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ONE-DIMENSIONAL PROBLEM OF RHEOLOGICAL LAW OF MOLECULAR AND MOLAR TRANSFER IN FLUIDS

Abstract: *The fluid flow is considered in the paper according to the rheological law of molecular and molar transfer in a flow. The obtained differential equation of the third order was solved analytically for the one-dimensional problem of fluid flow in a round pipe. The flow pattern obtained for the selected model according to the analytical solution was given.*

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Introduction

The fluid flow in various canals and pipelines has been sufficiently well studied with account for molecular transfer for layered flows in the framework of the Navier-Stokes equations. Numerous publications [1, 2, 3, 4, 5, 6, 7] were devoted to the solution of this equation. However, the improvement of the methods for studying flows under various conditions made it possible to identify a number of hydrodynamic features (for example, in velocity diagram) that cannot be explained by the Navier-Stokes equations.

Assuming that the fluid is a thermodynamic system, and its source of flow is the Gibbs free energy, more general equations of fluid motion were obtained when compared to theNavier-Stokes' ones [8]. In this paper, to take into account the group transfers of molecules in the flow, the stress is taken in direct proportion to the derivative of fluid acceleration. In a concurrent consideration of the mechanisms of individual molecules and their groupstransfer using the Navier-Stokes differential equations, the terms with a third-order derivative were formed.The stationary problem of fluid flow in a flat canal was solved for this model [9] using operational calculus.

Statement of problem. Consider the fluid flow in a cylindrical tube. According to the rheological law of molecular and molar transfer in a fluid

$$\tau = \mu \frac{\partial u}{\partial n} + m_l \frac{\partial w}{\partial n}, \qquad (1)$$

or in a component form:

$$\tau = \begin{cases} \mu\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) + m_l\left(\frac{\partial w_i}{\partial x_j} + \frac{\partial w_j}{\partial x_i}\right) & j \neq i, \\ -p + 2\mu\frac{\partial v_i}{\partial x_i} + m_l\frac{\partial w_i}{\partial x_i} & j = i \quad (\mathrm{i, j} = 1, 2, 3) \end{cases} \qquad (2)$$

the system of equations of fluid motion in cylindrical coordinate systems takes the following form:

$$\begin{cases} \dfrac{\partial v_1}{\partial t} = -\dfrac{dp}{dx_1} + \mu\left(\dfrac{\partial^2 v_1}{\partial x_2^2} + \dfrac{1}{x_2}\dfrac{\partial v_1}{\partial x_2}\right) + m_l\left[\dfrac{\partial^3 v_1}{\partial t \partial x_2^2} + \dfrac{1}{x_2}\dfrac{\partial^2 v_1}{\partial t \partial x_2} + \right. \\ \left. + v_1\left(\dfrac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + \dfrac{1}{x_2}\dfrac{\partial^2 v_1}{\partial x_1 \partial x_2}\right) + v_2\left(\dfrac{\partial^3 v_1}{\partial x_2^3} + \dfrac{1}{x_2}\dfrac{\partial^2 v_1}{\partial x_2^2}\right) + \dfrac{\partial v_1}{\partial x_2}\dfrac{\partial^2 v_1}{\partial x_1 \partial x_2} + \dfrac{\partial v_2}{\partial x_2}\dfrac{\partial^2 v_1}{\partial x_2^2}\right], \\ \dfrac{\partial v_1}{\partial x_1} + \dfrac{1}{x_2}\dfrac{\partial (x_2 v_2)}{\partial x_2} = 0. \end{cases} \tag{3}$$

A model one-dimensional problem is formulated.

The equations of motion under the assumptions $v_1 = v_1(x_2, t), \quad v_2 = const,$ $dp/dx_1 = N = const$, are reduced to the form:

$$m_l\left(\frac{\partial^3 v_1}{\partial t \partial x_2^2} + \frac{1}{x_2}\frac{\partial^2 v_1}{\partial t \partial x_2}\right) + m_l v_2\left(\frac{\partial^3 v_1}{\partial x_2^3} + \frac{1}{x_2}\frac{\partial^2 v_1}{\partial x_2^2}\right) + \mu\left(\frac{\partial^2 v_1}{\partial x_2^2} + \frac{1}{x_2}\frac{\partial v_1}{\partial x_2}\right) = N. \tag{4}$$

Equation (4) is solved under the following initial and boundary conditions

$$\left.\begin{array}{ll} v_1 = 0, \quad \dfrac{\partial v_1}{\partial x_2} = 0, \quad \dfrac{\partial^2 v_1}{\partial x_2^2} = 0 & t = 0, \\ \dfrac{\partial v_1}{\partial x_2} = 0, \quad v_1 < \infty & x_1 = 0, \\ v_1 = 0 \quad x_1 = R. & \end{array}\right\} \tag{5}$$

The boundary conditions of equation (4) are the cohesion conditions and axial symmetry. At initial time, the fluid in the infinitely long round pipe is at rest, and at time $t = 0$, a pressure drop dp/dx_1 occurs, that later remains constant in time.

