New Fuzzy Divergence Measure and its Applications: A New Approach

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Abstract- Various authors and researchers have established fuzzy divergence measures and their applications in multi-criteria decision making (MCDM), pattern recognition (PR), medical diagnosis (MD), fuzzy clustering, speech recognition etc. Here, we have derived a new fuzzy divergence measure and investigated their properties to existence and validity. Also investi-gated fuzzy divergence measure in PR, MCDM and MD. Compared the established results by various authors and researcher. Novelty of this research may be useful to industries for decision making, identifying the medical diseases and pattern recognition.

Key Words : Entropy, fuzzy sets, new f-divergence measure, MCDM, PR, MD. Mathematics Subject Classi cation (2010) 94A17, 03E72, 68T10, 90B50, 92C50.

Introduction

In information theory, Shannon [20] de ned the entropy measure for a prob-ability distribution. Fuzzy entropy, a fuzziness measure (cf.[13]) often used and cited in many literatures, was introduced by Zadeh [27, 28]; then De Luca and Termini [14] de ned an entropy of a fuzzy set based on Shannon's func-tion. Later on, many other researchers made more e ort in this particular area. In 1975, Kaufmann [11] proposed a new fuzziness measure of fuzzy set by a distance between its membership function and its nearest Classical(Ordinary) sets. Entropy, is a very important notion for measuring uncertain information and received great attention in the past decades. It has broad applications in many areas such as Pattern recognition, decision making, medical diagno-sis, signal and image processing, fuzzy clustering, speech recognition, feature selection, fuzzy aircraft control, bio-informatics etc.

Afterwards, a number of other researchers have studied the fuzzy diver-gence measures in di erent ways and provide their applications in di erent areas. Fuzzy divergence measure introduced by Fan and Xie [5] is based on ex-ponential operation and its relation with fuzzy divergence measure [1]. Prakash et al. [17] proposed two fuzzy divergence measures corresponding to Ferreri [7] probabilistic measure of directed divergence. Ghosh et al. [8] gave its appli-cation in the area of automated leukocyte recognition. The study submitted by Montes et al. [15] in 2002 was based on special classes of divergence mea-sures and used the link between fuzzy and probabilistic uncertainty. Bhatia and Singh [2] proposed the fuzzy divergence measure corresponding to Taneja Tomar and Ohlan [22] studied a sequence of fuzzy mean di erence di-vergence measures by establishing inequalities among them and provided their applications in the context of consistency in linguistic variables and pattern recognition. Tomar and Ohlan [24] introduced a new parametric generalized exponential fuzzy divergence measure corresponding to Verma and Sharma [25] with its application to strategic decision making.

In recent times, the literature on applications of information and divergence measures between fuzzy sets has extended considerably, still there is a possi-bility of developing better divergence measures can be developed which can be applied to various elds. Here we study a new symmetric generalized measure of fuzzy divergence and its essential properties to check its authenticity. The new generalized measure has elegant properties which are proven in the paper to present the e ciency of the proposed measure.

This paper is organized as follows. In Section 1, some important concepts of probability theory will be reviewed along with the illustration of fuzzy set theory using the membership degree and non-membership degree of fuzzy set. In Section 2, a new symmetric fuzzy divergence measure between fuzzy sets is proposed. Section 3 provides some more elegant properties of the proposed measure in a number of theorems. Finally, the applications of proposed gen-eralized fuzzy divergence measure to MCDM, PR and MD are illustrated by three numerical examples in Section 4. In section 5, results and discussion. Finally, some concluding remarks are drawn in Section 6.

1.1 Shannon Entropy

Shannon rstly use the word entropy to measure an uncertain degree of the randomness in a probability distribution and de ned the information contained in an experiment. It is given by

n

X_i

$H(P) =$ $p_i \log p_i$ (1.1)

