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MATHEMATICAL MODEL OF CURVILINE CREW MOTION ON CYLINDER WHEELS

Abstract: The paper proposes the conclusion of the kinematic equations of motion of the crew on balloon wheels along a curved path of sufficiently small curvature.

Key words: mathematical model, numerical algorithm, vehicles, computer technology, rolling theory, stability of motion, computational experiment.

Language: English

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Introduction

The main focus of this article emphasizes that create of a mathematical model with curved and straight lines, taking into account the elasticity and deformation of tires in motion during the study of the stability of the motion of dynamic systems. As well as there is the task of making software by using Maple, program.

As a result of the scientific research DIV (AUTO software - instrumental tool) has been created which is prevent accidents during the movement of the vehicle. The main goal of the DIV that automation of the process of studying the stability of the motion of dynamic systems.

THE MAIN RESULTS AND FINDINGS

Suppose that the conditions are satisfied under which the theory of rolling the wheel with a deformable tire is valid when the deformation of the tire is sufficiently small. The condition for the wheels to roll without slipping and the small deformation of the tire impose certain restrictions on the class of studied movements. In particular, the curvature of the path should be sufficiently small, and the speed of movement should not be too large [1].

We denote by q_1, q_2, \dots, q_n the generalized coordinates of the crew on m balloon wheels and introduce values that determine the position of the

wheel ($i=1,2,\dots,m$). Let x_i, y_i be the Cartesian coordinates of the point K_i of the meeting of the straight line of greatest inclination, drawn in the middle plane of the wheel through its center, with the road plane; θ_i - angle, formed by the middle plane of the wheel on the road and the Ox axis of the fixed coordinate system $Oxyz$, the Oxy plane of which coincides with the road plane, and the axis Oz is directed upwards; χ_i - is the angle between the axis Oz and the middle plane of the wheel. The quantities $x_i, y_i, \theta_i, \chi_i$ are known functions of the generalized coordinates q_1, q_2, \dots, q_n .

Suppose first that the crew's movement is given. This means that the quantities $x_i, y_i, \theta_i, \chi_i$ - are known functions of time. Then, according to the theory of rolling a wheel with a deformable tire, it is possible to determine the deformation of tires at any time. In accordance with the notation in pic 1, a condition for the absence of tire slippage during the lateral displacement of the i -the wheel and during its rotation around the vertical axis leads to the relations:

$$\begin{aligned} dx_i^* \sin(\theta_i + \varphi_i) - dy_i^* \cos(\theta_i + \varphi_i) &= 0, \\ d(\theta_i + \varphi_i) &= dS_i^* (\alpha_i \xi_i - \beta_i \varphi_i - \gamma_i \chi_i). \end{aligned} \quad (1)$$

here x_i^*, y_i^* - the coordinates of the point that coincided before the lateral deformation of the tire

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with the point K_i ; dS_i^* - arc element of the rolling line Γ_i (see Pic 1, a); ξ_i - lateral deformation of the tire; φ_i - deformation of the tire during twisting; $\alpha_i, \beta_i, \gamma_i$ - kinematic parameters of the tire related to its lateral deformation [2]. With lateral wheel displacement

$$\begin{aligned} x_i^* &= x_i + \xi_i \sin \theta_i, \\ y_i^* &= y_i - \xi_i \cos \theta_i \end{aligned} \quad (2)$$

