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MODIFIED LEAST COST METHOD FOR SOLVING TRANSPORTATION PROBLEM

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Transportation Problem; Least Cost Cell method; Modified Distribution method; Balanced and unbalanced transportation problem; Unique cell selection.

A B S T R A C T

Optimal route selection for delivering product is the key concern for companies related to supply chain management. Route selection plays an important part, as it greatly affects the financial section of such companies. This paper presents Two-step exact algorithm for transportation problem. It uses the basic ideas of Least Cost Cell and Modified Distribution method. The algorithm is equally effective for balanced and unbalanced transportation problems. The effectiveness of the algorithm is discussed by considering different problem types with experimental setup followed by result analysis.

1. INTRODUCTION

1.1 Literature Review

Transportation problem (Taha (2004), Winston et al. (2004), Greenberg (2012), Woodruff (2013)) has special data structure in solution characterized as a transportation graph and constitutes large area of integer linear programming. It was formalized by Monge (1781). Major advances were made during World War II by the Soviet/Russian mathematician Kantorovich, hence transportation problem is also known as the Monge-Kantorovich (Kantorovich (1960)) transportation problem. Transportation problem is explored extensively in the mathematical programming and engineering literatures sometimes referred to as the facility location and allocation problem. The first important contribution was made by Hitchcock (1941), which he presented in his study entitled “The Distribution of a Product from Several sources to numerous Localities”. The credit to development of transportation methods involving shipping sources and destinations goes to Hitchcock and Koopmans (Koopmans (1947)). Transportation problem received this name because many of its applications involve determining optimal transport route. George B. Dantzig (Dantzig (1951)) is credited for optimal solutions, as he applied the concept of Linear Programming in solving the Transportation models. Later he used simplex method on transportation problem as the primal simplex transportation method (Dantzig (1963)). In 1958 Reinfeld and Vogel developed Vogel’s Approximation Method which is one of the most popular heuristic algorithm based on penalty calculation (Reinfeld et al. (1958)). Later in 1963 another heuristic

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algorithm based on Greedy approach was developed by Dantzig known as Least Cost Method (Dantzig (1963b)). Williams (Abadie et al. (1963)) et al. used decomposition principle of Dantzing and Wolfe (Dantzig et al. (1960)) to Hitchcock's transportation problem solution. Efroymson and Ray (Efroymson et al. (1966)) discussed branch and bound algorithm to solve plant location problem- a special class of discrete programming problem. J. Frank Sharp. et.al (Sharp et al. (1970)) developed an optimal solution for production transportation problem. Roy and Gelders (Roy et al. (1981)) solved liquid bottled product transportation problem as a 0-1 integer programming model. However, optimal solution was obtained using branch and bound approach. Marcotte and Soland (Marcotte et al. (1986)) presented a simplified branch and bound algorithm. Lobel (Lobel (1997)) proposed optimal vehicle routing scheduling for public transportation. Equi et al. (Equi et al. (1997)) presented a combined model for transportation and scheduling problems. Ting et al. (Ting et al. (2003)) used dynamic programming model for route planning in shipping. Mixed constraints transportation problem was discussed by Veena and Kowalski (Veena et al. (2006)). A. C. Caputo. et.al. (Caputo et al. (2006)) presented optimal solution to road transport activities.

This paper propose an exact algorithm which is based on least cost selection strategy of Least Cost Method with required modifications to obtain better basic solution. The obtained solution is checked using loop formation strategy which is similar to Modified Distribution method but varies in searching for cells. The paper is divided into five sections; Section1 is concerned with literature review and formulation of transportation problem. Section 2 discusses problem types in transportation problem while Section 3 discusses solution types for the same. Section 4 focuses on some important terminologies related to initial solution. Section 5 discusses related work and proposed algorithm. Section 6 discusses algorithms for related and proposed work followed by examples in Section 7 and result and discussion of work in Section 8.

1.2 Overview of Transportation Problem

Transportation Problem is a special kind of the Network optimization problem (Phillips et al. (1981), Magnanti et al. (1984), Murty (1985), Ahuja et al. (1995)) aiming to find an optimal route. Network optimization problem belongs to special class of combinatorial optimization problem (Nemhauser et al. (1988),Wolsey et al. (1993), Papadimitriou et al. (1998), Sait et al. (1999), Hentenryck et al. (2009), Du et al. (2013)) which is a special class of Integer linear programming. TP can be treated as weighted graph with weights representing per unit shipping cost (Figure 1.1). Shipping charge for each route may not be the least one, but overall shipping cost is desired to be minimum called as the optimal path. As discussed earlier, it can be represented using transportation graph which is a bipartite graph between Demand and Supply, but for simplicity it is better to represent using Tabular structure as shown in Table 1.1, with cells representing per unit cost that will incur while shipping goods using that route.