=1

Directed divergence measure is a relative entropy measure which provides a distance formula between the two discrete probability distributions. Kullback and Leibler [12] rst proposed a measure of directed divergence between the two distributions $P = (p_1, p_2, p_3, ..., p_n)$ and $Q = (q_1, q_2, q_3, ..., q_n)$ as:

n

$D(P\ j\ Q) = {}^{X} p_i \log {}^{p_i}$

—

i=1 $\quad {}^{q_i}$

The New f-divergence contains several divergences used in determining the a nity/distance between two probability distributions. This divergence is in-

convex function f, de ned on (0,). Let n = P = (p ,

troduced by using a n 1 1

$p_2, p_3, ..., p_n$): p_i 0, ${}^{P} p_i = 1$, n 2 be a set of complete nite discrete

i=1

probability distributions. For a convex function, a new f-divergence measure is developed by Jain and Saraswat and is given by

n

$S_f\ {}^{(P;\,Q)\,=}\ {}_{i=1}$ $q_i f\ {}^{p}i_{2qi}$ i

+ q

X

It is well known that S_f (P, Q) is a multi-purpose functional form which results in number of general divergence measures. Most common choices of f satisfy f(1) = 0, so that S_f (P, Q) = 0. Convexity ensures that divergence measure S_f (P, Q) is non-negative.

1.2 Fuzzy Concepts

Fuzziness, a feature of uncertainty, results from the lack of sharp di erence of being or not being a member of the set, i.e., the boundaries of the set under consideration are not sharply de ned. A fuzzy set A de ned on a universe of discourse X is given by Zadeh and is de ned below:

A = fh x, $_A$(x) / x 2 Xig,

where $_A$(x) : X ! [0, 1] is the membership function of A. The membership value describes the degree of belongingness of X in A. When valued in [0, 1], it is the characteristic function of crisp (i.e. non fuzzy) set. Zadeh [28, 27] gave some notions related to fuzzy sets, some of which are listed below:

(1) Complement: A = Complement of A () $_A$(x) = 1 $_A$(x) for all x 2 X .

Union: A[B = Union of A and B () $_{A[B}$(x) = maxf $_A$(x), $_B$(x)g for all x 2 X

Intersection: A\B = intersection of A and B () $_{A\backslash B}$ (x)= minf $_A$(x), $_B$(x)g for all x 2 X

Considering the concept of fuzzy sets, De Luca and Termini [14] introduced the measure of fuzzy entropy corresponding to Shannon's entropy and given as

$$H(N) = \sum_{i=1}^{n} h\ _A(x_i)\log\ _A(x_i) + (1\ \ _A(x_i))\log(1\ \ _A(x_i))^{i}$$

X

This idea of divergence measure was extended from probabilistic to fuzzy set theory by Bhandari and Pal [1] by giving a fuzzy information measure for discrimination of a fuzzy set B relative to some other fuzzy set A. Let A and B be two fuzzy sets de ned in discrete universe of discourse X = fx_1, x_2, ...,

x_ng having the membership values $_A(x_i)$ and $_B(x_i)$, i = 1,2,...n respectively, then the fuzzy divergence measure of fuzzy set B relative to A is given by

$$D(A\ j\ B) = \sum_{i=1}^{n} A^{(x_i)\log}\ \frac{_A(x_i)}{_{B(xi)}} + (1\ \ A(xi))\log \frac{1\ \ _B(x_i)}{1\ \ _A(x_i)} \qquad (1.2)$$

X

Satisfying the condition

D(A, B) 0,

D(A, B) = 0 i A = B,

D(A, B) is a convex function of $_A(x_i)$.

If X is a universe of discourse and F(X) is the set of all fuzzy subsets, a mapping L : F(X) x F(X) ! R is a fuzzy divergence measure if and only if for each A, B, C 2 F(X), the following axioms hold:

(a_1) L(A, B) 0,

(a_2) L(A, A) = 0,

(a_3) max fL(A U C, B U C), L(A \ C, B \ C)gL(A, B).

Non-negativity of L(A, B) is the natural assumption.

Generliazed Symmetric fuzzy divergence mea-sure

Here, we shall propose a symmetric generalized measure of divergence between two fuzzy sets A, B of universe of discourse X = fx_1, x_2, x_3,...., x_ng having the

membership values 2 (0, 1) corresponding to [19] and is given by

$$N_t(A;B) = \frac{1}{2^{2t+1}} \sum_{i=1}^{n} \left({}_A(x_i) \quad {}_B(x_i) \right)^{2t} \frac{\left({}_A(x_i) \; {}_B(x_i) \right)^t}{\left({}_A(x_i) + {}_B(x_i) \right)} + \frac{\left(1 \; {}_A(x_i)\right)\left(1 \quad {}_B(x_i)\right)\right)^t}{\left(2 \quad {}_A(x_i) \quad {}_B(x_i)\right)}$$

(2.1)

t = 0, 1,....