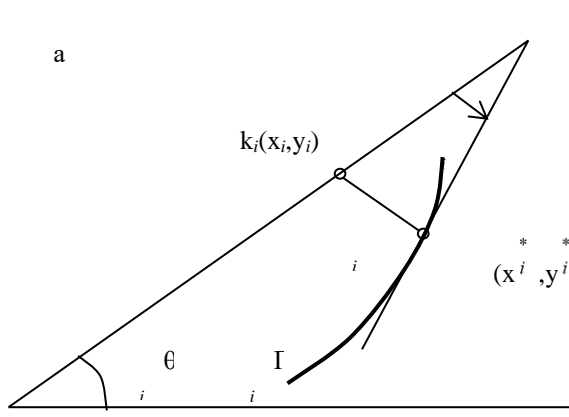


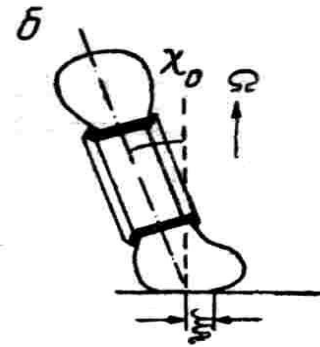
Figure 1

In the written relations, the small quantities are ξ_i, φ_i и χ_i . Using (2) and discarding small quantities of the second order and higher, from (1) we obtain

$$\begin{aligned} dx_i \sin(\theta_i + \varphi_i) - dy_i \cos(\theta_i + \varphi_i) + d_i &= 0, \\ d\theta_i + d\varphi_i - dS_i^* (\alpha_i \xi_i - \beta_i \varphi_i - \gamma_i \chi_i) &= 0. \end{aligned} \quad (3)$$

With the same degree of accuracy

$$dS_i^* = dx_i \cos(\theta_i + \varphi_i) + dy_i \sin(\theta_i + \varphi_i) \quad (4)$$



Pic 1.

The relation expresses the condition that the ... does not slip in the longitudinal direction

$$dS_i + r_i d\vartheta_i + d\eta_i + \lambda_i dS_i \eta_i - \nu_i dS_i (r_{0i} - r_i) = 0 \quad (5)$$

Here $dS_i = dx_i \cos \theta_i + dy_i \sin \theta_i$ - arc elements of the curve described by the point K_i ; η_i - tire longitudinal strain value; r_{0i} - radius of the uncompressed tire; r_i - distance from the center of the wheel to the reference plane; λ_i, ν_i - kinematic parameters of the tire related to its longitudinal deformation; $d\vartheta_i$ - an element of the angle when turning the wheel around the axis of its rotation.

Equations (3) - (5) represent the desired relationships for determining the deformation ξ_i, φ_i, η_i , if the movement of the wheel is known. Knowing the deformation of the tire, you can find the potential forces acting on the wheel. According to [4], they are equivalent to the transverse force F_i and the longitudinal force P_i , applied to the point K_i , the moment M_{θ_i} relative to the vertical axis, the moment

M_{χ_i} relative to the longitudinal horizontal axis, and the moment M_i relative to the transverse axis defined by the expressions

$$\begin{aligned} F_i &= a_i \xi_i + \sigma_i N_i \chi_i, & P_i &= K_{\tau_i} \eta_i, & M_{\theta_i} &= b_i \varphi_i, \\ M_{\chi_i} &= -\sigma_i N_i \xi_i - \rho_i N_i \chi_i, & M_i &= \mu_i N_i \eta_i, \end{aligned} \quad (6)$$

where N - is the load on the i - th wheel, a_i - is the

coefficient of lateral stiffness of the tire; K_{τ_i} - coefficient of longitudinal stiffness of the tire; b_i - tire stiffness coefficient.

We divide equations (3) and (5) by dt and replace them with dS^* its expression (4) - $dS_i = dx_i \cos \theta_i + dy_i \sin \theta_i$. As a result, we obtain the equations

$$\begin{aligned} \dot{x}_i \sin(\theta_i + \varphi_i) - \dot{y}_i \cos(\theta_i + \varphi_i) + \dot{\xi}_i &= 0, \\ \dot{\theta}_i + \dot{\varphi}_i - (\dot{x}_i \cos \theta_i + \dot{y}_i \sin \theta_i) (\alpha_i \xi_i - \beta_i \varphi_i - \gamma_i \chi_i) &= 0, \end{aligned} \quad (7)$$

$r_i \dot{\vartheta}_i + \dot{\eta}_i + (\dot{x}_i \cos \theta_i + \dot{y}_i \sin \theta_i) [1 + \lambda_i \eta_i - \nu_i (r_{0i} - r_i)] = 0$, which are the kinematic equations of the movement of the crew on balloon wheels along a curved path of sufficiently small curvature [5].

