When is the Optimal Time to Retire?

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Abstract- This paper investigates the optimal age of retirement by considering many factors. The main factor is the system in which the employee is working. We use the CALPERS (California Public Employees Retirement System) data in our study. The method and mathematical tools we use, however, is not restricted to CALPERS system and can be generalized to any other systems. In addition, we also examine the longevity risk factor that affects the optimal retirement age. Overall, we find the optimal age of retirement is determined by the benefit an employee receives after retirement and the longevity of the employee.

Keywords: CALPERS, Retirement age, benefit factor, longevity

Introduction

As people live longer, sufficient and efficient provisions for retirement are more important. A good retirement plan could save one from outliving his or her retirement wealth, or not able to collect sufficient benefits before death. In this paper, we calculate optimal age of retirement based on mathematical models. We use the data from CALPERS (California Public Employment Retirement System). The results are particularly important to CALPERS employees. While the specific results of this study are geared to CALPERS employees, the method and the mathematical tools can be easily generalized to any retirement system. We first derive the basic equation between retirement age, number of years of service, and benefits which is measured by percentage of gross salary. However, retirement depends not only on the amount one would get after retirement but also on the longevity of the person. Once longevity comes into the picture, risk factor comes into the corresponding mathematical model. To address this issue, we collect information on longevity study of employees. Then we derive a mathematical function between a person's retirement age and the number of years they live after retirement. These functions will then be combined to get an integrated function which yields the optimal age of retirement. On a plot, the optimal age of retirement is the intersection of the two functions. The problem becomes quite complex as the retirement benefits vary on the number of years of service. This research considers all the important and relevant information regarding retirement and longevity of the employee to calculate the optimal age of retirement.

CALPERS provides a table of benefit factors for the CALPERS employees with various years of service. This data is shown for the case of 2% at 55 (CALPERS, 2002). In addition, CALPERS also provides the percentage gross salary versus retirement age. These benefit factors are used to calculate the percentage of gross salary of an employee which is essentially the product of benefit factors and number of years of service. It is to be noted that the benefit factors vary with the age of retirement. The optimal age of retirement is determined by many factors. The main factor is the system in which the employee is working. In addition, there are other factors such as health, psychological and sociological factors (Putcha and Sloboda, 2015; Putcha and Sloboda, 2013). Two approaches can be used to solve the problem of optimal retirement age. Both approaches are discussed and the results are reported in this paper. Overall, we find the optimal age of retirement is determined by the benefit an employee receives after retirement and the longevity of the employee.

2. Data Collection

We obtain our primary data from CALPERS where benefit factors for different years of service are tabulated. CALPERS change retirement plans from time to time. Table 1 is CALPERS retirement plan released in 2002. This is also known as "the 2% at 55 formula" because the benefit factor is 2% when employees retire at age of 55. In addition, CALPERS also provides the percentage gross salary versus retirement age. The data is shown in Table 2.

Table 1: Retirement Age and Benefit Factor (CALPERS, 2002)

Retirement Age	Benefit Factor (%)
50	1.100
51	1.280
52	1.460
53	1.640
54	1.820
55	2.000
56	2.064
57	2.126
58	2.188
59	2.250
60	2.314
61	2.376
62	2.438
63+	2.500

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Figure 1 is a graphic representation of Table 1.

Figure 1: Retirement Age and Benefit Factor (CALPERS, 2002)

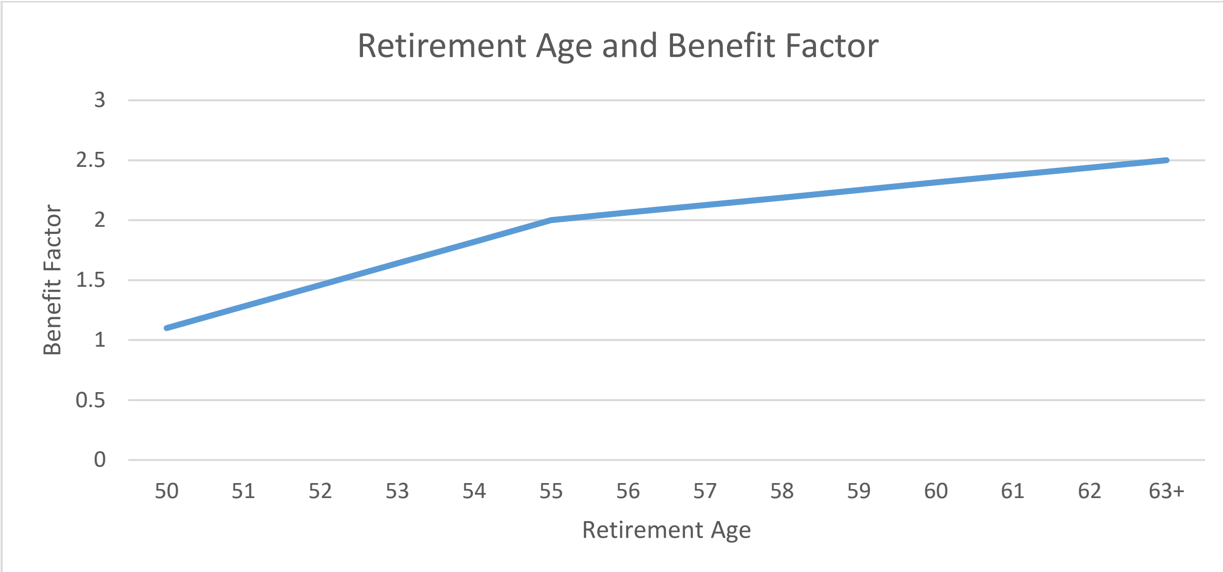

Table 2: Percentage of Gross salary for various years of service and retirement age (CALPERS, 2002)