2.1 Theoram

N_t(A, B) is a valid measure of fuzzy divergence.

Proof. It is clear From (2.1) that

N_t(A, B) 0

N_t(A, B) = 0 if ${}_A(x_i)$ = ${}_B(x_i)$,

max fN_t(A [C, B [C), N_t(A \ C, B \ C)g N_t(A, B) We divide X into six subsets:

X_1 = fx/x 2 X, ${}_A(x)$ ${}_B(x)$ ${}_C(x)$g

X_2 = fx/x 2 X, ${}_A(x)$ ${}_C(x) <$ ${}_B(x)$g

X_3 = fx/x 2 X, ${}_B(x) <$ ${}_A(x)$ ${}_C(x)$g

X_4 = fx/x 2 X, ${}_B(x)$ ${}_C(x) <$ ${}_A(x)$g

X_5 = fx/x 2 X, ${}_C(x) <$ ${}_A(x)$ ${}_B(x)$g

X_6 = fx/x 2 X, $_C$(x) < $_B$(x) < $_A$(x)g

In set X_1,

A [C = Union of A and B () $_{A[C}$(x) = maxf $_A$(x), $_C$(x)g = $_C$(x)

B [C = Union of B and C () $_{B[C}$(x) = maxf $_B$(x), $_C$(x)g = $_C$(x);

A \ C = intersection of A and C () $_{A\backslash C}$(x) = minf $_A$(x), $_C$(x)g = $_A$(x);

B \ C = intersection of B and C () $_{B\backslash C}$(x) = minf $_B$(x), $_C$(x)g = $_B$(x);

N_t(A [C, B [C) = N_t(C, C) = 0,

N_t(A \ C, B \ C) = N_t(A, B),

So, maxfN_t(A [C, B [C), N_t(A \ C, B \ C)g = N_t(A, B),

Similarly, in the sets X_2, X_3, X_4, X_5, X_6 we have

maxfN_t(A [C, B [C), N_t(A \ C, B \ C)g N_t(A, B).

Thus, max fN_t(A [C, B [C), N_t(A \ C, B \ C)g N_t(A, B) for all A, B, C 2 F(X)

Properties of proposed fuzzy divergence mea-sure

In this section we provide properties of the proposed generalized fuzzy diver-gence measure (2.1) in accordance with the following theorems. To prove these theorems we assume that X divided into two parts X_1 and X_2 such that the sets

X_1 = fx/x 2 X, $_A(x_i)$ $_B(x_i)$g

X_2 = fx/x 2 X, $_A(x_i)$ < $_B(x_i)$g

Theorem 1. (a)N_t(A, A \ B) = N_t(B, A [B).

N_t(A [B, A) + N_t(A \ B, A) = N_t(B, A).

N_t(A [B, A \ B) = N_t(A, B).

(d)N_t(A [B, C) + N_t(A \ B, C) = N_t(A, B) + N_t(B, C).

Theorem 2. (a) N_t(A, A) = N_t(A, A)

N_t(A, B) = N_t(A, B)

N_t(A, B) = N_t(A, B)

N_t(A, B) + N_t(A, B) = N_t(A, B) + N_t(A, B).

Applications of Proposed fuzzy divergence mea-sure

Here we are going to introduce the application in the context of MCDM, PR and MD.

4.1 Multi-Criteria Decision Making

Decision-making deals with the problem of choosing the best alternative with the highest degree of satisfaction for all the appropriate criteria or goals. Multi-criteria decision-making (MCDM) is the most well known branch of decision-making that allows decision-makers to rank and select alternatives according to di erent criteria. Using below method we can solve the MCDM problems with the help of proposed generalized fuzzy divergence measure.

Let R = (R_1, R_2, R_3,....., R_m) be a set of options, D = (D_1, D_2, D_3,....., D_m)

be the set of criteria. The Characteristics of the option R_i in term of criteria

D are represented by the following FSs:

R_i = f < D_j, $_{ij}$ >, D_j 2 Dg, i = f1, 2, 3,...., mg and j = f1, 2, 3,.., ng

Where $_{ij}$ indicates the degree that the option R_i satis es the criteria D_j.