To solve the posed problem, modifying (4) we obtain the equation convenient for integration

$$m_l \frac{1}{x_2}\frac{\partial}{\partial x_2}\left(x_2 \frac{\partial^2 v_1}{\partial t \partial x_2}\right) + m_l v_2 \frac{1}{x_2}\frac{\partial}{\partial x_2}\left(x_2 \frac{\partial^2 v_1}{\partial x_2^2}\right) + \mu \frac{1}{x_2}\frac{\partial}{\partial x_2}\left(x_2 \frac{\partial v_1}{\partial x_2}\right) = N. \tag{6}$$

Multiplying both sides of this equation by x_2 and integrating the obtained valuesby x_2 , we have:

$$m_l x_2 \frac{\partial^2 v_1}{\partial t \partial x_2} + m_l v_2 x_2 \frac{\partial^2 v_1}{\partial x_2^2} + \mu x_2 \frac{\partial v_1}{\partial x_2} = \frac{N x_2^2}{2} + c_1. \tag{7}$$

Applying this procedure for the second time, we arrive at the equation

$$\frac{\partial v_1}{\partial x_2}+\frac{1}{x_2}\frac{\partial v_1}{\partial t}+\frac{\mu}{m_l v_2}v_1=\frac{Nx_2^2}{4m_l v_2}+\frac{c_1}{m_l v_2}\ln x_2+\frac{c_2}{m_l v_2}. \tag{8}$$

After Laplace transform with respect to *t*, the sequential equation is written in the form

$$\frac{\partial \overline{v_1}}{\partial x_2}+\left(\frac{s}{v_2}+\frac{\mu}{m_l v_2}\right)\overline{v_1}=\frac{Nx_2^2}{4sm_l v_2}+\frac{c_1\ln x_2}{sm_l v_2}+\frac{c_2}{sm_l v_2}, \tag{9}$$

wheresis the transform parameter.

The conditions for $t=0$ from (5) were taken into account in passing to the images.

The obtained inhomogeneous equation (9) was solved by the method of variation of constants. The homogeneous part of the equation has the following solution

$$\overline{v_1}=c\ e^{-bx_2},$$

where $b=\frac{s}{v_2}+\frac{\mu}{m_l v_2}$.

If to consider *c* not as an arbitrary constant, but as some function of x_2, i.e. $c=c(x_2)$, then we can choose the function $c(x_2)$ so that function (8) becomes a solution to the inhomogeneous equation (9).

To find the function $c(x_2)$, we calculate the derivative of function $\overline{v_1}=c(x_2)e^{-bx_2}$, substitute the expressions $\overline{v_1}$ and $d\overline{v_1}/d x_2$ into equation (9) and require that it be satisfied identically. Since

$$\frac{d\overline{v_1}}{dx_2}=\frac{dc(x_2)}{dx_2}e^{-bx_2}-c(x_2)be^{-bx_2}, \tag{10}$$

then equation (9) goes over to equation

$$\frac{dc(x_2)}{dx_2}e^{-bx_2}=\frac{Nx_2}{4m_l v_2 s}+\frac{c_1\ln x_2}{m_l v_2 s}+\frac{c_2}{m_l v_2 s}. \tag{11}$$

The latter is an equation with separable variables and has a general solution

$$c(x_2)=\frac{N}{4m_l v_2 s}e^{bx_2}\left(\frac{x_2^2}{b}-\frac{2x_2}{b^2}-\frac{2}{b^3}\right)+\frac{c_1}{m_l v_2 s}\left[\frac{e^{bx_2}\ln x_2}{b}-\left(\ln x_2+\sum_{n=1}^{\infty}\frac{(bx_2)^n}{n\cdot n!}\right)\right]. \tag{12}$$

Substituting the found expression $c(x_2)$ into equality (8), we obtain the sought for solution to the inhomogeneous equation (9) in the form:

$$\begin{aligned}\overline{v_1}(x_2)&=\frac{N}{4sm_l v_2}\left(\frac{x_2^2}{b}-\frac{2x_2}{b^2}-\frac{2}{b^3}\right)+\\&+\frac{c_1}{m_l v_2 s}\left[\left(\frac{1}{b}-e^{-bx_2}\right)\ln x_2-e^{-bx_2}\sum_{n=1}^{\infty}\frac{(bx_2)^n}{n\cdot n!}\right]+\frac{c_2}{m_l v_2 bs}+\frac{c_3}{m_l v_2 s}e^{-bx_2}.\end{aligned} \tag{13}$$

