Impact Factor:

ISRA (India) = 4.971 | ISI (Dubai, UAE) = 0.829 | GIF (Australia) = 0.564 | JIF = 1.500
SIS (USA) = 0.912 | РИНЦ (Russia) = 0.126 | ESJI (KZ) = 8.716 | SJIF (Morocco) = 5.667
ICV (Poland) = 6.630 | PIF (India) = 1.940 | IBI (India) = 4.260 | OAJI (USA) = 0.350

QR – Issue QR – Article

SOI: 1.1/TAS DOI: 10.15863/TAS
International Scientific Journal
Theoretical & Applied Science

p-ISSN: 2308-4944 (print) e-ISSN: 2409-0085 (online)

Year: 2020 Issue: 02 Volume: 82

Published: 29.02.2020 http://T-Science.org

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SOLUTION OF PHYSICALLY NONLINEAR PROBLEMS OF THE ELASTICITY THEORY FOR BODIES FROM REINFORCED COMPOSITES

***Abstract**: The paper describes a method for constructing a solution to the problem of physically nonlinear deformation of transversely isotropic composite bodies, in which the stiffness of the reinforcing elements is much higher than the stiffness of the binder. A simplified model of plastic deformation is used. The technique is a synthesis of the Poincaré perturbation method and the energy method of boundary states. The problems for the cube and cylinder are solved, the accuracy analysis is carried out and conclusions are formulated.*

***Key words**: Boundary state method, perturbation method, transverse isotropy, composite materials, physical nonlinearity.*

***Language**: English*

***Citation**: Ivanychev, D. A., & Novikov, E. A. (2020). Solution of physically nonlinear problems of the elasticity theory for bodies from reinforced composites. *ISJ Theoretical & Applied Science, 02 (82),* 243-248.

***Soi**: http://s-o-i.org/1.1/TAS-02-82-43 ***Doi**: crossref https://dx.doi.org/10.15863/TAS.2020.02.82.43

***Scopus ASCC**: 2610.*

Introduction

Physically nonlinear problems are devoted to a lot of research. In [1], the basic principles of the theory of plasticity of anisotropic materials are presented. Rather new models of continuous media are stated. In [2], the process of elastoplastic deformation of transversely isotropic composites with cavities is studied. In [3], physical nonlinearity was considered together with material inhomogeneity. Homogeneous and inhomogeneous problems are solved in an axisymmetric formulation for a thick-walled cylinder. The geometrically and physically nonlinear problem of bending a three-layer plate with a soft anisotropic filler was considered in [4]. In [5], a solution to the problem of contact of plates with a physically nonlinear medium is presented. In [6], the resolving equations of the planar deformation theory of plasticity were constructed, which are described by mathematical models in which the physical relations are in the form of arbitrary cross-dependencies between invariants of stress and strain tensors.

The method of boundary states in the field of solving anisotropic problems has proved its effectiveness. For example, in [7] the plane problems of the theory of elasticity were solved for rectangular bodies with circular cutouts, and in [8], the Saint-Venant problem for an extended anisotropic cylinder was studied.

A number of works are devoted to solving boundary value problems of the theory of elasticity with the participation of mass forces [9-12].

This paper presents a methodology for solving physically nonlinear problems of the theory of elasticity for transversely isotropic composite bodies, in which the rigidity in one direction (z axis) is much higher than the rigidity in the other direction, as a result of which a simplified theory of plasticity can be applied.

Statement of the problem and the mathematical model of its solution

A physically nonlinear medium from a homogeneous continuous isotropic material is investigated. In the physically nonlinear theory, as well as in the theory of plasticity, the following concepts are used [13]:

stress intensity

$$\sigma_i = \frac{1}{\sqrt{2}}\sqrt{(\sigma_x-\sigma_y)^2+(\sigma_y-\sigma_z)^2+(\sigma_z-\sigma_x)^2+6(\tau_{xy}^2+\tau_{yz}^2+\tau_{zx}^2)}\,;$$

strain rate

$$\varepsilon_i = \frac{\sqrt{2}}{3}\sqrt{(\varepsilon_x-\varepsilon_y)^2+(\varepsilon_y-\varepsilon_z)^2+(\varepsilon_z-\varepsilon_x)^2+3/2(\gamma_{xy}^2+\gamma_{yz}^2+\gamma_{zx}^2)}\,.$$

The same quantities expressed in terms of principal stresses:

$$\sigma_i = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}\,; \quad (1)$$

$$\varepsilon_i = \frac{\sqrt{2}}{3}\sqrt{(\varepsilon_1-\varepsilon_2)^2+(\varepsilon_2-\varepsilon_3)^2+(\varepsilon_3-\varepsilon_1)^2}\,. \quad (2)$$

The intensity of tangential stresses (according to Huber-Mises [14]) is

$$\tau_i = \frac{\sigma_i}{\sqrt{3}}\,; \quad (3)$$

Shear strain rate

$$\gamma_i = \sqrt{3}\varepsilon_i\,. \quad (4)$$

Consider the process of deformation in the xy isotropy plane of a transversally isotropic body (the z axis is perpendicular to the isotropy planes).