	Years of service						
	5 years of service	10 years of service	15 years of service	20 years of service	25 years of service	30 years of service	35 years of service
Retirement Age	Percentage of final compensation (%)						
50	5.5	11	16.5	22	27.5	33	
51	6.4	12.8	19.2	25.6	32	38.4	
52	7.3	14.6	21.9	29.2	36.5	43.8	51.1
53	8.2	16.4	24.6	32.8	41	49.2	57.4
54	9.1	18.2	27.3	36.4	45.5	54.6	63.7
55	10	20	30	40	50	60	70
56	10.32	20.64	30.96	41.28	51.6	61.92	72.24
57	10.63	21.26	31.89	42.52	53.15	63.78	74.41
58	10.94	21.88	32.82	43.76	54.7	65.64	76.58
59	11.25	22.5	33.75	45	56.25	67.5	78.75
60	11.57	23.14	34.71	46.28	57.85	69.42	80.99
61	11.88	23.76	35.64	47.52	59.4	71.28	83.16
62	12.19	24.38	36.57	48.76	60.95	73.14	85.33
63+	12.5	25	37.5	50	62.5	75	87.5

3. Methodology

Two approaches can be used to solve the problem of optimal retirement age:

3.1 Approach I

We take the following steps for Approach 1:

1. Table 1 shows the data for retirement age as it varies with benefit factor. This data is taken from CALPERS (2002).
2. Establish a mathematical relation between final compensation and retirement age from 50 to 63+ for 5, 10, 15, 20, 25, 30 and 35 years of service.
3. Plot behavior curve between final compensation and retirement age from 50 to 63+ for 5, 10, 15, 20, 25, 30, and 35 years of service.

   Based on CALPERS data, before age 63, the percent of final compensation depends both on retirement age and number of years of service. For age 63 and beyond, the benefit factor is fixed at 2.5% and the final compensation is only determined by number of years of service and not by retirement age any more.
4. The behavior curve plotted in step 2 is superimposed on the longevity curve (Chung, 2011) which is essentially a behavior curve between employees' retirement age and age of death.
5. A functional relation is then developed between retirement age and age of death.
6. Superimposing the two behavior curves (developed in Step 2 and Step 3) and using the mathematical equations developed in Step 1 and Step 4, we derive local optimal retirement ages for service of 5, 10, 15, 20, 25, 30 and 35 years respectively.
7. We plot number of years of service against the local retirement ages. From this, we then obtain the global optimal retirement age.

We use the benefit factors in Table 1 to calculate percentage of gross salary of an employee which is defined as the product of benefit factors and number of years of service. It is to be noted that the benefit factors vary with retirement age. Fig. 1 plots the relation between benefit factor and retirement age, which is show to be almost linear. The equation for this relation is:

Benefit Factor and Retirement Age: $y = 0.1026x - 3.8277$ (1)

The coefficient of determination (R^2) is 0.9325 and the correlation coefficient (r) is 0.9656.

To check the statistical adequacy of this equation, two parameters are calculated in general. One is r (Correlation coefficient) and the other is $s_{y/x}$ which is standard error of estimate. For a good fit, $r >= 0.8$ and $s_{y/x} <= s_y$ in which $s_{y/x}$ is the standard error of estimate and s_y is the standard deviation of the dependent variable y. These expressions are taken from literature (Ang & Tang, 2007). To get an optimal age of retirement this curve is superimposed with the longevity curve. The longevity curve is shown in Fig. 2 based on the data available in literature (Chung, 2011).

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Figure 2: Longevity Curve (Chung, 2011)

