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INVESTIGATION OF A BOUNDARY-VALUE PROBLEM FOR A THIRD ORDER PARABOLIC HYPERBOLIC EQUATION IN THE FORM $\left(b\frac{\partial}{\partial y}+\mathrm{c}\right)(Lu)=0$

Abstract: *In the present paper in a pentagonal domain a boundary-value problem was set and investigated for a third order parabolic hyperbolic equation in the form* $\left(b\frac{\partial}{\partial y}+\mathrm{c}\right)(Lu)=0$ *. Unique solvability of the considered problem was proved by the method of construct solution and also by methods of integral equations and differential equations, method of continuity.*

Key words: *Differential equation, method of constructing solutions, method of continuity, boundary-value problem, parabolic hyperbolic type, unique solvability, pentagonal domain.*

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Introduction

Currently, research is actively developing non-classical equations of mathematical physics, in particular, equations of mixed, composite and mixed-composite types. One of the main reasons for this process is the appearance of applied applications of boundary value problems posed for such equations.

It is known that mixed equations of the second order of an elliptic-hyperbolic type were originally studied. Fundamental research on such equations was started in the 1920s by the Italian mathematician Tricomi [1] and was developed by Gellerstedt [2], A.V. Bitsadze [3], K.I. Babenko [4], I.L. Karol [5] , F.I. Frankl [6], M.M.Smirnov [7], M.S. Salakhitdinov [8], etc.

The main part

Studies of the equations of elliptic-parabolic and parabolic-hyperbolic types of the second order began in the 50-60s of the last century. In 1959, I.M. Gelfand [9] pointed out the need for a joint consideration of equations in one part of the parabolic region and the other part of the hyperbolic region. He gives an example related to the motion of a gas in a channel surrounded by a porous medium: in a channel, the gas motion is described by the wave equation, outside it by the diffusion equation. Then, in the 70-80s of the twentieth century, research began on the equations of the third and high orders of the parabolic-hyperbolic type. Boundary-value problems for such equations were posed and studied for the first time by T.D.Dzhuraev [10] and his students [11], [12], [18].

Over the past time, studies on boundary value problems for equations of the third and higher orders of parabolic-hyperbolic type have developed in a broad sense, and are currently expanding in the directions of complication of equations and their areas of consideration, as well as generalizations of the equations problems considered for them (for example, see [15], [16], [17] and others)

In the present work a boundary-value problem will be set for a third order parabolic-hyberboloc equation

$$\left(b\frac{\partial}{\partial y}+c\right)(Lu)=0 \qquad (1)$$

in a pentagonal domain G of the plane xOy, where $b,c \in R$, $G = G_1 \cup G_2 \cup G_3 \cup J_1 \cup J_2$,

$G_1 = \{(x,y) \in R^2 : 0 < x < 1,\ 0 < y < 1\}$,

$G_2 = \{(x,y) \in R^2 : -1 < y < 0,\ -1 - y < x < y + 1\}$

$G_3 = \{(x,y) \in R^2 : -1 < x < 0,\ 0 < y < 1\}$,

$J_1 = \{(x,y) \in R^2 : y = 0,\ 0 < x < 1\}$,

$J_2 = \{(x,y) \in R^2 : y = 0,\ -1 < x < 1\}$

$$Lu = \begin{cases} u_{xx} - u_y, & (x,y) \in G_1, \\ u_{xx} - u_{yy}, & (x,y) \in G_i \quad (i = 2,3), \end{cases}$$

We will study the following problem for the equation (1):