1.2.1 Formulation of Transportation Problem

Let the transportation problem consist of m origins and n destinations, where

x_{ij} = the amount of goods transported from the i_{th} origin to the j_{th} destination.

c_{ij} = the cost involved in transporting per unit product from the i_{th} origin to the j_{th} destination.

a_i = the number of units available at the i_{th} origin.

b_j = the number of units required at the j_{th} destination.

Consider the linear transportation problem as:

$$Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

subject to the constraints

$$a_i \geq \sum_{j=1}^{n} x_{ij} \text{ ; for all } i \text{ €} I = (1,2,\ldots..,m)$$

$$b_j \leq \sum_{i=1}^{m} x_{ij} \text{ ; for all } j \text{ € } J = (1, 2,\ldots.,n)$$

and $x_{ij} \geq 0$, for all (i, j) € I X J. for unbalanced TP, $a_{ij} < b_{ij}$ or $a_{ij} > b_{ij}$.

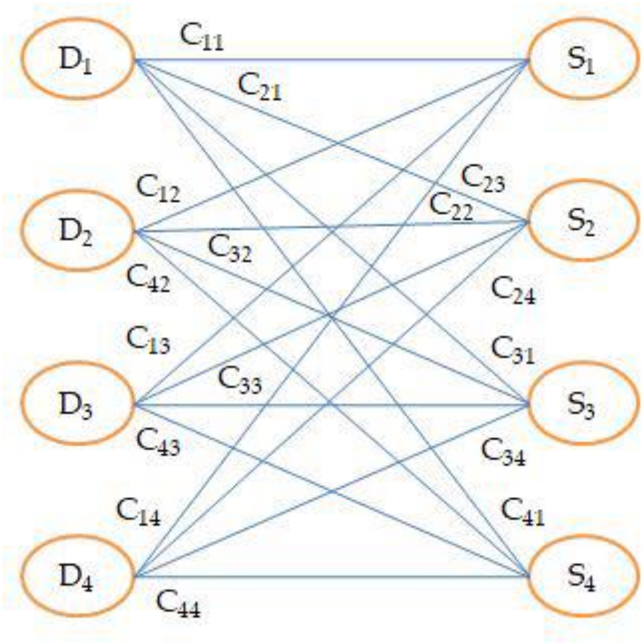

Figure 1.1 Weighted graph for Transportation problem with weight representing per unit shipping cost

Table 1.1. Transportation table with m Source and n Destination

Destination→ Source ↓	1	2	…	N	Supply
1	c_{11}	c_{22}	…	c_{in}	s_i
2	c_{21}	c_{22}	…	c_{2n}	s_2
⋮	…	…	…	…	⋮
m	c_{m1}	c_{m2}	…	c_{mn}	s_m
Demand	d_1	d_2	…	d_n	$\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j$

2. PROBLEM TYPES

Above minimization objective function is valid for total demand (Demand) and total supply (Supply) value being equal. However, Demand and Supply values may not be equal, which is a common scenario in real life problems, and requires balancing the problem. Thus, transportation problem is divided into two types: Balanced and Unbalanced.

2.1 Balanced Problem

Balanced problem have equal demand and supply values. This is the ideal situation where Supply is exactly equal to Demand occurred. Such situation can occur for small sized problems, but for large sized problems like delivering product in entire city, it is usually uncommon.

2.2 Unbalanced Problem

In many real life problems, like transportation of products within entire city, Demand and Supply values are not equal, hence the problem becomes unbalanced. To make it balance, dummy row/column with *0* cell costs is introduced if demand is greater/lesser than supply. Introduction of dummy row or column is required for above minimization objective equation to hold good. Introduction of dummy does not mean extra supply is generated or extra demand has incurred. Balakrishnan (1990) has discussed unbalanced transportation problem in his work "Modified Vogel's approximation method for the unbalanced transportation problem". Calculating basic feasible solution without balancing may produce different result for LCM, as it selects least cost cell for assignment and in this case it will always select dummy row/column, as it will contain zeros only.