The dependence of the intensity of shear stresses on the intensity of shear deformations is shown in Figure 1. Curve 2 corresponds to a linear dependence, 2 to a nonlinear one.

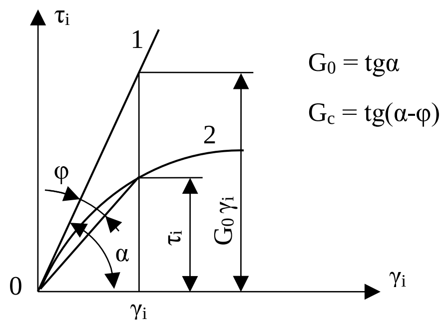

Figure 1 - Dependence between stress and strain intensities

In figure 1 G is the shear modulus for the plane of isotropy, G_c is the secant shear modulus for the same plane, and

$$G_c = \frac{\tau_i}{\gamma_i}\,. \quad (5)$$

We introduce a small parameter β characterizing the deviation of the secant shear modulus from the shear modulus:

$$G_c = G(1-\beta)\,. \quad (6)$$

Let the material have a nonlinear net shear diagram described by the relation

$$\tau_i = A\gamma_i - B\gamma_i^k\,, \quad (7)$$

where A, B, k – are material constants determined from the experiment on shear in the plane of isotropy. From (6) it follows

$$\beta = 1-\frac{G_c}{G}\,. \quad (8)$$

Substituting equalities (5), (8) into dependence (7), we obtain

$$\beta = 1-\frac{A}{G}-\frac{B}{G}\gamma_i^{k-1}\,. \quad (9)$$

The relationship between the intensity of shear stresses and the intensity of shear deformations does not depend on the type of stress state. From this it follows that the dependence $\tau_i = f(\gamma_i)$ is the same for any combination of stresses and strains and can be determined from any experiment, for example, by a pure shift. Having stresses and strains from the simplest experiment, using formulas (1), (2), (3), (4), we can obtain the dependence $\tau_i \sim \gamma_i$ and calculate the small parameter by the formula (9).

In a similar way, we can introduce a small parameter α for planes perpendicular to the isotropy planes:

$$G_z^c = G_z(1-\alpha)\,;\ \alpha = 1-\frac{C}{G_z}-\frac{D}{G_z}\gamma_i^{h-1}\,, \quad (10)$$

where C, B, h – are material constants determined from the experiment on shear in a plane perpendicular to the isotropy plane, G_z^c, G_z – is the secant shear modulus and shear modulus in the same plane.

The state of the medium in the simplified theory of plasticity is subject to the generalized Hooke law [15]:

$$\sigma_{xx} = (\lambda_2 + \lambda_4)\theta + \lambda_3\varepsilon_x + 2\lambda_4(1-\pi(p))\frac{\varepsilon_{xx}-\varepsilon_{yy}}{2};$$

$$\sigma_{yy} = (\lambda_2 + \lambda_4)\theta + \lambda_3\varepsilon_x + 2\lambda_4(1-\pi(p))\frac{\varepsilon_{yy}-\varepsilon_{xx}}{2};$$

$$\sigma_{zz} = \lambda_3\theta + \lambda_1\varepsilon_x;\ \theta = \varepsilon_{xx} + \varepsilon_{yy}; \qquad (11)$$

$$\sigma_{xy} = 2\lambda_4(1-\pi(p))\varepsilon_{xy};$$

$$\sigma_{xz} = 2\lambda_5(1-\chi(q))\varepsilon_{xz};$$

$$\sigma_{yz} = 2\lambda_5(1-\chi(q))\varepsilon_{yz},$$

where $\pi(p)$ and $\chi(q)$ are plasticity functions of type A.A. Ilyushin, equal to zero in the elastic zone; λ_i – parameters of a transversely isotropic medium associated with technical constants by the following expressions:

$$\lambda_1 = E_z(1-\nu)/l;\ \lambda_2 = E(\nu + k\nu_z^2)/l[(1+\nu)l];$$

$$\lambda_3 = E\nu_z/l;\ \lambda_4 = G = E/[2(1+\nu)];$$

$$\lambda_5 = G_z;\ l = 1-\nu-2\nu_z^2 k;\ k = E/E_z;$$

here E_z and E are the elastic modules, respectively, in the direction of the z axis and in the isotropy plane, ν_z is the Poisson's ratio characterizing compression along r during tension along the z axis, ν is the Poisson's ratio characterizing lateral compression in isotropic planes under tension in the same planes, G and G_z – shear modulus in isotropy planes and perpendicular to them.

If, in Hooke's law (11), instead of shear modules, secant modules (6), (10) are used, and discrete values $\pi(p)$, $\chi(q)$ are assigned to functions β, α, respectively, then it will have:

$$\sigma_{xx} = [\lambda_2 + 2\lambda_4(1-\beta)]\varepsilon_{xx} + \lambda_2\varepsilon_{yy} + \lambda_3\varepsilon_{zz};$$

$$\sigma_{yy} = [\lambda_2 + 2\lambda_4(1-\beta)]\varepsilon_{yy} + \lambda_2\varepsilon_{xx} + \lambda_3\varepsilon_{zz};$$

$$\sigma_{zz} = \lambda_3\theta + \lambda_1\varepsilon_{zz}; \qquad (12)$$

$$\sigma_{xy} = 2\lambda_4(1-\beta)\varepsilon_{xy};$$

$$\sigma_{xz} = 2\lambda_5(1-\alpha)\varepsilon_{xz};$$

$$\sigma_{yz} = 2\lambda_5(1-\alpha)\varepsilon_{yz}.$$

This purpose allows us to describe the actual behavior of a physically nonlinear transversely isotropic medium through the constants of a certain elastic medium and small parameters β and α, the zero values of which correspond to a linear medium.

Next, asymptotic series are introduced:

$$u_i = \sum_{n=0}^{\infty}\beta^n u_i^{(n)};\ \varepsilon_{ij} = \sum_{n=0}^{\infty}\beta^n\varepsilon_{ij}^{(n)};\ \theta = \sum_{n=0}^{\infty}\beta^n\theta^{(n)};$$

$$\sigma_{ij} = \sum_{n=0}^{\infty}\beta^n\sigma_{ij}^{(n)}.$$

Hooke's law (12) after replacing the summation and postulate variables with zero values for any formally non-existent decomposition element for which the index has a negative value ($n<0$) leads to the corollary:

$$\sigma_{xx}^{(n)} = \lambda_2\theta^{(n)} + 2\lambda_4\varepsilon_{xx}^{(n)} + \lambda_3\varepsilon_{zz}^{(n)} + \tilde{\sigma}_{xx}^{(n)};$$

$$\sigma_{yy}^{(n)} = \lambda_2\theta^{(n)} + 2\lambda_4\varepsilon_{yy}^{(n)} + \lambda_3\varepsilon_{zz}^{(n)} + \tilde{\sigma}_{yy}^{(n)};$$

$$\sigma_{zz}^{(n)} = \lambda_3\theta^{(n)} + \lambda_1\varepsilon_{zz}^{(n)} + \tilde{\sigma}_{zz}^{(n)};$$

$$\sigma_{xy}^{(n)} = 2\lambda_4\varepsilon_{xy}^{(n)} + \tilde{\sigma}_{xy}^{(n)};$$

$$\sigma_{yz}^{(n)} = 2\lambda_5\varepsilon_{yz}^{(n)} + \tilde{\sigma}_{yz}^{(n)};$$

$$\sigma_{xz}^{(n)} = 2\lambda_5\varepsilon_{xz}^{(n)} + \tilde{\sigma}_{xz}^{(n)};$$

$$\tilde{\sigma}_{xx}^{(n)} = 2\lambda_4\varepsilon_{xx}^{(n-1)};$$

$$\tilde{\sigma}_{yy}^{(n)} = 2\lambda_4\varepsilon_{yy}^{(n-1)};$$

$$\tilde{\sigma}_{zz}^{(n)} = 0;$$

$$\tilde{\sigma}_{xy}^{(n)} = -2\lambda_4\varepsilon_{xy}^{(n-1)};$$

$$\tilde{\sigma}_{yz}^{(n)} = -2\lambda_5\varepsilon_{yz}^{(n-1)};$$

$$\tilde{\sigma}_{xz}^{(n)} = -2\lambda_5\varepsilon_{xz}^{(n-1)}.$$

Then you can write:

$$s_{ij}^{(n)} = \sigma_{ij}^{(n)} - \tilde{\sigma}_{ij}^{(n)}. \qquad (13)$$

and get for the decomposition elements the familiar form of the generalized Hooke law for a transversely isotropic body:

$$s_{xx}^{(n)} = \lambda_2\theta^{(n)} + 2\lambda_4\varepsilon_{xx}^{(n)} + \lambda_3\varepsilon_{zz}^{(n)};$$

$$s_{yy}^{(n)} = \lambda_2\theta^{(n)} + 2\lambda_4\varepsilon_{yy}^{(n)} + \lambda_3\varepsilon_{zz}^{(n)};$$

$$s_{zz}^{(n)} = \lambda_3\theta^{(n)} + \lambda_1\varepsilon_{zz}^{(n)}; \qquad (14)$$

$$s_{xy}^{(n)} = 2\lambda_4\varepsilon_{xy}^{(n)};$$

$$s_{yz}^{(n)} = 2\lambda_5\varepsilon_{yz}^{(n)};$$

$$s_{xz}^{(n)} = 2\lambda_5\varepsilon_{xz}^{(n)}.$$

Cauchy's ratio:

$$\varepsilon_{ij}^{(n)} = \frac{1}{2}\left(u_{i,j}^{(n)} + u_{j,i}^{(n)}\right). \qquad (15)$$

Denoting through X_i^0 volume forces and assuming the series to be known

$$X_i^0 = \sum_{n=0}^{\infty}\beta^n X_i^{0(n)},$$

we rewrite the equilibrium equations in the form

$$s_{ij,j}^{(n)} + X_i^{(n)} = 0;\ X_i^{(n)} = X_i^{0(n)} + \tilde{\sigma}_{ij,j}^{(n)}. \qquad (16)$$

Relations (14) - (16) in shape correspond to the deformed state of a linear transversely isotropic elastic body.

The method of boundary states with perturbations

Any internal state of a linear isotropic elastostatic medium constitutes a set of displacements, strains, and stresses agreed upon by the governing relations $\xi = \{u_i, \varepsilon_{ij}, \sigma_{ij}\} \in \Xi$. Their trace at the boundary ∂V of region V with a single external normal n_j contains information about displacements and forces along the boundary $\gamma = \{u_i, p_i\} \in \Gamma$, $p_i = \sigma_{ij} n_j$ and corresponds to the boundary state. The spaces of possible internal and boundary states are Hilbert and isomorphic [16]: $\Xi = \{\xi^{(1)}, \xi^{(2)}, ..., \xi^{(n)}\} \leftrightarrow \Gamma = \{\gamma^{(1)}, \gamma^{(2)}, ..., \gamma^{(n)}\}$.

$a_1\xi^{(1)} + a_2\xi^{(2)} \leftrightarrow a_1\gamma^{(1)} + a_2\gamma^{(2)}$, $(\xi^{(1)}, \xi^{(2)})_\Xi = (\gamma^{(1)}, \gamma^{(2)})_\Gamma$.

Any correct problem reduces to an infinite system of linear algebraic equations

$$Q\mathbf{c} = \mathbf{q}, \tag{17}$$

with respect to the vector of Fourier coefficients $\mathbf{c}$ of the expansion of the desired state in a series along an orthonormal basis

$$\xi = \sum_l c_l \xi^{(l)}. \tag{18}$$

Matrix Q is structurally determined only by the type of boundary conditions and numerically through an orthonormal basis. In the first and second main problems, matrix Q is the identity matrix. The vector of the right-hand sides includes information on the specific filling of the boundary conditions.

At each step, an infinite system of equations (17) is formulated in accordance with the boundary conditions of this iteration. In practice, it is enough to consider real boundary conditions only at $n = 0$, solving only the main problem with $Q \equiv E$ in subsequent iterations and taking into account the corrections on the right-hand side caused by the appearance of fictitious volume forces in the relations, which in the general case are not potential, but have a polynomial character.

Before performing iterations, the following actions are performed: on the basis of the general solution and the basis of harmonic functions in $V \cup \partial V$, the bases of spaces Ξ and Γ are formed; isomorphic orthonormal bases are constructed; the members of $X_i^{0(n)}$ expansion series for X_i^0 are established. Due to the independence of the initial basis from small parameters, the orthonormal basis is constructed exactly once and then used in each iteration.

