thirty years using the "Natural Logarithm" transformation technique, the "No Transformation" technique and as a reference, the raw ARIMA-GARCH-MCS method.

From Figure 5, one can clearly see that the 2.5% percentile level (or the lower bound of the 95% confidence interval) went negative after the 150 month forecast which is economically incorrect. This negative lower bound of the 95% confidence interval is not acceptable, the raw ARIMA-GARCH-MCS method is considered bad and should not be used to forecast non-negative time series variables (especially when the final value of the time series variable data set in close to zero).

When comparing Figure 9 and Figure 10, the difference between the forecasts (and confidence intervals) is not that pronounced (economic wise). Both, the "Natural Logarithm" transformation technique and the "No Transformation" technique were able to produce economically meaningful results. But, since the "Natural Logarithm" transformation technique does not preserve the patterns of the time series variable, the "No Transformation" technique forecasts must be considered superior.

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6. CONCLUSION

In this paper, the disadvantage of the "Natural Logarithm" transformation technique that prevents generating negative values in the forecast horizon was discussed. An alternative technique, the "No Transformation" technique, that does not suffer from the disadvantage of the "Natural Logarithm" transformation technique was presented. Both methods were applied to forecast the average monthly USD deposits rate offered in the Lebanese retail banking industry. Given that the "Natural Logarithm" transformation technique does not preserve the patterns of the time series variable, the "No Transformation" technique forecasts must be considered superior.

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