3. SOLUTION TYPES

3.1 Degenerate Solution

The term degenerate solution was coined for Simplex method (Zoutendijk (1960), Klee et al. (1970)) when one of the basic variables become zero. and may cause cycling, i.e. initial table is again achieved. However, pivoting rules (Gal (1993)) such as Bland's rule (Pan (1990)) can avoid degeneracy. In case of transportation problem, degeneracy occurs when number of assignments made is less than $m + n - 1$, where m and n are number of rows and columns respectively. This demands need for one extra assignment, $\in$ *to* least cost unassigned cell with non-loop condition to proceed for optimality check. Optimality check is performed using Stepping stone (Chranes et al. (1954)) or Modified distribution method (Dantzig (1963)). This extra assignment helps in forming closed path (loop) in each and every vacant cell for Stepping stone method and in calculation of u and v values for Modified distribution method. Shafaat et al. (1988) showed that degeneracy does not means that initial solution obtained is non-optimal and proposed algorithm for making this extra assignment.

3.2 Non-Degenerate Solution

Non-degeneracy is the ideal condition when total assignments equals $m + n - 1$ with m rows and n columns. This condition is required for calculation of u and v values for Modified distribution method as assignments less than $m + n - 1$ will halt the process for calculation of u-v values needed.

4. SOME TERMINOLOGY

4.1 Feasible Solution

While obtaining initial solution for transportation problem, all basic variables must be non-negative. This is required as it restricts illegal assignments, i.e. assignment made to a specific cell must be exactly equal to minimum of demand and supply values for that particular cell.

4.2 Basic Solution

Initial solution should satisfy constraints described in Section1.2.1, i.e. total assignments should sum up to total of Demand or Supply values.

4.3 Basic Feasible Solution

Basic Feasible solution satisfies feasibility and basic solution conditions, i.e. non-negative restriction and constraint conditions must be satisfied. In other words, only legal assignments (feasible solution) should be made which sums up to Demand or Supply values.

5. RELATED WORK

5.1 Least Cost Method (LCM)

Least Cost Method is one of the popular heuristic algorithms developed by Dantzig (1963). It is based on greedy approach for making assignments. It is somewhat same as Kruskal's minimum spanning tree approach (Ahuja (2017)), where least weight edge is selected until spanning tree is formed. LCM searches for the least cost cell in the entire matrix and makes maximum possible assignment which is the minimum of demand and supply value for that particular selected cell. In case of non-unique least cost cell, selection is made randomly. It is one of the simplest heuristic approaches, but is time consuming, as searching least cost cell in the entire matrix each time is cumbersome. Algorithm for LCM is discussed in Section 6.1.

5.2 Modified Distribution Method (MODI)

Modified Distribution Method (MODI) or U-V method (Dantzig (1963)) is used to check optimality of basic feasible solution obtained using any transportation method like LCM. It only works on non-degenerate solutions; hence obtained solution is converted to non-degenerate solution. If solution is non-optimal, it shifts allocation to non-allocated cell in-order to optimize the initial solution. Shifting is made using loop formation strategy which requires certain parameters discussed below:

• The values of variables u_i and v_j are calculated using following formula:

$$c_{ij} = u_i + v_j,$$

where c_{ij} represents allocated cells;

The process starts with assigning either u_i or v_j value to 0 depending on whether maximum assignment is in i^{th} row or j^{th} column.

• Loop is created if one of the opportunity cost, d_{ij} value is negative. If so, then loop is created for cell with most negative d_{ij} value. Calculation of d_{ij} is made using following formula:

$$d_{ij} = c_{ij} - (u_i + v_j).$$

In case if any of d_{ij} value becomes 0, it shows that alternate path do exist.

• Minimum of four cells is needed for loop formation with even number of cells.
• Only horizontal and vertical movements are allowed using assigned cells.
• Assignments are modified only for the boundary cells forming loop. Assignment of even cells (counted from starting cell) is decreased and odd cell's assignments are increased by value equal to minimum assignment of even cells.

Soul of MODI is loop construction. Loop construction is somewhat Semi-Cycle formation (SCF) where some vertices participate in loop/cycle construction unlike cycle - formation in graph (Hamiltonian Cycle (Wilson (1972))), where each vertex participates in cycle formation. Prasad et al. (2018b) proposed Semi-Cycle loop formation approach in $O(n^2)$ time, where n represents number of assigned cell + 1. Loop may form more than once depending upon quality of initial solution. Algorithm for MODI is discussed in Section 6.2.