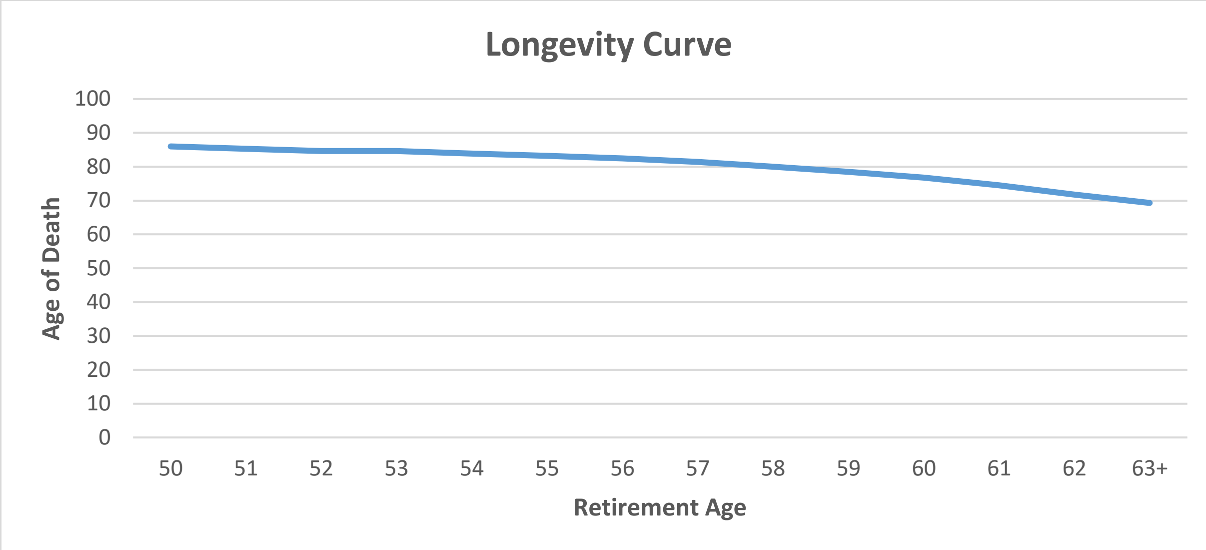
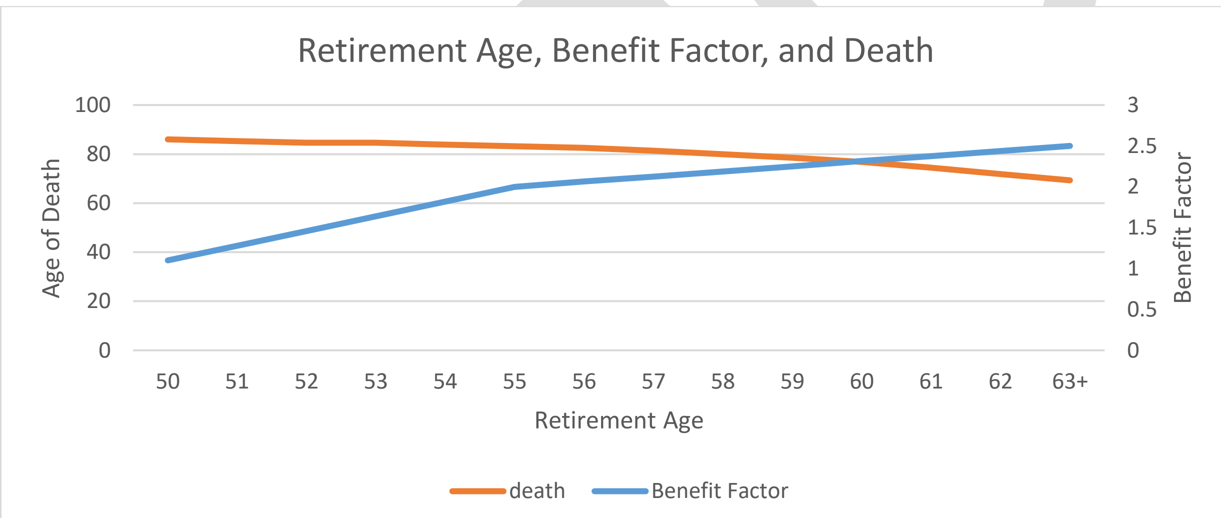


Figure 3 shows combined curve of benefit factor and longevity curve.

Figure 3: Retirement Age, Benefit Factor, and Death



Using the benefit factor information, we plot percentage of gross salary against retirement age for service years of 5, 10, 15, 20, 25, 30, and 35 respectively. These are shown in Figs. 4-10.

A function relation has been developed for each of these plots based on the concept of regression analysis. For a good fit, the correlation coefficient r >= 0.8. The steps outlined in Approach I are implemented and the resulting Figures and Equations are shown below.

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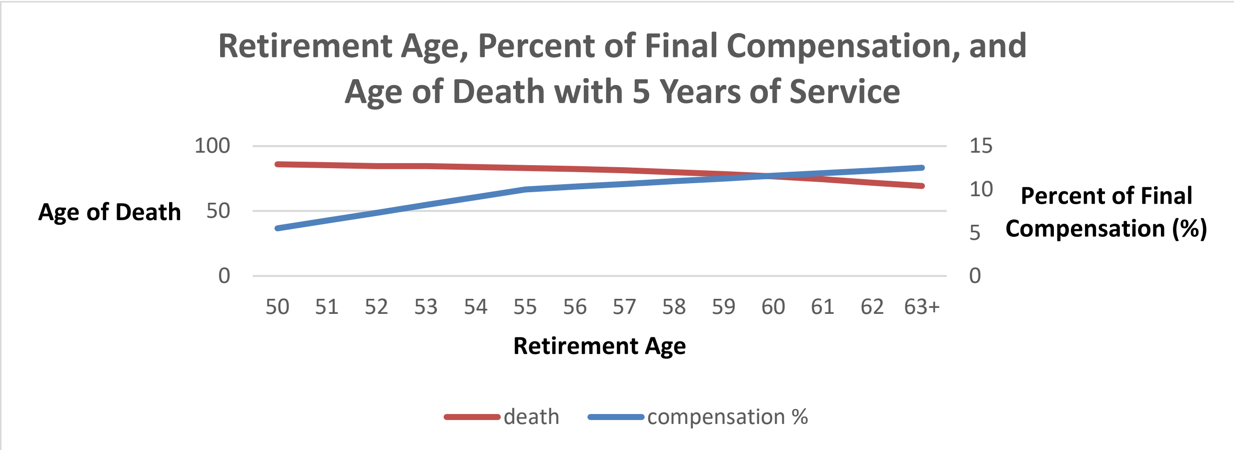
5 years of service:

Figure 4: Retirement Age, Percent of Final Compensation, and Age of Death with 5 year of Service

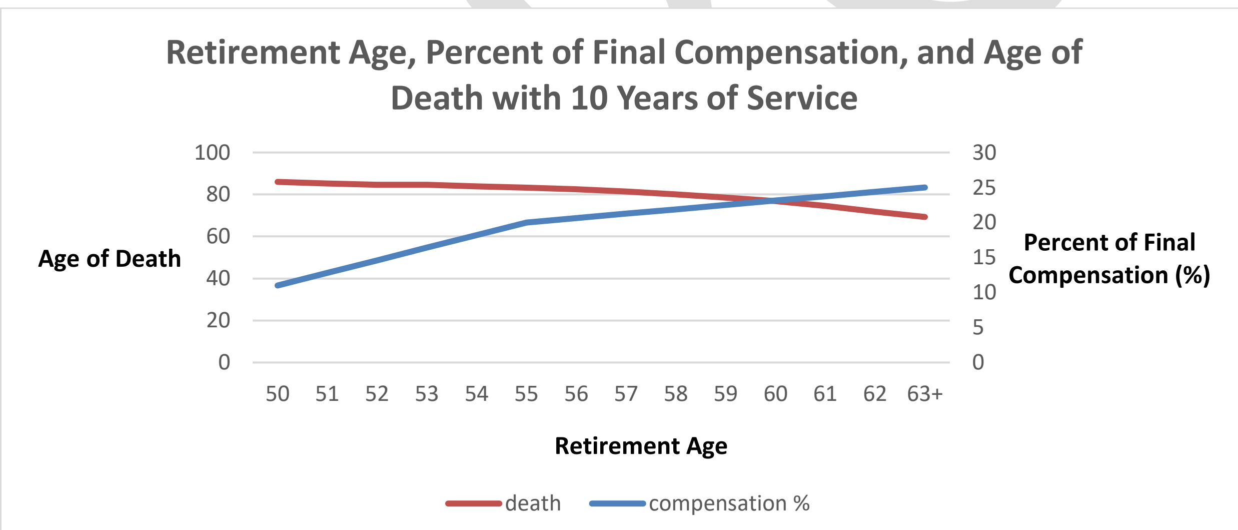


Compensation and retirement age: $y=-20.54+0.54x$ **(2)**

$R^2 = 0.9342$

10 years of service:

Figure 5: Retirement Age, Percent of Final Compensation, and Age of Death with 10 year of Service



Compensation and retirement age: $y=-41.08+1.08x$ **(3)**

$R^2 = 0.9342$

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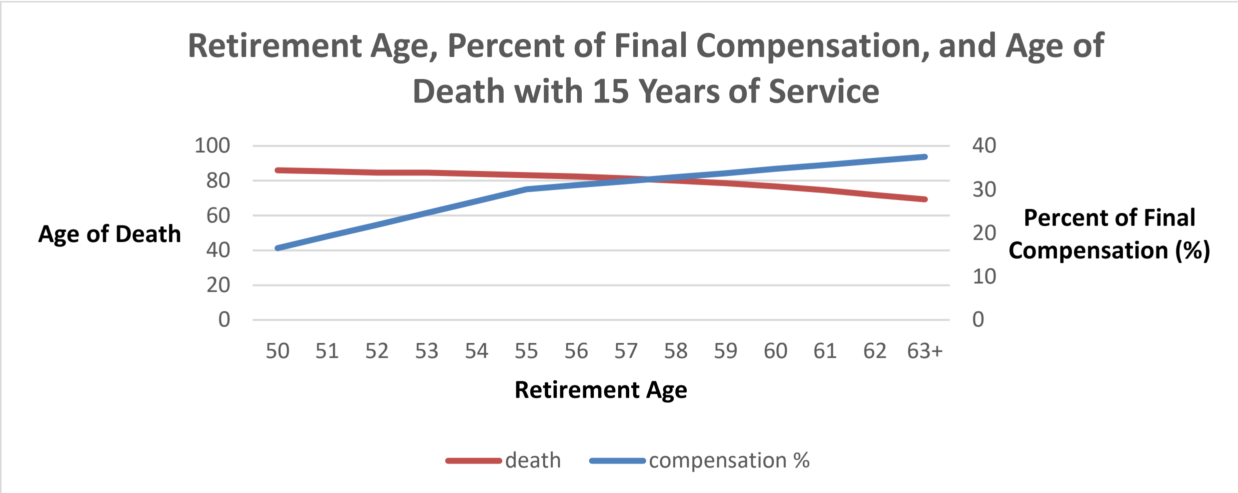
15 years of service:

Figure 6: Retirement Age, Percent of Final Compensation, and Age of Death with 15 year of Service

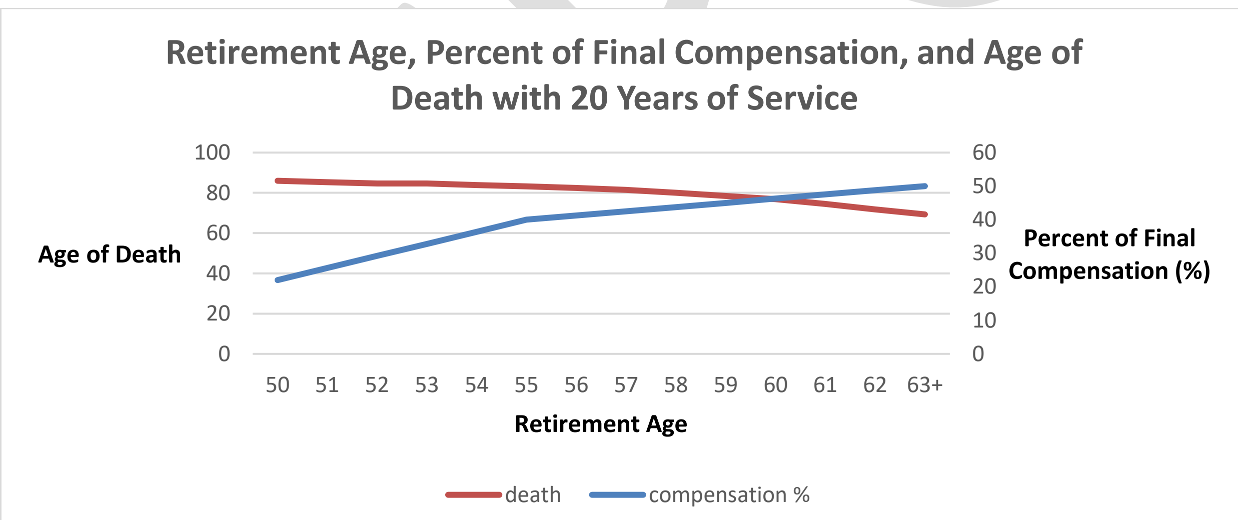


Compensation and retirement age: y=-61.62+1.62x (4)

R^2 = 0.9342

20 years of service:

Figure 7: Retirement Age, Percent of Final Compensation, and Age of Death with 20 year of Service

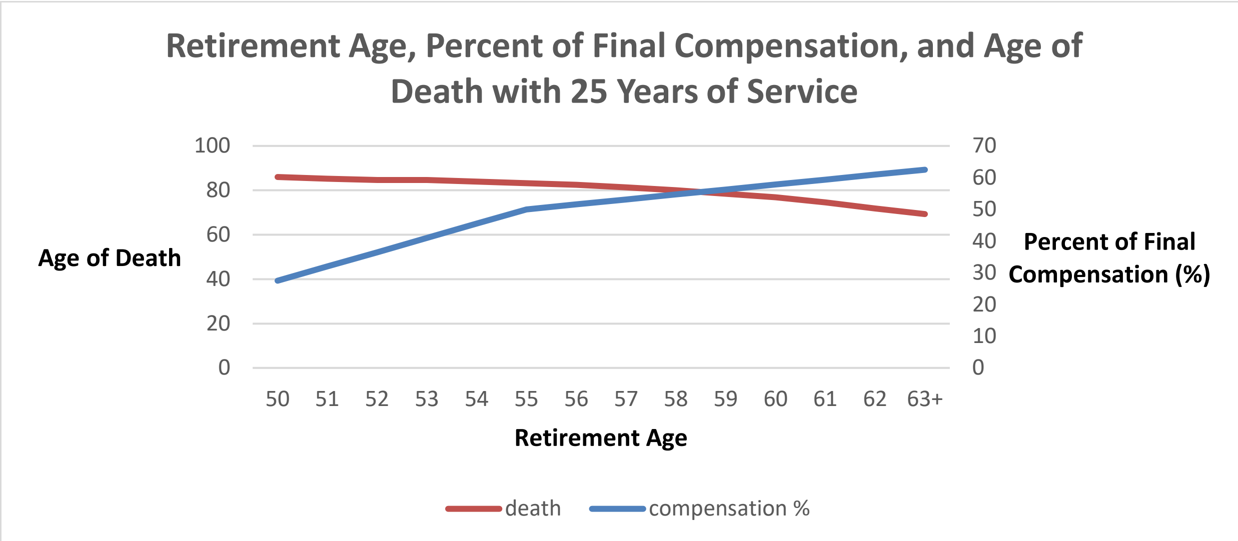


Compensation and retirement age: y=-82.17+2.16x (5)