Problem 1. Find a function $u(x,y)$, with properties: 1) continuous in the closed domain $\overline{D}$ and in the domain $G \setminus J_1 \setminus J_2$ has continuous derivatives which is participating in equation (1), here u_x and u_y are continuous in G up to the bound of the domain G, which is shown in boundary condition; 2) satisfies equation (1) in the domain $G \setminus J_1 \setminus J_2$; 3) satisfies the following boundary - value conditions:

$$u(1,y) = \varphi_1(y), \quad 0 \le y \le 1; \tag{2}$$

$$u(-1,y) = \varphi_2(y), \quad 0 \le y \le 1; \tag{3}$$

$$u\big|_{BC} = \psi_1(x), \quad 0 \le x \le 1; \tag{4}$$

$$u\big|_{DF} = \psi_2(x), \quad -1 \le x \le -1/2; \tag{5}$$

$$\left.\frac{\partial u}{\partial n}\right|_{BC} = \psi_3(x), \quad 0 \le x \le 1; \tag{6}$$

$$\left.\frac{\partial u}{\partial n}\right|_{DC} = \psi_4(x), \quad -1 \le x \le 0; \tag{7}$$

4) satisfies the following glying conditions on the line of type changing:

$$u(x,+0) = u(x,-0) = T(x), \quad -1 \le x \le 1; \tag{8}$$

$$u_y(x,+0) = u_y(x,-0) = N(x), \quad -1 \le x \le 1; \tag{9}$$

$$u_{yy}(x,+0) = u_{yy}(x,-0) = M(x), \quad -1 < x < 1; \tag{10}$$

$$u(+0,y) = u(-0,y) = \tau_3(y), \quad 0 \le y \le 1; \tag{11}$$

$$u_x(+0,y) = u_x(-0,y) = \nu_3(y), \quad 0 \le y \le 1. \tag{12}$$

where φ_i and $\psi_j\left(i = \overline{1,2}; j = \overline{1,4}\right)$ are given sufficiently smooth functions, $\tau_i,\ \nu_i (i = 1,2,3), \mu_1, \mu_2$ are temporarily unknown but smoothly functions, $n -$ is internal normal of $x + y = -1$ (DC) or $x - y = 1$ (BC), $F(-1/2,-1/2)$. Together with the introduced notations (8) - (12) the following notation are used as well:

$$T(x) = \begin{cases} \tau_1(x), & if\ 0 \le x \le 1, \\ \tau_2(x), & if\ -1 \le x \le 0; \end{cases}$$

$$N(x) = \begin{cases} \nu_1(x), & if\ 0 \le x \le 1, \\ \nu_2(x), & if\ -1 \le x \le 0; \end{cases}$$

$$M(x) = \begin{cases} \mu_1(x), & if\ 0 < x < 1, \\ \mu_2(x), & if\ -1 < x < 0, \end{cases}$$

$u(x,y) = u_j(x,y),\ (x,y) \in G_j \quad (j = \overline{1,3})$, where it is assumed that $\tau_1(0) = \tau_2(0)$, $\nu_1(0) = \nu_2(0)$.

Theorem. If $\varphi_1, \varphi_2 \in C^3[0,1]$, $\psi_1 \in C^3[0,1]$, $\psi_2 \in C^3[-1,-1/2]$, $\psi_3 \in C^2[0,1]$, $\psi_4 \in C^2[-1,0]$, and the agreeing conditions $\varphi_1(0) = \psi_1(1)$, $\psi_2(-1) = \varphi_2(0)$, $\psi_4'(0) = \psi_3'(0)$ are fulfilled then the problem 1 will have unique solution. For this aim based on introduced.

Proof. We will prove the theorem by the method of construct solution. notations, we will rewrite equation (1) as

$$u_{1xx} - u_{1y} = \omega_1(x) e^{-\frac{c}{b}y}, \quad (x,y) \in G_1; \tag{13}$$

$$u_{ixx} - u_{iyy} = \omega_i(x) e^{-\frac{c}{b}y}, \quad (x,y) \in G_i \quad (i = 2,3), \tag{14}$$

where $\omega_i(x)$ $\left(i = \overline{1,3}\right)$ are unknown and should be defined functions but we will assume that they are sufficient smooth functions.