5.3. Proposed Algorithm

The theme of optimization in transportation problem is not selecting least cost cells for each route, but an optimal route selection, that might include least cost cell for some routes. Optimization should be achieved within desirable time, as time is another key factor besides obtaining optimal solution. Least Cost Method suffers the same problem as that by Prim's (Cormen et al. (2009)) and Kruskal's (Ahuja (2017)), if both are used for travelling salesman problem. Greedy approach unlike Dynamic approach makes selection without bothering effect of present selection on future selections that might result in selecting some higher cost cells, and thus making solution non-optimal. Second important point of interest is time, time required for obtaining basic feasible solution using LCM and then checking its optimality using MODI is time consuming. We tried to work on both aspects by modifying LCM's least cost selection step using KVP approach (Prasad et al. (2018a)), and calculation of d_{ij} values by checking only two least cost non-assigned cells for each row. Loop formation step is optimized by Semi-Cycle loop formation approach (Prasad et al. (2018b)). The proposed algorithm searches some unique cell based on LCM strategy with some more cell selection by searching one more cell in corresponding row, if there is more than one selection in corresponding column. This step gives fair chance to those cells that are not least in value, but might receive an assignment. This step can be related with penalty calculation step of Vogel's Approximation Method (Reinfeld et al. (1958)), that selects two least cost cells for penalty calculation of each row and column. If more assignments are required with no unique cells remaining, assignments are made row-wise on first come first serve basis using LCM approach. This step is greedy but is somewhat not, as this step is performed when most of the assignments are already made. The reason why assignments are not made to least cost cells of unique cells is to reduce complexity. Regarding loop formation step, proposed algorithm just checks two least cost un-assigned cells of each row reducing time in comparison to MODI's checking of each non-assigned cell. Algorithm and examples are discussed in Section 6.3 and 7.

6. ALGORITHM

6.1 Least Cost Method

***Step*1.** If the given transportation problem is balanced go to *step*3, else *step*2.

***Step*2.** Balance the given TP by introducing dummy row or column (whichever is required) with zero cell costs.

***Step*3.** Select the valid-least cost cell in the matrix and make an assignment which is the minimum of demand and supply value for that cell.

***Step*4.** Adjust the demand and supply value, cells having zero of demand or supply value will not participate in further allocation.

***Step*5.** Repeat *step* 3 *and* 4 until all the demand and supply values become zero.

***Step*6.** Compute the total transportation cost for the allocations made.

Step1 checks for balanced transportation problem which is also termed as rim condition. If balanced, algorithm move to *step3*, else move to *step2*. *Step3* selects least cost cell for assignment and brakes tie arbitrarily in case more there are more than one cell with same least value. *Step4* adjusts demand and supply value for corresponding assigned cell. *Step3* and *4* are repeated until all assignments are made. Finally *step6* calculates basic feasible solution's cost.

6.2 Modified Distribution Method

Step1. Check for non-degeneracy, if found degenerate, introduce infinitesimally small value ∈ to least cost non-assigned cell.

Step2. Calculate u_i and v_j ($i = 1, 2, 3...m$ and $j = 1, 2, 3...n$) such that for each occupied cell $c_{ij} = u_i + v_j$. Assign value *0* to any arbitrary u_i to start the process.

Step3. Calculate cell evaluations (opportunity cost) d_{ij} for each non-assigned cell, by using the formula:

$d_{ij} = c_{ij} - (u_i + v_j)$.

Step4. Check value of d_{ij}'s such that,

(i) if all $d_{ij} > 0$, solution is optimal and unique,

(ii) if at least one $d_{ij} = 0$, with and remaining d_{ij}'s > 0, solution is optimal having alternate solution,

(iii) if at least one $d_{ij} < 0$, solution is not optimal.

Step5. If step4 (iii) is satisfied go to *step6*, else exit.

Step6. Construct a loop with most negative d_{ij} value cellas the starting point such that,

(i) only even number of assigned cells can take part in loop formation with minimum of four cells,

(ii) route can be changed from assigned cells only,

(iii) assign $+\theta$ and $-\theta$ to most negative d_{ij} value cell and assigned cells of loop such that $+\theta$ and $-\theta$ are assigned in alternate fashion starting from to most negative d_{ij} value cell,

(iv) add value, val to $+\theta$ cells and subtract val from $-\theta$ cells where val is the minimum of assignment value among $-\theta$ cells.

Step7. Repeat *step2* to *6* until all d_{ij}*'s* are non-negative.

Step1 deals with degeneracy condition by introducing ∈ to least cost non-assigned cell, this is required as without this, all *u-v* values cannot be accurately calculated. Step2 calculates all *u-v* values by assigning any u_i value to *0* to start the process. Step3 is concerned with calculation of opportunity cost values, d_{ij} for non-assigned cells. If any d_{ij} found to be less than 0, then we move to step4, else basic feasible solution is optimal. Main and most complex step in MODI is *step6*, which is the loop formation step, loop must be formed if any d_{ij} is found to be negative, and sub-steps of *step6* are the loop formation rules. Solution obtained after loop formation step does not guarantees that solution is optimized, and we need to again follow *step2* to *6* in order to check for optimality.

6.3 Proposed Algorithm

Step1. Check for problem type, if balanced, go to *step3* else *step2*.