R^2 = 0.9343

25 years of service:

Figure 8: Retirement Age, Percent of Final Compensation, and Age of Death with 25 year of Service

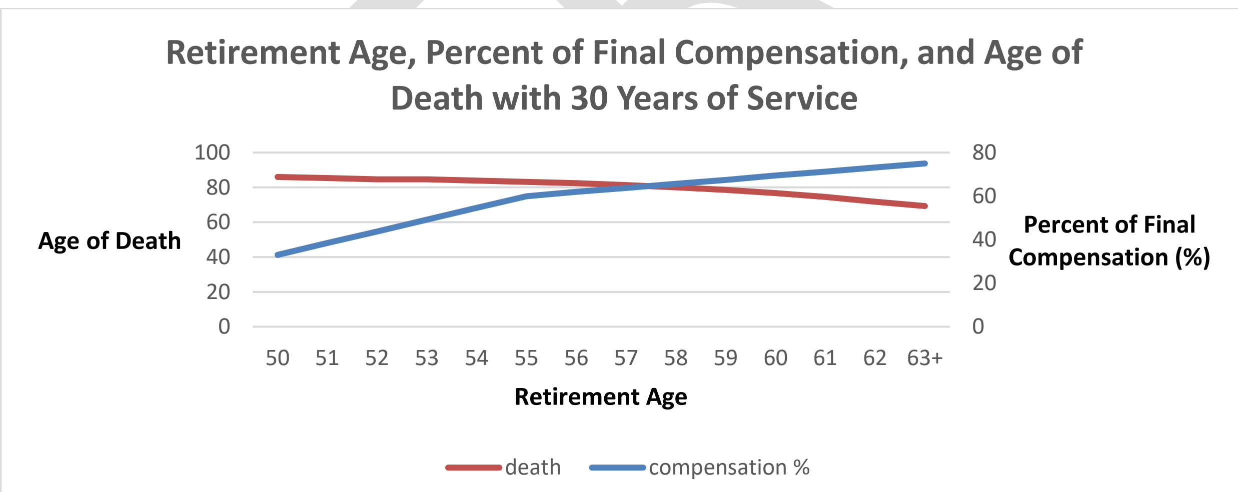


Compensation and retirement age: y=-102.71+2.69x (6)

R^2 = 0.9343

30 years of service:

Figure 9: Retirement Age, Percent of Final Compensation, and Age of Death with 30 year of Service

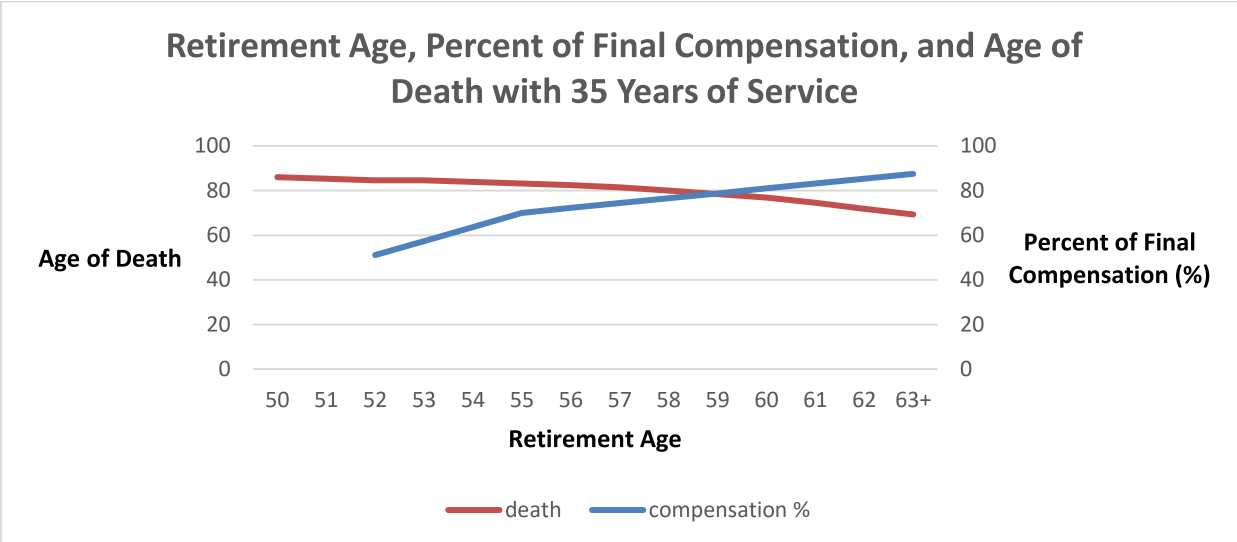


Compensation and retirement age: y=-123.23+3.23x (7)