Firstly we will carry on investigation in the domain G_2. A solution of the equation (14) $(\text{for } i = 2)$, satisfying conditions (8), (9), is represented in the form

$$u_2(x,y) = \frac{1}{2}\left[T(x+y) + T(x-y)\right] + \frac{1}{2}\int_{x-y}^{x+y} N(t)\,dt - \frac{1}{2}\int_0^y e^{-\frac{c}{b}\eta}\,d\eta \int_{x-y+\eta}^{x+y-\eta} \omega_2(\xi)\,d\xi \tag{15}$$

Substituting (15) into conditions (6) and (7), after some computations we get

$$\omega_2(x) = -\sqrt{2}\,\psi_3'(x) e^{\frac{c}{b}(x-1)}, \quad 0 \le x \le 1, \tag{16}$$

$$\omega_2(x)=\sqrt{2}\psi_4'(x)e^{-\frac{c}{b}(x+1)},\ -1\le x\le 0. \qquad (17)$$

Substituting (15) into condition making some transformations (4), we get

$$T'(x)+N(x)=\alpha_1(x),\ -1\le x\le 1, \qquad (18)$$

where $\alpha_1(x)=\psi_1'\left(\frac{x+1}{2}\right)-\int\limits_0^{(x-1)/2} e^{-\frac{c}{b}\eta}\omega_2(x-\eta)\,d\eta$.

In case $-1\le x\le 0$ equation (18) has the form

$$\tau_2'(x)+\nu_2(x)=\alpha_1(x),\ -1\le x\le 0. \qquad (19)$$

Further, substituting (15) in the condition (5), after some computations, we come

$$\tau_2'(x)-\nu_2(x)=\delta_1(x),\ -1\le x\le 0, \qquad (20)$$

where

$$\delta_2(x)=\psi_2'\left(\frac{x-1}{2}\right)-\int\limits_0^{-(x+1)/2} e^{-\frac{c}{b}\eta}\omega_2(x+\eta)\,d\eta.$$

Form (19) and (20) we will find functions $\tau_2'(x)$ and $\nu_2(x)$ as follows

$$\tau_2'(x)=\frac{1}{2}\left[\alpha_1(x)+\delta_1(x)\right],$$

$$\nu_2(x)=\frac{1}{2}\left[\alpha_1(x)-\delta_1(x)\right]. \qquad (21)$$

Integrating the first equality of (21) from -1 to x, we obtain

$$\tau_2(x)=\frac{1}{2}\int\limits_{-1}^{x}\left[\delta_1(t)+\delta_2(t)\right]dt+\psi_2(-1).$$

In case $0\le x\le 1$ equation (18) has the form

$$\tau_1'(x)+\nu_1(x)=\alpha_1(x),\ 0\le x\le 1. \qquad (22)$$

Now, in the domain G_1 we rewrite equation (1) in the form

$$bu_{1xxy}-bu_{1yy}+cu_{1xx}-cu_{1y}=0.$$

Passing to the limit in the last and equation (22) $(i=2)$ for $y\to 0$, we get

$$b\nu_1''(x)-b\mu_1(x)+c\tau_1''(x)-c\nu_1(x)=0,\ 0\le x\le 1, \qquad (23)$$

$$\mu_1(x)=\tau_1''(x)-\omega_2(x). \qquad (24)$$

Eliminating functions $\nu_1(x)$ and $\mu_1(x)$ from (22), (23) and (24), then integrating from 0 to Z receive resulting equation after that changing z by x, we arrive equation

$$\tau_1''(x)+\left(1-\frac{c}{b}\right)\tau_1'(x)-\frac{c}{b}\tau_1(x)=\alpha_2(x)+k_1,\ 0\le x\le 1 \qquad (25)$$

where $\alpha_2(x)=\alpha_1'(x)+\frac{1}{b}\int\limits_0^x\left[b\omega_2(t)-c\alpha_1(t)\right]dt$ and k_1 is unknown constant.