We can solve the above MCDM problem using Eq. (2.1).

Step 1: Find out the positive-ideal solution R^+ and negative-ideal solution

R :

R^+ = f< $_1{}^+$>, < $_2{}^+$>,...,< $_n{}^+$>g

= f< $_1$ >, < $_2$ >..., < $_n$ >g

where for each j = 1, 2, 3,.... , n.

$_j{}^+$ = < $\max_i$ $_{ij}$

$_j$ = < $\min_i$ $_{ij}$

Step 2: Calculate $N_t(R^+, R_i)$ and $N_t(R , R_i)$ using Eq. (2.1)

Step 3: Calculate the relative fuzzy divergence measure $N_t(R_i)$ of alterna-tive R_i with respect to R^+ and R , where

$$N_t(R_i) = \frac{N_t(R^+;R_i)}{N_t(R^+;R_i)+N_t(R\ ;R_i)}, \quad i = 1, 2, ..., m.$$

Step 4: Rank the preference order of all alternatives according to the rel-ative fuzzy divergence measure.

Step 5: Select the best alternative R_k with the smallest $N_t(R_k)$.

To solve a real problem related to MCDM, we establish the applicability of new fuzzy divergence measure. For this we are going to consider customer decision-making problem given below.

Case study Consider a customer who wants to by a car . let there be ve types of cars , the alternatives R = fR_1, R_2, R_3, R_4, R_5 g be available in the market to by a car the customer takes the following four criteria : (i) Quality of product (D_1), (ii) price (D_2), (iii) Technical capability (D_3) and (iv) Fuel

Economy (D_4).

The ve possible options are to be evaluated by using decision making un-der the above four criteria in the following form:

$R_1 = f< D_1, .5>, <D_2, .3>, <D_3, .4>, <D_4, .7> g$

$R_2 = f<D_1, .2>, <D_2, .7>, <D_3, .6>, <D_4, .6>g$

$R_3 = f<D_1, .8>, <D_2, .5>, <D_3, .9>, <D_4, .2>g$

$R_4 = f<D_1, .6>, <D_2, .4>, <D_3, .7>, <D_4, .5>g$

$R_5 = f<D_1, .6>, <D_2, .5>, <D_3, .5>, <D_4, .7>g$

The stepwise computational procedure to solve the above multi-criteria fuzzy decision- problem now goes as follows.

Step 1: The positive-ideal solution (R^+) and negative-ideal solution (R)

respectively are

$R^+ = f<D_1, .8>, <D_2, .7>, <D_3, .9>, <D_4, .7>g$

$= f<D_1, .2>, <D_2, .3> ,<D_3, .4>, <D_4, .2>g$

Step 2: Table (1) and (2) shows the calculated numerical values of $N_t(R^+, R_i)$ and $N_t(R , R_i)$ using Eq. (2.1) for t 0

Step 3: Calculated values of the relative fuzzy divergence measure $N_t(R_i)$

for i= 1, 2, 3, 4, 5 with t are presented in table (3)

Step 4: According to the calculated numerical values of relative divergence measure for di erent values of t ranking order of alternative as follows:

For t = 1, $R_3 > R_4 > R_5 > R_2 > R_1$

For t = 5, $R_3 > R_4 > R_5 > R_2 > R_1$

For t = 10, $R_3 > R_4 > R_5 > R_2 > M_1$

For t = 100, $R_3 > R_4 > R_5 > R_2 > R_3$

Table 1: Calculated numerical values of $N_t(R^+, R_i)$, t 0

	t=1	t=5	t=10	t=100
$N_t(R^+, R_1)$	.78309	.42980	.52645	20.74649
$N_t(R^+, R_2)$	.74554	.07039	.00396	1.02861e-25
$N_t(R^+, R_3)$	.38793	.00864	.00014	4.25021e-36
$N_t(R^+, R_4)$	.27570	.00082	3.38702e-06	3.88065e-49
$N_t(R^+, R_5)$	.39811	.09831	.03322	6.11111e-11

Table 2: Calculated numerical values of $N_t(R , R_i)$, t 0

	t=1	t=5	t=10	t=100
$N_t(R , R_1)$	.45944	.00884	.00014	4.25021e-36
$N_t(R , R_2)$	.44046	.00192	6.77459e-06	7.76130e-49
$N_t(R , R_3)$	1.08374	.47986	.52961	1.02861e-25
$N_t(R , R_4)$	.43525	.00187	6.77459e-06	7.76130e-49
$N_t(R , R_5)$	.60647	.01031	.00014	4.25021e-36

Then by table (3) variation in values of t brings change in ranking, but leaves the best choice unchanged. So R_3 is the most perfectly alternative.