R^2 = 0.9342

35 years of service:

Figure 10: Retirement Age, Percent of Final Compensation, and Age of Death with 35 year of Service



Compensation and retirement age: $y=-108.13+3.16x$ **(8)**

$R^2 = 0.9342$

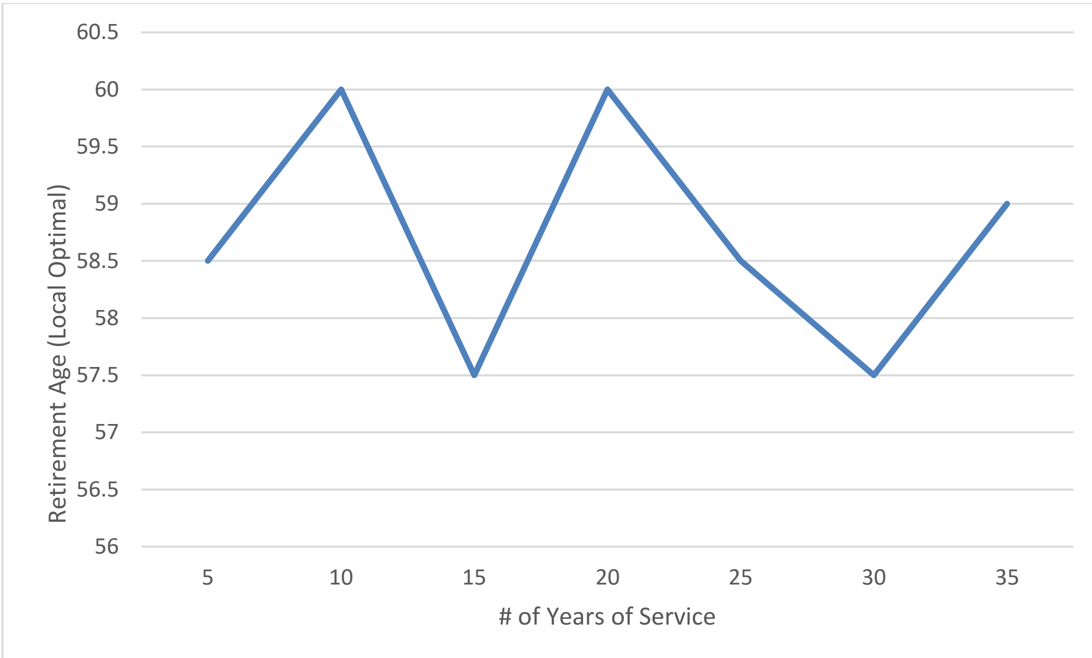
Figures 4 through 10 show that the optimal retirement age varies based on number of years of service. We calculate and tabulate those local optimal retirement age as below in Table 3.

Table 3: Number of Years of Service and Retirement Age (Local Optimal)

# of Years of Service	Retirement Age (Local Optimal)
5	58.5
10	60.0
15	57.5
20	60.0
25	58.5
30	57.5
35	59.0

We then plot the data in Table 3 in Figure 11 below.

Figure 11: Number of Years of Service and Retirement Age (Local Optimal)



We can conclude from Figure 11 that the optimal age of retirement is 60.

3.2 Approach II

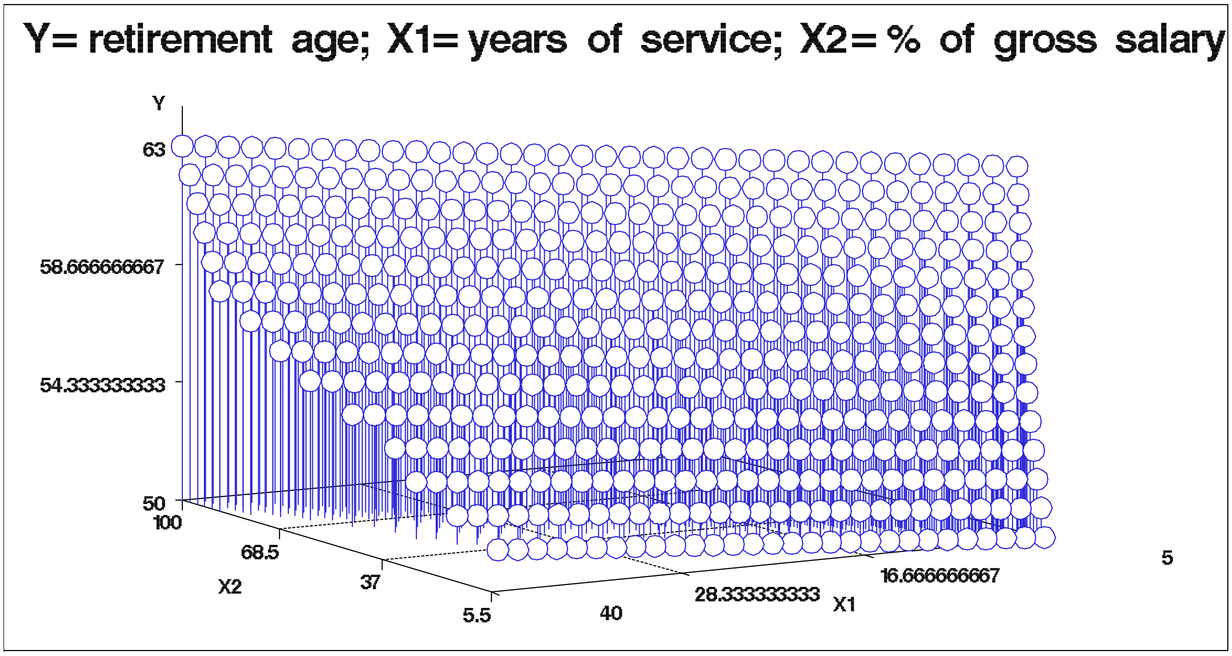
It is noted that Approach I treats data for 5 years of service and 10 years of service separately. In Approach II, both the percentage of gross salary and number of years of service are treated as independent variables (as x_1 and x_2) and the retirement age as a dependent variable (y). We first derive a relation between y, x_1 and x_2 by using the concept of multiple linear regression analysis. The following equation has been developed between y (retirement age), x_1 (years of service) and x_2 (% gross salary).