Step2. Balance the table by introducing dummy column if Supply > Demand, else dummy row.

Step3a. Select unique cell (least cost cell greater than *0*) from each row with First Come First Serve Basis in case of non-uniqueness satisfying *step3b*. For dummy row, there will be no selection.

Step3b. Select one more cell, if there is already an assignment in that column for given row.

Step4. If entire cells of a column get selected, go to step 5, else step 6.

Step5. Make an assignment to least cost cell greater than *0* from unselected cells.

Step6. Make assignment to selected cells right from first row of the table, with smallest cell being used first. Make assignment to least cost cells starting from first row in case more assignment is needed, and there is no assigned cell.

Step7. Repeat *step6* until Supply/Demand becomes zero.

Step8. If degeneracy condition occurs, make assignment of ∈ to least cost un-allocated cell.

Step9. Check for loop formation for two least cost unassigned cells of each row. If found, adjust assignment value (if total cost is less) and repeat the process again.

Step10. Calculate total transportation cost.

Step3a of the algorithm borrows the idea from Least Cost Cell method with some modification provided in *step3b*. The term unique cell selection is used because *step3a* selects some unique cells that start the process. Although selecting least cost cell is just like sorting of array in increasing order for each row but can be done effectively using Key-Value pair approach (Prasad et al. (2018a)), which will allow to make relationship between sorted and unsorted array. *Step4* checks room for any other cell which is needed to be assigned and if so *step5* is called. *Step6* makes assignment to selected cells starting from row one and if needed makes assignment to non-selected cells to make sure Supply/Demand is satisfied. *Step7* is normal step used in case of degeneracy condition which arises when total assignments made is less than (total row + total column - 1). *Step9* borrows the idea from Modified Distribution method, with some variation in searching for non-allocated cell for assignment. It calculates d_{ij} value for two least cost non-assigned cells in each row which is different from MODI as it calculates d_{ij} value for all non-allocated cells. Loop formation step is same as discussed in MODI algorithm inn Section 6.2.

7. EXAMPLES

(I)

	W1	**W2**	**W3**	**W4**	**W5**	**Supply**
F1	8	8	2	10	2	**40**
F2	11	4	10	9	4	**70**
F3	5	2	2	11	10	**35**
F4	10	6	6	5	2	**90**
F5	8	11	8	6	4	**85**
Demand	**80**	**55**	**60**	**80**	**45**	**320/320**

(Das et al. (2014)) Since it is balanced, unique cell selection step can be performed.

	W1	**W2**	**W3**	**W4**	**W5**	**Supply**
F1	8	8	2 $\boxed{40}$	10	2	**40**
F2	11	4 $\boxed{55}$	10	9	4 $\boxed{15}$	**70**
F3	5 $\boxed{15}$	2	2 $\boxed{20}$	11	10	**35**
F4	10	6	6	5 $\boxed{60}$	2 $\boxed{30}$	**90**
F5	8 $\boxed{65}$	11	8	6 $\boxed{20}$	4	**85**
Demand	**80**	**55**	**60**	**80**	**45**	**320/320**

Above table contains unique cells (marked with red color) and assignments with total cost 1475.

	W1	**W2**	**W3**	**W4**	**W5**	u_i
F1	8	8	2 $\boxed{40}$	10	2	**-4**
F2	11	4 $\boxed{55}$	10	9	4 $\boxed{15}$	**0**
F3	5 $\boxed{15}$	2	2 $\boxed{20}$	11	10	**-4**
F4	10	6	6	5 $\boxed{60}$	2 $\boxed{30}$	**-2**
F5	8 $\boxed{65}$	11	8	6 $\boxed{20}$	4	**-1**
v_j	**9**	**4**	**6**	**7**	**4**	

Since all d_ij values are positive, hence the basic feasible solution is optimal with total cost 1475.

(II)

	W1	W2	W3	W4	W5	Supply
F1	73	40	9	79	20	**8**
F2	62	93	96	8	13	**7**
F3	96	65	80	50	65	**9**
F4	57	58	29	12	87	**3**
F5	56	23	87	18	12	**5**
Demand	**6**	**8**	**10**	**4**	**4**	**32/32**

(Russell (1969)) Since it is balanced, unique cells can be selected.

	W1	W2	W3	W4	W5	Supply
F1	73	40	9 $\boxed{8}$	79	20	**8**
F2	62	93	96	8 $\boxed{4}$	13 $\boxed{3}$	**7**
F3	96 $\boxed{1}$	65 $\boxed{8}$	80	50	65	**9**
F4	57 $\boxed{1}$	58	29 $\boxed{2}$	12	87	**3**
F5	56 $\boxed{4}$	23	87	18	12 $\boxed{1}$	**5**
Demand	**6**	**8**	**10**	**4**	**4**	**32/32**

Above table contains unique cells (marked with red color) and assignments with total cost 1110.