For solving equation (25) we will consider the following cases: 1°. $c\ne 0$, $c\ne -b$; 2°. $c=-b$; 3°. $c=0$.

Let's consider case 1°. (25) under conditions

$$\tau_1(0)=\frac{1}{2}\int\limits_{-1}^{0}\left[\delta_1(t)+\delta_2(t)\right]dt+\psi_2(-1)$$

$$\tau_1'(0)=\frac{1}{2}\left[\alpha_1(0)+\delta_1(0)\right],\ \tau_1(1)=\varphi_1(0), \qquad (26)$$

We get

$$\tau_1(x)=\frac{b}{b+c}\int\limits_0^x\left[e^{\frac{c}{b}(x-t)}-e^{t-x}\right]\alpha_2(t)\,dt+\frac{bk_1}{b+c}\left[\frac{b}{c}\left(e^{\frac{c}{b}x}-1\right)-\left(1-e^{-x}\right)\right]+k_2e^{\frac{c}{b}x}+k_3e^{-x}$$

where

$$k_2=\frac{b}{2(b+c)}\left\{\int\limits_{-1}^{0}\left[\alpha_1(t)+\delta_1(t)\right]dt+2\psi_2(-1)+\alpha_1(0)+\delta_1(0)\right\}$$

$$k_3=\frac{b}{2(b+c)}\left\{\frac{c}{b}\int\limits_{-1}^{0}\left[\alpha_1(t)+\delta_1(t)\right]dt+\frac{2c}{b}\psi_2(-1)-\alpha_1(0)-\delta_1(0)\right\}$$

$$k_1=\left[\frac{b}{c}\left(e^{\frac{c}{b}}-1\right)-\left(1-e^{-1}\right)\right]^{-1}\cdot\left\{\frac{b+c}{b}\left[\varphi_1(0)-k_2e^{\frac{c}{b}}-k_3e^{-1}\right]-\int\limits_0^1\left[e^{\frac{c}{b}(1-t)}-e^{t-1}\right]\alpha_2(t)\,dt\right\}$$

Now, we consider 2° case. In this case equation (25) has the form

$$\tau_1''(x)+2\tau_1'(x)+\tau_1(x)=\alpha_2(x)+k_1,\ 0\le x\le 1.$$

By solving this equation under conditions (26), we obtain

$$\tau_1(x)=\int\limits_0^x(x-t)e^{t-x}\alpha_2(t)\,dt+k_1\left(1-e^{-x}-xe^{-x}\right)+\left(k_2+k_3x\right)e^{-x}$$

where

$$k_2=\frac{1}{2}\int\limits_{-1}^{0}\left[\alpha_1(t)+\delta_1(t)\right]dt+\psi_2(-1),$$

$$k_3=\frac{1}{2}\left[\alpha_1(0)+\delta_1(0)\right]+k_2,$$

$$k_1=\frac{1}{e-2}\left[\varphi_1(0)e-k_2-k_3-\int\limits_0^1(1-t)e^t\alpha_2(t)\,dt\right].$$

Finally, we consider the last 3°case. In this case equation (25) after integrating from 0 to x, has the form

$$\tau_1'(x)+\tau_1(x)=\alpha_3(x)+k_1x+k_2,\ 0\le x\le 1,$$

where $\alpha_3(x)=\int_0^x \alpha_2(t)dt$ and k_2 is temporarily unknown constant.