Table 3: Computed values of relative divergence measure $N_t(R_i)$ for i = 1,2,3,4,5

	t=1	t=5	t=10	t=100
$N_t(R_1)$	.63024	.97985	.99970	1
$N_t(R_2)$	.62681	.97345	.99830	1
$N_t(R_3)$	.26359	.01769	.00030	4.13199e-11
$N_t(R_4)$	.38779	.30483	.33330	.33330
$N_t(R_5)$	.39629	.90508	.99580	1

Case study by using weight vector

Fuzzy MCDM problems consist of m alternatives a_i(i=1(1)m) such that alternative is achieved by means of n criteria b_j(j=1(1)m) $@_{ij}$ is constructed by alternative a_i with respect to criterion b_j, are fuzzy values (FVs). Let w_j

n

be weight of criterion such that w_j 0, ^{P} w_j = 1, W = $(w_1, w_2, ..., w_n)^T$

i=1

It is worth mentioning that proposed method is appropriate for circum-stances where the number of decision experts is small such that they assess the criterion based on their experience and knowledge and the alternatives could be of any type, then these assessment of alternatives can be converted to FVs.

If the information regarding the criterion weight vector is not completely known or only partially known, then this proposed method can used to solve the MCDM problems. This method consists of the following steps:

Step 1: Generate decision matrix G = (@$_{ij}$)m n

The decision experts furnish all the feasible evaluations regarding the alter-native a_i with respect to criterion b_j mentioned by fuzzy values @$_{ij}$ (i=1(1)m, j=1(1)n)

Step 2: Compute PIS and NIS.

The solution which result in optimum value for each criterion is ideal so-lution. The optimal values (PIS) for diverse criterion are altered and pointed out as

8

max @ijf orbenif itcreterionbj

<

i=1(1)m

"$^+$ = (4.1)

min @$_{ij}$f orcostcreterionb$_j$

i=1(1)m

for j=1(1)n.

and

8

max @ijf orbenif itcreterionbj

<

i=1(1)m

" = (4.2)

min @$_{ij}$f orcostcreterionb$_j$

i=1(1)m

for j=1(1)n.

Step 3: Compute the weight vector

Overall performance of the alternative a_i computed by given formula

n

X$_j$ ^{k}ij

k(a_i) = $w_j k_{ij}$; wherek$_{ij}$ = + (4.3)

k + k

=1 ij ij

Apparently, the larger value of $k(a_i)$ shows the better the alternative. There-fore, all the alternatives are measured as a whole to construct a combined weight vector. Thus, linear programming model is demonstrated as follows:

m m n

X_i X X

$\max k = K(a_i) = w_j k_{ij};$ (4.4)

8

w 2 W

<

where; ^{n}

P

wj = 1

j=1

Step 4: Compute the closeness degree of the alternatives.

Based on (4.3), closeness degree $k(a_i)$ of each alternative a_i (i=1(1)m) with respect to the ideal solution is evaluated.

Step 5: Rank the alternatives.

Choose the highest value, denoted by $k(a_i)$ among the values $k(a_i)$, (i=1(1)m)

Hence a_d is the optimal choice.

Case Study Evaluation and assessments of the organizations in the nan-cial system based on nancial criteria are very indispensable. In this section, proposed method is implemented by via real data to the ranking of organiza-tions listed below:

Bajaj Steel (a_1), HDFC Bank (a_2), Tata Steel (a_3), and InfoTech Enterprises (a_4)

Four alternatives fa$_1$, a_2, a_3, a_4 g are considered for their performance on the basis of given inter-independent criterion set fb$_1$, b_2, b_3, b_4, b_5g, b_1; Earn-ings per share (EPS);2 b_2: Face value; b_3: P/C (Put-Call) ratio; b_4 : Dividend; b_5 : P/E (Price-to-earning) ratio. Out of these rst two are bene t criteria, i.e., higher quantity shows good growth prospects and the remaining are cost criteria, i.e., lower quantity shows good growth prospects.