Let now $T = T(q, \dot{q}, t)$ - kinetic energy of the crew, $Q_j = Q_j(q, \dot{q}, t), (j = \overline{1, n})$ - specified generalized forces; $R_j = R_j(\xi, \varphi, \eta, \chi)$ - generalized forces due to tire deformation. To find the expressions R_j we calculate the virtual work of the deformation forces:

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$$\begin{aligned}
 \delta A = & \sum_{i=1}^m [F_i(\delta x_i \sin \theta_i - \delta y_i \cos \theta_i) + \\
 & + P_i(\delta x_i \cos \theta_i + \delta y_i \sin \theta_i + r_i \delta \mathcal{G}_i) \\
 & + M_{\theta_i} \delta \theta_i + M_{x_i} \delta x_i + M_i \delta \mathcal{G}_i] = \dots = \sum_{j=1}^n \sum_{i=1}^m \left[(F_i \sin \theta_i + P_i \cos \theta_i) \frac{\partial x_i}{\partial q_j} + \right. \\
 & \left. + (P_i \sin \theta_i - F_i \cos \theta_i) \frac{\partial y_i}{\partial q_j} + M_{\theta_i} \frac{\partial \theta_i}{\partial q_j} + M_{x_i} \frac{\partial x_i}{\partial q_j} + (r_i P_i + M_i) \frac{\partial \mathcal{G}_i}{\partial q_j} \right]. \\
 \text{Отсюда } R_j = & \sum_{i=1}^m \left[(F_i \sin \theta_i + P_i \cos \theta_i) \frac{\partial x_i}{\partial q_j} + (P_i \sin \theta_i - F_i \cos \theta_i) \frac{\partial y_i}{\partial q_j} + \right. \\
 & \left. + M_{\theta_i} \frac{\partial \theta_i}{\partial q_j} + M_{x_i} \frac{\partial x_i}{\partial q_j} + (r_i P_i + M_i) \frac{\partial \mathcal{G}_i}{\partial q_j} \right] \quad (8).
 \end{aligned}$$

Here the forces F_i, P_i and moments $M_{\theta_i}, M_{x_i}, M_i$ are determined by expressions (6). After taking into account all the forces acting on the system, including the forces of interaction of the tires with the road, the equations of the dynamics of the crew on the balloon wheels are written in general form

$$\frac{d}{dt} \frac{\partial T}{\partial \dot{q}_j} - \frac{\partial T}{\partial q_j} = Q_j + R_j \quad (j = \overline{1, n}) \quad (9)$$

where the generalized forces R_j are determined by expressions (8). Equations (9) together with (7) describe the movement of the crew on balloon wheels along a curved path of sufficiently small curvature [6].

To study the stability of movement around a circle with a constant speed V , we compose the equations of motion of the crew with its small deviations from stationary motion around the circle.

Let be $\theta_i = \theta_i^0 + \theta_i'$, here $\theta_i^0 = \Omega t$ ($\Omega = \text{const}$) - the angle θ_i on the trajectory of the unperturbed motion; θ_i' - small deviation θ_i . Instead of quantities, \dot{x}_i, \dot{y}_i we introduce u_i, v_i by means of the relations.

$$\begin{aligned}
 \dot{x}_i &= v_i \cos \theta_i^0 + u_i \sin \theta_i^0; \\
 \dot{y}_i &= v_i \sin \theta_i^0 - u_i \cos \theta_i^0. \quad (10)
 \end{aligned}$$