	W1	W2	W3	W4	W5	u_i
F1	73	40	9 $\boxed{8}$	79	20	**-20**
F2	62	93	96	8 $\boxed{4}$	13 $\boxed{3}$	**0**
F3	96 $\boxed{1}$	65 $\boxed{8}$	80	50	65	**39**
F4	57 $\boxed{1}$	58	29 $\boxed{2}$	12	87	**0**
F5	56 $\boxed{4}$	**23(-2)**	87	18	12 $\boxed{1}$	**-1**
v_j	**57**	**26**	**29**	**8**	**13**	

d_52 contains negative value, hence loop can be performed between cells (F5,W2), (F3,W2), (F3,W1) and (F5,W1).

	W1	W2	W3	W4	W5	u_i
F1	73	40	9 $\boxed{8}$	79	20	**-18**
F2	62	93	96	8 $\boxed{4}$	13 $\boxed{3}$	**0**
F3	96 $\boxed{5}$	65 $\boxed{4}$	80	50	65	**41**
F4	57 $\boxed{1}$	58	29 $\boxed{2}$	12	87	**2**
F5	56	**23** $\boxed{4}$	87	18	12 $\boxed{1}$	**-1**
v_j	**55**	**24**	**27**	**8**	**13**	

Since all d_ij values are positive, hence the basic feasible solution is optimal with total cost 1102.

(III)

	W1	W2	W3	W4	Supply
F1	4	6	8	13	**500**
F2	13	11	10	8	**700**
F3	14	4	10	13	**300**
F4	9	11	13	3	**500**
Demand	**250**	**350**	**1050**	**200**	**1850/2000**

(Kumaraguru et al. (2014)) Since given problem is unbalanced, with Supply > Demand, we first need to introduce dummy column, C1 to make problem balanced.

	W1	W2	W3	W4	C1	Supply
F1	4 250	6 250	8	13	0	**500**
F2	13	11	10 500	8 200	0	**700**
F3	14	4 100	10 200	13	0	**300**
F4	9	11	13 350	3	0 150	**500**
Demand	**250**	**350**	**1050**	**200**	**150**	**2000/2000**

Above table contains unique cells (marked with red color) and assignments with total cost 16050.

	W1	W2	W3	W4	C1	u_i
F1	4 250	6 250	8	13	0	**2**
F2	13	11	10 500	8 200	0	**0**
F3	14	4 100	10 200	13	0	**0**
F4	9	11	13 350	3	0 150	**3**
v_j	**2**	**4**	**10**	**8**	**-3**	

d_13 and d_44 contain values -4 and -8, hence loop can be performed between cells (F4,W4), (F2,W4), (F2,W3) and (F4,W3).

	W1	W2	W3	W4	C1	u_i
F1	4 250	6 250	8	13	0	**2**
F2	13	11	10 700	8	0	**0**
F3	14	4 100	10 200	13	0	**0**
F4	9	11	13 150	3 200	0 150	**3**
v_j	**2**	**4**	**10**	**0**	**-3**	

Since all d_ij values are positive, hence the basic feasible solution is optimal with total cost13650.

(IV)

	W1	W2	W3	W4	W5	Supply
F1	5	8	6	6	3	**800**
F2	4	7	7	6	5	**500**
F3	8	4	6	6	4	**900**
Demand	**400**	**400**	**500**	**400**	**800**	**2500/2200**

(Girmay et al. (2013)) Since given problem is unbalanced with Supply < Demand, we first need to introduce dummy row R1 to make problem balanced.

	W1	W2	W3	W4	W5	Supply
F1	5	8	6	6	3 [800]	**800**
F2	4 [400]	7	7	6 [100]	5	**500**
F3	8	4 [400]	6 [500]	6	4	**900**
R1	0	0	0	0 [300]	0	**300**
Demand	**400**	**400**	**500**	**400**	**800**	**2500/2500**

Above table contains unique cells (marked with red color) and assignments with total cost 9200.

	W1	W2	W3	W4	W5	u_i
F1	5	8	6	6	3 [800]	**-1**
F2	4 [400]	7	7	6 [100]	5	**0**
F3	8	4 [400]	6 [500]	6	4 ∈	**0**
R1	0	0	0 ∈	0 [300]	0	**-6**
v_j	**4**	**4**	**6**	**6**	**4**	

Since all d_ij values are positive, hence the basic feasible solution is optimal with total cost 9200. However alternate solution do exist, as d_34 is 0 and will form loop between cells (F3,W4), (F3,W3), (R1,W3) and (R1,W4) respectively.