To determine the integration constants c_1, c_2, c_3, we use the boundary conditions from (5). Sincefor $x_2\to\infty$ the following is appropriate

$$\frac{c_1}{m_l v_2 s}\left[\left(\frac{1}{b}-e^{-bx_2}\right)\ln x_2 - e^{-bx_2}\sum_{n=1}^{\infty}\frac{(bx_2)^n}{n\cdot n!}\right]\to\infty,$$

then from the boundedness condition of axial velocity $c_1 = 0$ is determined. From the conditionof cohesion, i.e. $\overline{v}_1 = 0$, at $x_2 = R$ the following relation is obtained

$$c_2 = \frac{Nb}{4}-\left(\frac{R^2}{b}-\frac{2R}{b^2}-\frac{2}{b^3}\right)-c_3 b e^{-bR}. \quad (14)$$

The value of the constant c_3 is determined from the condition of axial symmetry, i.e. $d\overline{v}_1/d\,x_2 = 0$ at $x_2 = 0$:

$$c_3 = -\frac{N}{2b^3}.$$

Substituting the value of c_3 into (14), c_2 is determined in the final form

$$c_2 = -\frac{N}{4}\left(R^2-\frac{2R}{b}-\frac{2}{b^2}\right)+\frac{N}{2b^2}e^{-bR}.$$

After simple modifications, we obtain an expression for the velocity in the images

$$\overline{v}_1(x_2) = \frac{N}{4m_l v_2}\left[\left(x_2^2-R\right)\frac{v_2}{s(s+\mu/m_l)}-2(x_2-R)\frac{v_2^2}{s(s+\mu/m_l)^2}+\right.$$
$$\left.+2\frac{v_2^3}{s(s+\mu/m_l)^3}\left(e^{-\left(\frac{s}{v_2}+\frac{\mu}{m_l v_2}\right)R}-e^{-\left(\frac{s}{v_2}+\frac{\mu}{m_l v_2}\right)x_2}\right)\right]. \quad (15)$$

In (15)we turn now to the original. From the table of originals and images [10] for the first two terms we have

$$\frac{1}{s\left(s+\frac{\mu}{m_l}\right)} \doteq \frac{m_l}{\mu}\left(1-e^{-\frac{\mu}{m_l}t}\right),$$
$$\frac{1}{s\left(s+\frac{\mu}{m_l}\right)} \doteq \left(\frac{m_l}{\mu}\right)^2\left(1-e^{-\frac{\mu}{m_l}t}-\frac{\mu}{m_l}te^{\frac{\mu}{m_l}}\right). \quad (16)$$

For the last term of equation (15) the Duhamel integral [10] is used:

$$sG(s)F(s) \doteq f(0)g(t)+\int_0^t f'(\tau)g(t-\tau)d\tau, \quad (17)$$

in this case

$$F(s) = \frac{e^{-\frac{s}{v_2}x_2}}{s^2} \doteq \begin{Bmatrix} 0 & 0<t<\frac{x_2}{v_2} \\ t-\frac{x_2}{v_2} & t>\frac{x_2}{v_2}\end{Bmatrix} = f(t),$$
$$G(s) = \frac{1}{\left(s+\frac{\mu}{m_l}\right)^3} \doteq \frac{t^2}{2}e^{-\frac{\mu}{m_l}t} = g(t),$$
$$f(0) = \frac{x_2}{v_2}, f'(\tau) = 1. \quad (18)$$

We substitute (18) into (17) and perform the integration. Considering the obtained formulas and (16), we have

$$v_1(t,x_2)=\frac{dp}{dx_1}\frac{1}{4\mu}\left\{\left(x_2^2-R\right)\left(1-e^{-\frac{\mu}{m_l}t}\right)-\right.$$
$$-2\left(x_2-R\right)v_2\frac{m_l}{\mu}\left(1-e^{-\frac{\mu}{m_l}t}-t\frac{\mu}{m_l}e^{-\frac{\mu}{m_l}t}\right)+$$
$$+2v_2^2e^{-\frac{\mu x_2}{m_l v_2}}\left[e^{-\frac{\mu}{m_l}t}\left(\frac{t^2}{2}+\frac{t^2}{2}\frac{\mu x_2}{m_l v_2}+t\frac{m_l}{\mu}+\frac{m_l^2}{\mu^2}\right)-\frac{m_l^2}{\mu^2}\right]- \tag{19}$$
$$\left.-2v_2^2e^{-\frac{\mu x_2}{m_l v_2}}\left[e^{-\frac{\mu}{m_l}t}\left(\frac{t^2}{2}+\frac{t^2}{2}\frac{\mu R}{m_l v_2}+t\frac{m_l}{\mu}+\frac{m_l^2}{\mu^2}\right)-\frac{m_l^2}{\mu^2}\right]\right\}.$$