Actual numerical values of the alternatives with respect to criterion set are adopted from Joshi and Kumar (2014) and Mishra et al (2017) and their average information values are depicted in the Table 4. Based on knowledge and experience of experts regarding the criterion set, partial information of the weights is given by

W = f$(w_i)^T$ j .2 w_1 .35, .1 w_2 .27, .15 w_3 .25, w_1 .2w_4, .8 w_4 .15,

n

P

.2 w_5 .4, w_2 - w_5 w_3g such that w_j = 1

i=1

Step 1: Crisp values in Table 4 are fuzzi ed by generating the form FSs for each criterion and decision matrix (Table 5) for FSs is constructed.

A_1 = 0.2846 / 20.5 + 0.75 /10 + 0.234 / 2.21 + 0.23 / 2 + 0.24 / 4.8, A_2 = 0.318 / 23.31 + 0.248 / 2 + 0.766 / 24.7 + 0.097 / 0.67 + 0.76 / 27,

Table 4: Average actual numerical value of criteria

	b_1	b_2	b_3	b_4	b_5
a_1	20.50	10.00	2.21	2.00	4.80
a_2	23.31	2.00	24.7	.67	27.00
a_3	60.06	10.00	5.65	2.92	6.5
a_4	16.86	5.00	9.70	1.25	11.40

A_3 = 0.759 / 60.06 + 0.75 /10 + 0.315 / 5.65 + 0.32 / 2.92 + 0.278 / 6.5, A_4 = 0.241/16.86 + 0.437 / 5 + 0.41/ 9.7 + 0.155 /1.25 + 0.394 /11.4

Table 5: Fuzzy decision matrix

	b_1	b_2	b_3	b_4	b_5
a_1	.2846	.75	.234	.23	.24
a_2	.318	.248	.766	.097	.76
a_3	.759	.75	.315	.32	.278
a_4	.241	.437	.41	.155	.394

Step 2: PIS and NIS are calculated by using (4.1) and (4.2) are as follows:

"$^+$ = f0.759,0.75,0.234,0.23,0.24g

" = f0.241,0.248,0.766,0.32,0.76g

Step 3: Calculating k^+_{ij}(@$_{ij}$, "$^+$) and k $_{ij}$(@$_{ij}$, , "):

k_{11}^+ = 0.2919, k_{12}^+ = 0.0000, k_{13}^+ = 0.0000, k_{14}^+ = 0.0000, k_{15}^+ =

0.0000,

k_{2} $^+$ = 0.2449, k_{22} $^+$ = 0.3369, k_{23} $^+$ = 0.3948, k_{24} $^+$ = 0.0377, k_{25} $^+$ = 0.3706, 1

$^k3$

1 $^+ = 0.0000$, k_{32} $^+ = 0.0000$, k_{33} $^+ = 0.0084$, k_{34} $^+ = 0.0104$, k_{35} $^+ = 0.0019$,

$^k41$ $^+ = 0.3667$, k_{42} $^+ = 0.1151$, k_{43} $^+ = 0.0376$, k_{44} $^+ = 0.0093$, k_{45} $^+ = 0.0287$

and

$^k11$ 0.0025, k_{12} = 0.3369, k_{13} = 0.3948, k_{14} = 0.0104, k_{15} = 0.3706,

$^k2$

1 = 0.0075, k_{22} = 0.0000, k_{23} = 0.0000, k_{24} = 0.0995, k_{25} = 0.0000,

$^k3$

1 = 0.3667, k_{32} = 0.3369, k_{33} = 0.2597, k_{34} = 0.0000, k_{35} = 0.3039,

$^k41$ = 0.0000, k_{42} = 0.0421, k_{43} = 0.1539, k_{44} = 0.0416, k_{45} = 0.1619

Next, the overall performances, by using (4.3), of alternative are calculates as follows:

k_{11} = 0.0085, k_{12} = 1.0000, k_{13} = 1.0000, k_{14} = 1.0000, k_{15} = 1.0000, k_{21} = 0..0297, k_{22} = .0000, k_{23} = .0000, k_{24} = .7252, k_{25} = .0000,

k_{31} = 1.0000, k_{32} = 1.0000, k_{33} = 0.9687, k_{34} = 0.0000, k_{35} = 0.9938, k_{41} = 0.0000, k_{42} = 0.2678, k_{43} = 0.8037, k_{44} = 0.8173, k_{45} = 0.8494