8. RESULT AND DISCUSSION

The reason of using heuristic algorithms is because they are supposed to provide near-by optimal solution with healthy time rate. This is because checking optimality using MODI or Stepping stone methods is time consuming process. The reason for success of LCM method is its simple algorithm, which makes assignment ot least cost cells using Greedy approach, but this simple process is itself time consuming process because of searching time. Moreover, checking optimality generally requires more time as optimal solution is not obtained in single iteration using MODI or Stepping stone method. This paper propose Two-step Exact algorithm for solving Transportation Problem. First step uses basic idea of cell selection for assignment from Least Cost Method while second step borrows loop formation step from Modified Distribution Method. In the proposed algorithm, unique cells (least cost cell greater than 0) from each row are selected with First Come First Serve basis in case of non-uniqueness. For columns with multiple assignments, one more cell is selected in corresponding rows. We used Least Cost Method approach for assignment from selected unique cells. Remaining cells are used for assignment if needed using Least Cost Method approach. Loop formation is checked by considering two least cost un-assigned cells from each row. If formed, obtained solution is modified using rules of Modified Distribution

Method. Proposed algorithm is easy to implement and has its own in-built optimality checker to test for optimality. Unique cell selection can be selected efficiently using Key-Value Pair approach (Prasad et al. (2018a)). Loop formation step which uses basic idea of Modified Distribution method can be handled effectively using work of Prasad et al. (2018b). The algorithm is tested over *500* problem sets with matrix size varying from *5x5* to *100x100*, for both balanced and unbalanced problem. Sample problems were discussed to show the working of the algorithm.

References:

Abadie, J. M. & Williams, A. C. (1963). Dual & parametric methods in decomposition. *Recent advances in mathematical programming*, 149-158.

Adlakha, V., Kowalski, K. & Lev, B. (2006). Solving transportation problems with mixed constraints. *International journal of management Science & Engineering management*, *1*(1), 47-52.

Ahuja, R. K. (2017). Network flows: theory, algorithms, & applications. Pearson Education.

Ahuja, R. K., Magnanti, T. L., Orlin, J. B. & Reddy, M. R. (1995).Applications of network optimization. Handbooks in *Operations Research and Management Science*, *7*, 1-83.

Balakrishnan, N. (1990). Modified Vogel's approximation method for the unbalanced transportation problem. *Applied Mathematics Letters*, *3*(2), 9-11.

Caputo, A. C., Fratocchi, L. & Pelagagge, P. M. (2006). A genetic approach for freight transportation planning. *Industrial Management & Data Systems*, *106* (5), 719-738.

Charnes, A. & Cooper, W. W. (1954). The stepping stone method of explaining linear programming calculations in transportation problems. *Management science*, *1* (1), 49-69.

Cormen, T. H., Leiserson, C. E., Rivest, R. L. & Stein, C. (2009). *Introduction to algorithms*. MIT press.

Dantzig, G. B. (1951). Application of the simplex method to a transportation problem. Activity Analysis & Production & Allocation.

Dantzig, G. B. (1963). *Linear programming & extensions*. Princeton U. Press, Princeton, N.J.

Dantzig, G. B. & Wolfe, P. (1960). Decomposition principle for linear programs. *Operations research*, *8*(1), 101-111.

Das, U. K., Babu, M. A., Khan, A. R. & Uddin, M. S. (2014). Advanced Vogel's Approximation Method (AVAM): a new approach to determine penalty cost for better feasible solution of transportation problem. *International Journal of Engineering Research & Technology* (IJERT), *3*(1), 182-187.

Du, D. Z. & Pardalos, P. M. (Eds.) (2013). *Handbook of combinatorial optimization: supplement*. Springer Science & Business Media, 1.

Efroymson, M. A. & Ray, T. L. (1966). A branch-bound algorithm for plant location. *Operations Research*, *14*(3), 361-368.

Equi, L., Gallo, G., Marziale, S. & Weintraub, A. (1997). A combined transportation & scheduling problem. *European journal of operational research*, *97*(1), 94-104.

Gal, T. (1993). Selected bibliography on degeneracy. *Annals of Operations Research*, *46*(1), 1-7.

Girmay, N. & Sharma, T. (2013). Balance an Unbalanced Transportation Problem by a Heuristic approach. *International Journal of Mathematics & Its Application*, *1*(1), 16.

Greenberg, H. J. (2012). A Computer-Assisted Analysis System for Mathematical Programming Models & Solutions: A User's Guide for ANALYZE. Springer Science & Business Media, *1*.