Solution of the last equation satisfying conditions (26) is represented as

$$\tau_1(x)=\int_0^x e^{t-x}\alpha_3(t)dt+k_1(x-1+e^{-x})+k_2(1-e^{-x})+k_3e^{-x},$$

where

$$k_3=\frac{1}{2}\int_{-1}^{0}[\alpha_1(t)+\delta_1(t)]dt+\psi_2(-1),$$

$$k_2=\frac{1}{2}[\alpha_1(0)+\delta_1(0)]+k_3,$$

$$k_1=\varphi_1(0)e-k_2(e-1)-k_3-\int_0^1 e^t\alpha_3(t)dt.$$

Now, we consider the domain D_3. Passing to the limit for the equations we find(15) $(i=3)$ in (15) $(i=2)$ $y\to 0$, we get

$$\omega_3(x)=\omega_2(x),\ -1\le x\le 0.$$

Now, we will consider the following auxiliary problem:

$$\begin{cases} u_{3xx}-u_{3yy}=\omega_3(x)e^{-\frac{c}{b}y}, \\ u_3(x,0)=\tau_2(x),\ u_{3y}(x,0)=\nu_2(x), & -1\le x\le 0, \\ u_3(-1,y)=\varphi_2(y),\ u_3(0,y)=\tau_3(y), & 0\le y\le 1. \end{cases} \tag{27}$$

We will look for solution of the problem in the form

$$u_3(x,y)=u_{31}(x,y)+u_{32}(x,y)+u_{33}(x,y), \tag{28}$$

where $u_{31}(x,y)$ is a solution of the problem

$$\begin{cases} u_{3xx}-u_{3yy}=0, \\ u_3(x,0)=\tau_2(x),\ u_{3y}(x,0)=0, & -1\le x\le 0, \\ u_3(-1,y)=\varphi_2(y),\ u_3(0,y)=\tau_3(y), & 0\le y\le 1; \end{cases} \tag{29}$$

$u_{32}(x,y)$ is a solution of the problem

$$\begin{cases} u_{3xx}-u_{3yy}=0, \\ u_3(x,0)=0,\ u_{3y}(x,0)=\nu_2(x), & -1\le x\le 0, \\ u_3(-1,y)=0,\ u_3(0,y)=0, & 0\le y\le 1; \end{cases} \tag{30}$$

$u_{33}(x,y)$ is a solution of the problem

$$\begin{cases} u_{3xx}-u_{3yy}=\omega_3(x)e^{-\frac{c}{b}y}, \\ u_3(x,0)=0,\ u_{3y}(x,0)=0, & -1\le x\le 0, \\ u_3(-1,y)=0,\ u_3(0,y)=0, & 0\le y\le 1. \end{cases} \tag{31}$$

We will find solution of the problems (29)- (31) by the method of continuity. They are respectively has the forms:

$$u_{31}(x,y)=\frac{1}{2}[T_2(x+y)+T_2(x-y)], \tag{32}$$

$$u_{32}(x,y)=\frac{1}{2}\int_{x-y}^{x+y}N_2(t)dt$$

$$u_{33}(x,y)=-\frac{1}{2}\int_0^y e^{-\frac{c}{b}\eta}d\eta\int_{x-y+\eta}^{x+y-\eta}\Omega_3(\xi)d\xi,$$

Where

$$T_2(x)=\begin{cases} 2\varphi_2(-1-x)-\tau_2(-2-x), & -2\le x\le -1, \\ \tau_2(x), & -1\le x\le 0, \\ 2\tau_3(x)-\tau_2(-x), & 0\le x\le 1; \end{cases} \tag{33}$$

$$N_2(x)=\begin{cases} -\nu_2(-2-x), & -2\le x\le -1, \\ \nu_2(x), & -1\le x\le 0, \\ -\nu_2(-x), & 0\le x\le 1; \end{cases} \tag{34}$$

The first two conditions of problem (31) are fulfilled automatically. By satisfying the third condition, we get relation

$$\int_0^y e^{-\frac{c}{b}\eta}\Omega_3(y-1-\eta)d\eta=-\int_0^y e^{-\frac{c}{b}\eta}\Omega_3(\eta-1-y)d\eta.$$