Discussion of obtained solution (19). Function (19) is a general solution of equation (4) and describes the flow rate distribution in a round pipe. Here the expressions in square brackets for $t>\frac{x_2}{v_2}$ are equal to zero, and as $t\to\infty$ the expression for the stationary distribution of the fluid velocity in the pipe is:

$$v_1=\frac{dp}{dx_1}\frac{1}{4\mu}\left[\left(x_2^2-R\right)-2\left(x_2-R\right)v_2\frac{m_l}{\mu}+2v_2^2\frac{m_l^2}{\mu^2}\left(e^{-\frac{\mu R}{m_l v_2}}-e^{-\frac{\mu x_2}{m_l v_2}}\right)\right]. \tag{20}$$

In the absence of molar transfer in motion, i.e. for $m_l v_2=0$, it is possible to derive a formula from (20) for the velocity distribution of a viscous fluid in a pipe [11].

Results of computational experiments. Analysis of numerical calculations carried out for the stationary case based on the obtained analytical solution of the model problem of fluid flow in a round tubeaccording to the selected rheological law showed that a set of parameters $a_1=m_l v_2/\mu$ plays a characterizing role in the flow motion.

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Figure 1 shows the velocity profiles in dimensionless coordinates for various values of a_1.

Curve 1 corresponds to a zero valueof a_1, i.e. to the velocity distribution of viscous Newtonian fluid (the Poiseuille solution).

Curves 2-5 are obtained at valuesof $a_1 =$ 0.36; 0.73; 1.45; 2.18, respectively.

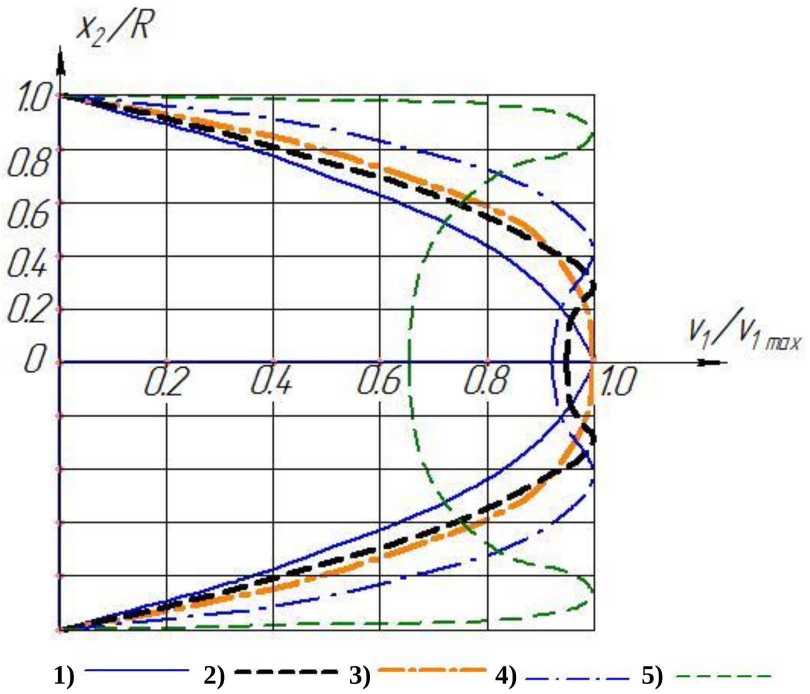

Fig. 1.Velocity distribution.

As follows from the presented velocity distribution curves, as the molar transfer coefficient of the momentum increases, the region of maximum velocities moves from the middle of the pipe to the peripheral region. Curve 2 corresponds to the fluid flow with the formation of a flow core. This nature of the flow is manifested at values of a_1 in the range from 0.01 to 0.6. Starting from $a_1 > 0{,}6$, the velocity along the flow axis decreases, the region of maximum velocity moves toward the wall. The decrease in velocity along the flow axis, according to the author, is associated with an increase in the molar force of internal friction.

Conclusions

Thus, the calculations based on equation (20) showed:

• the consistency of the selected rheological model for describing the distribution of flow velocity;

• the consistency of the model for determining other hydrodynamic flow parameters depending on the physicomechanical properties of fluids.

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