Hitchcock, F. L. (1941). The distribution of a product from several sources to numerous localities. *Journal of mathematics & physics*, *20*(1-4), 224-230.

Hentenryck, P. V. & Michel, L. (2009). Constraint-based local search. The MIT press.

Kantorovich, L. V. (1960). Mathematical methods of organizing & planning production. *Management science*, *6*(4), 366-422.

Klee, V. & Minty, G. J. (1970). *How good is the simplex algorithm* (No. TR-22). Washington univ seattle dept of mathematics.

Koopmans, T. C. (1947). Optimum Utilization of the Transportation system proceeding of the International statistical conference. Washington DC.

Kumaraguru, S., Kumar, B.S. & Revathy, M. (2014). Comparative study of various methods for solving transportation problem, *3*(9), 246.

Lobel, A. (1997). Optimale vehicle scheduling in public transit.

Magnanti, T. L. & Wong, R. T. (1984). Network design & transportation planning: Models & algorithms. Transportation science, *18*(1), 1-55.

Marcotte, O. & Sol&, R. M. (1986). An interactive branch-&-bound algorithm for multiple criteria optimization. *Management Science*, *32*(1), 61-75.

Monge, G. (1781). Mémoire sur la théorie des déblais et des remblais. Histoire de l'Académie Royale des Sciences de Paris.

Murty, K. G. (1985). Linear and combinatorial programming. Robert E. Krieger Publishing Company, Melbourne, FL(USA), 592.

Nemhauser, G. L.& Wolsey, L. A. (1992). Integer programming and combinatorial optimization. Wiley, Chichester. GL Nemhauser, MWP Savelsbergh, GS Sigismondi. Constraint classification for mixed integer programming formulations. COAL Bulletin, 20, 8-12.

Pan, P. Q. (1990). Practical finite pivoting rules for the simplex method. *Operations-Research-Spektrum*, *12*(4), 219-225.

Papadimitriou, C. H. & Steiglitz, K. (1998). Combinatorial optimization: algorithms and complexity. Courier Corporation.

Phillips, D. T. & Garcia-Diaz, A. (1981). *Fundamentals of network analysis*. Prentice Hall.

Prasad, A. K. & Pankaj (2018). A Study of KVP Approach on Some Transportation Models. In *2018 International Conference on Current Trends towards Converging Technologies (ICCTCT)*. IEEE, 1-5.

Prasad, A. K. & Pankaj (2018). Loop formation in Transportation Problem: Semi Cycle Formation in Restricted Graph. Journal of Advanced Research in Dynamical & Control Systems, *10*(13), 1625-1632.

Reinfeld, N. V. & Vogel, W. R. (1958). *Mathematical programming*. Prentice-Hall.

Sait, S. M. & Youssef, H. (1999). Iterative computer algorithms with applications in engineering: solving combinatorial optimization problems. IEEE Computer Society Press.

Shafaat, A. & Goyal, S. K. (1988). Resolution of degeneracy in transportation problems. *Journal of the Operational Research Society*, *39*(4), 411-413.

Sharp, J. F., Snyder, J. C. & Greene, J. H. (1970). A decomposition algorithm for solving the multifacility production-transportation problem with nonlinear production costs. *Econometrica: Journal of the Econometric Society*, 490-506.

Taha, H. A. (2004). Operations research: An introduction (for VTU). Pearson Education India.

Ting, S. C. & Tzeng, G. H. (2003). Ship scheduling & cost analysis for route planning in liner shipping. *Maritime Economics & Logistics*, *5*(4), 378-392.

Van Roy, T. J. & Gelders, L. F. (1981). Solving a distribution problem with side constraints. European Journal of Operational Research, *6*(1), 61-66.

Wilson, R. J. (1972). Introduction to graph theory. London: Longman, 107.

Wolsey, L. & Rinaldi, G. (1993). Integer programming and combinatorial optimization.

Winston, W. L. & Goldberg, J. B. (2004). Operations research: applications & algorithms (Vol. 3). Belmont: Thomson Brooks/Cole.

Woodruff, D. L. (Ed.). (2013). Advances in computational & stochastic optimization, logic programming, & heuristic search: interfaces in computer science & operations research (Vol. 9). Springer Science & Business Media.

Russell, E. J. (1969). Letters to the Editor—Extension of Dantzig's Algorithm to Finding an Initial Near-Optimal Basis for the Transportation Problem. Operations Research, *17*(1), 188-191.

Zoutendijk, G. (1960). *Methods of feasible directions: a study in linear & non-linear programming*. Elsevier.

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