After some simplification and computations, we get

$$\Omega_3(-1-y)=-\omega_3(y-1).$$

Setting in (34) $x\to 0$, after some transformation, we get

$$\Omega_3(y)=-\omega_3(-y).$$

Hence, we found that

$$\Omega_3(x)=\begin{cases} -\omega_3(-2-x), & -2\le x\le -1, \\ \omega_3(x), & -1\le x\le 0, \\ -\omega_3(-x), & 0\le x\le 1. \end{cases}$$

Substituting (32), (33) and (34) into (28), we come

$$u_3(x,y)=\frac{1}{2}[T_2(x+y)+T_2(x-y)]+\frac{1}{2}\int_{x-y}^{x+y}N_2(t)dt-\frac{1}{2}\int_0^y e^{-\frac{c}{b}\eta}d\eta\int_{x-y+\eta}^{x+y-\eta}\Omega_3(\xi)d\xi. \tag{35}$$

Differentiating (35) with respect to x and passing to the limit $x\to 0$, in the taken equation, we find the relation between unknown functions $\tau_3(y)$ and $\nu_3(y)$:

$$\nu_3(y)=\tau_3'(y)+\beta_1(y), \tag{36}$$

where

$$\beta_1(y)=\tau_2'(-y)-\nu_2(-y)+\int_0^y e^{-\frac{c}{b}\eta}\omega_3(\eta)d\eta\cdot$$

Now, we will investigate to the problem in the domain D_1. Passing to the limit $y\to 0$ in equation (14), we get

$$\omega_1(x)=\tau_1''(x)-\nu_1(x).$$

Further, we write solution of equation satisfying (14), satisfying conditions (2), (8), (11):

$$u_1(x,y)=\int_0^y\tau_3(\eta)G_\xi(x,y;0,\eta)d\eta-\int_0^y\varphi_1(\eta)G_\xi(x,y;1,\eta)d\eta+$$
$$+\int_0^1\tau_1(\xi)G(x,y;\xi,0)d\xi-\int_0^y e^{-\frac{c}{b}\eta}d\eta\int_0^1\omega_1(\xi)G(x,y;\xi,\eta)d\xi\cdot$$

Differentiating this solution with respect to x and tending x to zero, we get a relation between unknown functions $\tau_3(y)$ and $\nu_3(y)$. Eliminating function $\nu_3(y)$ from taken relation and (36), we arrive the second kind Volterra integral equation with respect to $\tau_3'(y)$:

$$\tau_3'(y)+\int_0^y K(y,\eta)\tau_3'(\eta)d\eta=g(y), \tag{37}$$

where $K(y,\eta)=N(0,y;0,\eta)$,

$$g(y)=-\beta_1(y)+\int_0^y\varphi_1'(\eta)N(0,y;1,\eta)d\eta+\int_0^1\tau_1'(\xi)N(0,y;\xi,0)d\xi+$$
$$+\int_0^y e^{-\frac{c}{b}\eta}d\eta\int_0^1\omega_1(\xi)N_\xi(0,y;\xi,\eta)d\xi,$$

$$\left.\begin{matrix}G(x,y;\xi,\eta)\\N(x,y;\xi,\eta)\end{matrix}\right\}=\frac{1}{2\sqrt{\pi(y-\eta)}}\sum_{n=-\infty}^{+\infty}\left\{\exp\left[-\frac{(x-\xi-2n)^2}{4(y-\eta)}\right]\mp\exp\left[-\frac{(x+\xi-2n)^2}{4(y-\eta)}\right]\right\}$$

is

Green's function of the first and second boundary-value problems for equation Furier.

By solving equation (37), we find function $\tau_3'(y)$ and using this functions we will find functions

$\nu_3(y)$, $T_2(x)$, $u_1(x,y)$, $u_3(x,y)$.

Conclusion

In conclusion we note that in the work [1, 2] some boundary-value problems were considered for the third and fourth order more general equations of parabolic- hyperbolic type I the domains with a line of type changing.

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