Here v_i - is the speed of the longitudinal movement of the i - the wheel; u_i - the speed of its transverse movement, which is a small value of the order of the remaining small quantities. Substituting relations (10) into (7), after linearization of relatively small quantities, we obtain the kinematic equations of the crew on balloon wheels in the form

$$\begin{aligned}
 u_i + \dot{\xi}_i + V_i \theta_i' + V_i \varphi_i &= 0, \\
 \dot{\theta}_i + \dot{\varphi}_i - \alpha_i V_i \xi_i + \beta_i V_i \varphi_i + \gamma_i V_i \chi_i &= 0, \quad (11) \\
 \dot{\eta}_i + v_i' - \omega_i^0 r_i' - r_i^0 \omega_i' + \lambda_i V_i \eta_i' + v_i V_i r_i' &= 0.
 \end{aligned}$$

The last equation (11) is obtained from equation (7) using the expressions

$$\begin{aligned}
 \dot{v}_i &= -\omega_i^0 - \omega_i'; \quad r_i = r_i^0 + r_i'; \\
 v_i &= V_i + v_i'; \quad \eta_i = \eta_i^0 + \eta_i'
 \end{aligned}$$

where the values of the corresponding variables in stationary motion are marked with a null, and small deviations from stationary values are marked with a dash [7]. The dynamic equations of motion in this case retain the form (9). As in the case of rectilinear stationary motion, the equations of small deviations from circular motion can be simplified at sufficiently high velocities V_i or at sufficiently large kinematic parameters $\alpha_i, \beta_i, \gamma_i$. However, it should be remembered that in the case of curvilinear motion, the velocities v_i must be limited from above, which follows from the requirement that the deformation is small, those small quantities ξ_i, φ_i, χ_i .

RESULTS AND DISCUSSION

As a result of scientific research in the Laboratory of Mathematical Modeling and Machine Dynamics under the Department of Mathematical Modeling of Samarkand State University named after Alisher Navoi to study the stability of dynamic systems, AUTO software-instrumental tool (DIV) was created. The purpose of the DIV is to automate the process of studying the motion stability of dynamic systems.

With the help of the DIV, the following was done:

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- 1) a communication environment in which the user can enter the terms of a particular issue into a computer based on their field terms;
- 2) the ability to indirectly enter a sequence of partial problems into the computer, using a communication environment to generate a mathematical model of the problem;
- 3) to derive a characteristic determinant, a characteristic equation based on the generated mathematical model and calculation of the values of the coefficients of this equation;

- 4) According to the scheme of conducting a computational experiment it is necessary to study in what parameter plane to construct the stagnation field and what structural parameters affect the boundary of the stagnation field, determination of the base of nominal values of constructive parameters, the criterion of stability for research, the area of stability (by graph analytic or by pressing conditional symbols on the plane of parameters) and a tool for selecting the method of interpreting the analysis of the results (using an interval or graph).

The DIV consists of 5 modules, the functional diagram of which is given in Figure 2.

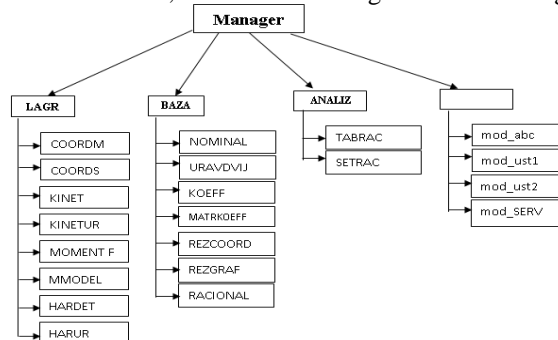


Figure 2

ANALYSIS, BASE, LAGR, EAVTO modules are libraries of practical modules that represent the basic ideas and algorithms for studying the motion stability of dynamic systems. A communication environment (interface) consisting of 5 options controlled using the **MANAGER** module has been created to address the **DIV** application modules.