$$\mathbf{y = 56.8864 - 0.7723\, x_1 + 0.3795\, x_2} \qquad \textbf{(9)}$$

The correlation coefficient for this regression equation is 0.489954.

We then use the principles of calculus to get an optimal value of percentage of gross salary x_1, optimal number of years of service x_2, and the corresponding optimal retirement age. The plot between the variables y, x_1 and x_2 is shown in Fig. 12.

Fig. 12. Variation of retirement age, years of service, and % gross salary



3.3 Discussion of Results

Table 4 presents the correlation coefficients (r) and the standard error of estimates ($s_{y/x}$).

Table 4: Correlation coefficients and standard error of estimates for regression equations developed

Eq. #	r	$S_{y/x}$	S_y
1	0.965636	0.007970438	0.120218933
2	0.965578	0.039852188	0.601094667
3	0.965565	0.079704377	1.202189334
4	0.965513	0.119556565	1.803284453
5	0.965488	0.159408753	2.404378667
6	0.965447	0.199260942	3.005473334
7	0.965422	0.239113134	3.606568001
8	0.965396	0.243456333	3.753223466
9	0.489954	$S_{Y/X1}$=0.214242 $S_{Y/X2}$=0.190994	0.128195765

Table 4 shows that the correlation coefficient for all the regression Equations (Eq. 1-8) is high. In addition, the condition that the standard error of estimate ($s_{y/x}$) is less than s_y is also satisfied. Hence, we can safely state that all the equations adequately represent the data for which these equations are developed.

From Fig. 11 we can see that the optimal retirement age based on Approach I is 60. Similarly, from Fig. 12 that uses multiple linear Regression analysis and Equation 9, we can see that the optimal retirement age is 56.8864 based on Approach II. The results are very close for both approaches.

The slight difference between the two sets of results from Approach I and Approach II is probably result from the fact that the multiple linear regression obtained in Equation 9 doesn't satisfy the adequacy tests for regression analysis. In Approach I the fitted relation for linear regression (Eq. 1-8) used between the dependent variable y and the independent variable x satisfies the adequacy tests, which show very high correlation coefficient r and $s_{y/x}$ is less than s_y. However, in Approach II the multiple linear regression (Equation 9) has a very low correlation coefficient r and $s_{y/x}$ is greater than s_y. It is expected that the results would be closer if a better fit for multiple linear regression is used.

4. Conclusions

Hence, our study shows that the optimal age of retirement is 60 based on CSU retirement system. It is to be noted that a mathematical approach developed to obtain retirement age based on certain given specifications (CALPERS data in this case). But our approach is very general and can be used for any practical system. Therefore, our research has lot of practical applications and is useful to both academia and general public.

Additional mandatory information

A. Compliance with Ethical Standards

There is no potential conflicts of interest related to my paper.

This research does not involve any human/animal participants.

We give consent to this Journal to publish my paper.

B. Funding

No funding was received for this research work.

C. Conflict of Interest

There is no conflict of interest for any author.

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REFERENCES:

1. Ang, A.H-S and Tang, W. (2007). Probability Concepts in Engineering. John Wiliey & Sons.
2. Chung, E (2011). Actuarial Study of life span vs. retirement age.
3. http://faculty.kfupm.edu.sa/coe/gutu...sc/Retire1.htm
4. CALPERS (1981). 2% at 55 Formula. Published by CALPERS.
5. Putcha, C.S. and Sloboda, B. (2015). A New Approach for a Forecasting Model in the Estimation of Social Security Benefits", (with B. Sloboda). 35th International Symposium on Forecasting, Riverside, CA, June 21-24.
6. Putcha, C.S. and Sloboda, B (2013). Predicting Unemployment Rate in The United States Since 2001. Chandrasekhar Putcha, Brian Sloboda and Kalamogo Coulibaly (Editors), Edwin Mellen Press.