Step 4: Construct the model and compute the weight vector.

max k = 1.0382 w_1 + 2.2678w_2 + 2.7784w_3 + 2.5425w_4 + 2.8432w_5

:25 w_1 :4; :16 w_2 :27;

8 :15 w_3 :25; w_1 :2w_4;

>

>

>

>

>

<

= s:t :1 w_4 :18; :2 w_5 :35; w_2 w_5 w_3 (4.5)

> $_n$

>

P

$w_j = 1$

$>_:$

j=1

$(w_j)^T = (.25, .16, .165, .1, .325)^T$

Step 5: Calculation of closeness degree of the alternatives.

$k(a_1) = .6521$, $k(a_2) = .0799$, $k(a_3) = .8928$, $k(a_4) = .5322$.

Step 6: Rank the alternatives.

$a_3 > a_1 > a_4 > a_2$. Hence, optimum alternative is a_3.

The ranking of four alternatives is also acquired by the TOPSIS, F-TOPSIS, intuitionistic fuzzy TOPSIS and proposed, methods and is compared in Table 6 given below:

Table 6: Comparison of ranking order of alternatives from various methods

Methods	Ranking	Optimal alternative
TOPSIS method proposed by Hwang, Yoon (1981)	$a_3 > a_4 > a_1 > a_2$	a_3
F-TOPSIS proposed by Chen (2001)	$a_3 > a_1 > a_4 > a_2$	a_3
IF-TOPSIS proposed by Joshi, Kumar (2014)	$a_3 > a_1 > a_4 > a_2$	a_3
Mishra et al method (2017)	$a_3 > a_1 > a_4 > a_2$	a_3
D.S. Hooda mathod (2018)	$a_3 > a_1 > a_4 > a_2$	a_3
Proposed method	$a_3 > a_1 > a_4 > a_2$	a_3

It is worth mentioning that there is no discrepancy in the ranking order of the alternatives by TOPSIS method, F-TOPSIS method, IF-TOPSIS method, Mishra et al (2017), Hooda (2018) and proposed method.

4.2 Pattern Recognition

Now, we will discuss how pattern recognition become simple by demonstrating application of the proposed generalised fuzzy divergence measure given as be-low.

Assume, we are have m known patterns $P_1, P_2, P_3, ..., P_m$ having the clas-si cation $D_1, D_2, D_3, ..., D_m$: respectively.

The Pattern are represented by the following fuzzy set in the universe of discourse $K = fk_1, k_2, k_3, .., k_ng$:

P_i = fh k_j, $_{pi}$ (k_j) i/ k_j 2 K g

where i = 1, 2, . . ., m and j = 1, 2, . . ., n.

Given an unknown pattern Q, represented by the fuzzy set Q_i = fh k_j, $_{Qi}$ (k_j)i / k_j 2 K g

Our main motive is to classify Q to one of the classes D_1, D_2, D_3, ...,

D_m.The process of assigning Q to D_X described below is as per the principal of minimum divergence/discrimination information between fuzzy sets

X = arg min fM(P_X , Q)g.

x

Above algorithm, the given pattern can be recognized so that the best class can be selected. It is considered as a practical application of minimum diver-gence measure principal [22] for pattern recognition.

Case Study Consider a problem having four known patterns P_1, P_2, P_3 and P_4 which have classi cations D_1, D_2, D_3 and D_4 respectively. These are represented by the following fuzzy sets in the universe of discourse K = fk_1, k_2, k_3g.