Software-tool tool integrated environment

Within a single human-machine system, an integrated environment that allows the user to communicate with the computer is intended to perform three groups of tasks:

- the user is provided with the task of setting the problem only on the basis of queries or on the basis of the selection of tools specified in the menus, without giving the computer a problem-solving program. In this case, the problem can be

solved in several parts, which allows for an indirect way of solving the problem in advance;

- allows the user to independently create an operational environment for problem solving, using terms and concepts in their field of expertise;
- the user is provided with natural forms of expression of the information exchanged in the process of solving the problem with computer technology and in this exchange the user can choose convenient ways of organizing communication;
- the user can change the form of communication, that is, make a system of possible changes in it using the "menu" type of communication or on the basis of various requests;
- a help system has been created for users to get comments on mistakes made in the communication process.

The Integrated Environment (IE) consists of 5 options.

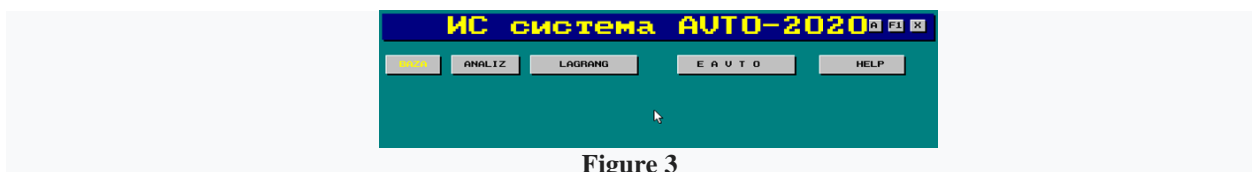


Figure 3

BASE - creates an environment for creating and editing databases;

"ANALYSIS" - analyzes the results of the computational experiment and creates an environment

for communication with the module, which rationally determines the parameters;

LAGRANG - creates an environment of communication with the module, which creates a

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mathematical model of the problem on the basis of the algorithm with the help of the Maple system of computer algebra;

“EAVTO” - conducts computational experiments based on algorithms and programs, creating an environment for determining the area of stagnation of a particular system;

“HELP” is a system for using the components of the DIV.

Tasks performed using options are displayed directly as a question-and-answer dialogue or using menus. Menu items can be activated by pressing the cursor control keys, function keys, or the button that indicates the first letter of the option.

CONCLUSION

The AVTO software and tool system has been developed for automating scientific research on the choice of mathematical models, algorithms, the process of composing applied modules, conducting computational experiments and determining rational values of parameters and rational areas of stability.

The elasticity and deformation of the tires in car movement are taken into account and it will be protected against wheel wear. Economic efficiency has been achieved, it means that tires do not quickly become unusable.

The main results of the work are as follows:

1. A mathematical model was developed for the linear motion of the car on a straight road, taking into account the lateral and longitudinal crane angles of the body (Model 1);

2. A mathematical model was developed for the motion of a car in an inclined plane, taking into account the angles of the lateral and longitudinal cranes (Model 2);

3. A mathematical model was developed for the horizontal and oblique motion of the vehicle, taking into account the deformation and flexibility of the tires, as well as the lateral and longitudinal crane angles of the vehicle (Model 3);

4. A nonlinear mathematical model of the curvilinear motion of the car was developed, taking into account the deformation and flexibility of the tires, as well as the non-potential forces in the tire material (Model 4);

5. A linear mathematical model of the curvilinear motion of the car was developed, taking into account the deformation and flexibility of the tires, as well as the non-potential forces in the tire material (Model 5);

6. A mathematical model of the linear motion of a car was developed, taking into account the deformation and flexibility of the tires, as well as the non-potential forces in the tire material (Model 6);

7. A mathematical model of the linear motion of a car with the same radius of the wheels, as well as the deformation and flexibility of the tires, as well as the non-potential forces in the tire material, was developed (Model 7);

8. A mathematical model of the linear motion of a vehicle with the same deformation and flexibility of the tires, as well as the non-potential forces in the tire material with the same radius of the wheels and the angles of rotation of the front wheels was developed (Model 8);

9. In the automation of scientific research, an AVTO software tool has been created that allows the selection of mathematical models, algorithms and a set of modules, conducting computational experiments and determining the area of rational stagnation and rational values of parameters based on their results;

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