At step $n = 0$: state $\tilde{\xi}^{(0)}$ is sought due to volume forces $X_i^{0(n)}$; a correction corresponding to this state is worn into real boundary conditions, an infinite system of algebraic equations is formed $Q\mathbf{c}^{(0)} = \mathbf{q}$; its solution and linear combination (18) give the internal state $\xi^{(0)}$; its sum with the state of the bulk forces prepares the initial approximation for ξ: $\xi = \xi^{(0)} + \tilde{\xi}^{(0)}$. According to the previous (13) formulas, the tensor $\tilde{\sigma}_{ij}^{(0)}$.

At $n > 0$: tensor $s_{ij}^{(n)} = \sigma_{ij}^{(n)} - \tilde{\sigma}_{ij}^{(n)}$ and vector $X_i^{(n)}$ are constructed; state $\tilde{\xi}^{(n)}$ is sought due to volume forces $X_i^{(n)}$ in accordance with (16); the correction value from them is introduced into the boundary conditions and the first main problem for the system of equations (14) - (16) is solved; summing with the state of fictitious volume forces and adjusting the stress field in accordance with (13) $\sigma_{ij}^{(n)} = s_{ij}^{(n)} + \tilde{\sigma}_{ij}^{(n)}$ allow this additive to be included in the accumulated resulting state with coefficients β^n, α^n.

After performing a sufficient number of approximations, it is necessary to carry out the final substitution of the values of small parameters and go to dimensional quantities.

The solution of the problem

The task for the body in the form of a cube (the Cartesian coordinate system is used). The body occupies the region $V = \{(x, y, z) \mid -1 \le x, y, z \le 1,\}$ and the technical constants of the material [17]: $E = 1.3992$; $E_z = 2.6682$; $\nu = 0.0682$; $\nu_z = 0.248$; $G = 0.6549$; $G_z = 0.5396$; $A = 0.5$; $B = 1.2$; $C = 0.4$; $D = 1.1$; $k = 2$; $\varepsilon_i = 0.1$. Small parameters:; $\beta = 0.053339$; $\alpha = 0.054855$.

The body is loaded along the faces with uniform unitary forces, causing comprehensive tension and shear. Volume forces are absent: $X_i^0 = 0$.

We show the expressions for strains and stresses for n = 3:

$$\varepsilon_{xx} = \varepsilon_{yy} = 0.573 + 0.49984\beta + 0.43601\beta^2 + 0.38033\beta^3;$$

$$\varepsilon_{zz} = 0.18889 - 0.13952\beta - 0.1217\beta^2 - 0.10616\beta^3; \tag{19}$$

$$\varepsilon_{yz} = \varepsilon_{xz} = 0.92661\sum_{n=1}^{3}\alpha^n;$$

$$\varepsilon_{xy} = 0.763436\sum_{n=1}^{3}\beta^n;$$

$$\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = \sigma_{yz} = \sigma_{xz} = \sigma_{xy} = 1.$$

After substituting small deformation parameters:

$$\varepsilon_x = \varepsilon_y = 0.600964; \ \varepsilon_z = 0.181087;$$

$$\varepsilon_{yz} = \varepsilon_{xz} = 0.980383; \ \varepsilon_{xy} = 0.806445.$$

The error will be estimated by comparing the strains of the obtained state with the strains of the elastic state of the material, the technical constants of which:

$$E_r = \frac{4\lambda_4(\beta - 1)(\lambda_3^2 - \lambda_1(\lambda_2 + \lambda_4 - \beta\lambda_4))}{\lambda_3^2 - \lambda_1\lambda_2 + 2(\beta - 1)\lambda_1\lambda_4} = 1.32899;$$

$$E_z = \lambda_1 - \frac{\lambda_3^2}{\lambda_2 + \lambda_4(1 - \beta)} = 2.65922;$$

$$\nu_r = \frac{\lambda_3^2 - \lambda_1\lambda_2}{\lambda_3^2 - \lambda_1(\lambda_2 - 2\lambda_4(\beta - 1))} = 0.07176;$$

$$\nu_z = \frac{\lambda_3}{2(\lambda_2 + \lambda_4(1-\beta))} = 0.25922; \qquad (20)$$

$$G_z = \lambda_5(1-\alpha) = 0.51; \; G_r = \frac{E_r}{2(1+\nu_{r\theta})} = 0.62.$$

For the last state:

$$\varepsilon_x = \varepsilon_y = 0.600967; \; \varepsilon_z = 0.181086;$$

$$\varepsilon_{yz} = \varepsilon_{xz} = 0.980392; \; \varepsilon_{xy} = 0.806452.$$

For deformations, the errors were: ε_{xx} and ε_{yy} – 0.00047 %; ε_z – 0.00043%, ε_{yz} and ε_{xz} – 0.0009 %; ε_{xy} – 0.00081%.