P_1 = fhk_1, .7i, hk_2, .3i, hk_3, .1ig

P_2 = fhk_1, .4i, hk_2, .2i, hk_3, .5ig

P_3 = fhk_1, .6i, hk_2, .3i, hk_3, .8ig

P_4 = fhk_1, .7i, hk_2, .5i, hk_3, .8ig

Given an unknown pattern Q, represented by the fuzzy set

Q = fhk_1,.5i, hk_2,.4i, hk_3,.9ig

Table 7: Calculated numerical values of $N_t(P_X , Q)$, X = 1, 2, 3, 4 for any t 0

	t=1	t=2	t=10	t=50
$N_t(P_1, Q)$	1.83275	3.16293	315.33686	3117982410207.97
$N_t(P_2, Q)$	.36451	.20263	.03221	4.28174e-06
$N_t(P_3, Q)$	.04307	.00258	1.39698e-10	1.05097e-46
$N_t(P_4, Q)$	.07572	.00474	1.39698e-10	1.05097e-46

Our aim here is to classify Q to one of the classes D_1, D_2, D_3 and D_4. From the formula (2.1) , we can compute the values of generalized fuzzy divergence measure $N_t(P_i, Q)$, i = f1, 2, 3, 4g for any t 0 and are presented in Table

(4) as follows. It is observed that Q can be classi ed to D_3 correctly.

4.3 Medical Diagnosis

The application of fuzzy sets theory has been found in the diagnosis of disease by many researchers. Pattern recognition algorithms is appropriate tool to re-solve medical diagnosis problems for recognizing disease which is a challenging research area according to practical point of view.

Below example elaborate the e ciency of the proposed generalized fuzzy di-vergence measure in resolving medical diagnosis problems, using the algorithm given by Ohlan [17]

Suppose that the universe of discourse X is a set of symptoms

$K = k_1$ (Temperature), k_2 (Headache), k_3 (Stomach Pain), k_4 (Cough), k_5 (Chest Pain)

Consider a set of diagnosis

$B = B_1$ (Viral), B_2 (Malaria), B_3 (Typhoid), B_4 (Stomach Problem), B_5(Chest Problem)

whose elements are presented by the following FSs, respectively,

$B_1 = f<k_1, 0.7>, <k_2, 0.2>, <k_3, 0.0>, <k_4, 0.7>, <k_5, 0.1>g$

$B_2 = f<k_1, 0.4>, <k_2, 0.3>, <k_3, 0.1>, <k_4, 0.4>, <k_5, 0.1>g$

$B_3 = f<k_1, 0.1>, <k_2, 0.2>, <k_3, 0.8>, <k_4, 0.2> <k_5, 0.2>g$

$B_4 = f<k_1, 0.1>, <k_2, 0.0>, <k_3, 0.2>, <k_4, 0.2>, <k_5, 0.8>g$

$B_5 =f<k_1, 0.3>, <k_2, 0.6>, <k_3, 0.2>, <k_4, 0.2> <k_5, 0.1>g$

Table 8: Calculated numerical values of N_t (P, B_i), i = f1, 2, 3, 4g for any t 0

	t=2	t=3	t=10	t=50
N_t(P, B_1)	1	1	1	1
N_t(P,B_2)	.45538	.41498	.52645	2.69467
N_t(P,B_3)	1.37102	1.80589	31.90312	804484348.54885
N_t(P,B_4)	1	1	1	1
N_t(P,B_5)	.28844	.09772	.00014	1.38297e-18

The aim here is to assign a patient

$P = f<k_1, .8>, <k_2, .2>, <k_3, .6>, <k_4, .6>, <k_5, .1>g$, to one of the above mentioned diagnosis B_1, B_2, B_3, B_4 and B_5. We proceed by considering the criteria min N_t(P, B_i) with t 2 N

i

Table (5) presents the values of using N_t(P, B_i), i = f1, 2, 3, 4g for t 2 N using the measure (2.1). It has been observed that the proper diagnosis for patient P is B_5 (Chest problem).

Results and Discussion

In some engineering problems, the decision based on crisp values is not possi-ble. therefore, the use of fuzzy MCDM may be more bene cial. The proposed approach can be used in various decision-making problems, PR and MD. The most of the existing MCDM approaches do not consider the vagueness and uncertainty in the problem and fail to make a good decision. Here MCDM, PR and MD will consider both the gaps, it can be further used in other de-cision making problems like supplier selection, project selection, location, face detection, medical and other such problems.

Conclusion

Same techniques may be adopt for other decision-making, medical problems like supplier selection, project selection, location selection and others more real life problems. Proposed symmetric fuzzy divergence measure with proof of valid-ity and some more e cient properties of this divergence measure also proven. Thus, it is concluded that the proposed divergence measure and method of MCDM, PR and MD require no computation. Finally, we observe that that proposed divergence measure is very appropriate measure to solve real-world problems related to MCDM, PR and MD.

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