We study the convergence of the obtained series with a significant increase in small parameters. Let now $A = C = 0.3$; $B = 0.2$; $D = 0.1$; $k = 2$; $\varepsilon_0 = 0.1$, $\beta = 0.511401$ and $\alpha = 0.4255$. Here, a comparison must be made with the state for a material for which $E = 0.723381$, $E_z = 2.52729$, $\nu = 0.13028$; $\nu_z = 0.424089$; $G_z = 0.31$; $G = 0.32$. For this strain state:

$$\varepsilon_{xx} = \varepsilon_{yy} = 1.03449; \; \varepsilon_z = 0.60072;$$

$$\varepsilon_{yz} = \varepsilon_{xz} = 1.6129; \; \varepsilon_{xy} = 1.5625.$$

After substituting small parameters in series (19), the strains amounted to:

when n = 3: $\varepsilon_{xx} = \varepsilon_{yy} = 0.99352$; $\varepsilon_z = 0.060072$; $\varepsilon_{yz} = \varepsilon_{xz} = 1.56$; $\varepsilon_{xy} = 1.45563$;

when n = 14: $\varepsilon_{xx} = \varepsilon_{yy} = 1.03449$; $\varepsilon_z = 0.060074$; $\varepsilon_{yz} = \varepsilon_{xz} = 1.6129$; $\varepsilon_{xy} = 1.56243$.

Thus, the accuracy of calculations is ensured by increasing the number of iterations.

We now consider the asymmetric problem for a cylinder (a cylindrical coordinate system is used). The body occupies the area $V = \{(r,z) |\; 0 \le r \le 1, -2 \le z \le 2\}$ and material technical constants: $E = E_r = 1.3992$; $E_z = 2.6682$; $\nu = \nu_r = 0.0682$; $\nu_z = 0.248$; $G = G_r = 0.6549$; $G_z = 0.5396$; $A = 0.5$; $B = 0.2$; $C = 0.4$; $D = 0.1$; $k = 2$; $\varepsilon_i = 0.1$. Small parameters: $\beta = 0.206026$; $\alpha = 0.240178$.

The body is loaded along the faces S_1 and S_2 with the efforts of:

$$\{p_r, p_z\} = \begin{cases} \{1,0\}, | \; r = 1, -2 \le z \le 2; \\ \{0,-1\}, | \; z = -2, 0 \le r \le 1; \\ \{0,1\}, | \; z = 2, 0 \le r \le 1. \end{cases}$$

$$X_i^0 = 0.$$

Solution (13) in the third approximation (n = 3) is the series:

$$u = 0.573r + 0.49984r\beta + 0.43601r\beta^2 + 0.38033r\beta^3;$$

$$w = 0.18889z - 0.13952z\beta - 0.1217z\beta^2 - 0.10616z\beta^3;$$

$$\sigma_r = \sigma_\theta = \sigma_z = 1; \; \sigma_{rz} = \sigma_{r\theta} = \sigma_{z\theta} = 0.$$

After substituting the small parameter, the strains are equal:

$$\varepsilon_r = \varepsilon_\theta = 0.697819; \; \varepsilon_z = 0.154051;$$

$$\varepsilon_{rz} = \varepsilon_{r\theta} = \varepsilon_{z\theta} = 0.$$

We estimate the error in a similar way, only now we need a comparison with a state whose technical constants are (20): $E_r = 1.12778$; $E_z = 2.62834$; $\nu_r = 0.084403$; $\nu_z = 0.297818$; $G_z = 0.42$; $G_r = 0.52$.

For the last state:

$$\varepsilon_r = \varepsilon_\theta = 0.698547; \; \varepsilon_z = 0.153848;$$

$$\varepsilon_{rz} = \varepsilon_{r\theta} = \varepsilon_{z\theta} = 0.$$

For deformations, the errors were: ε_r and ε_θ – 0.1 %; ε_z – 0.13%..

Conclusion

An analysis of the foregoing allows us to conclude that the method of boundary states with perturbations has proven to be an effective means of writing out an explicit solution to physically nonlinear problems of mechanics for and transversely isotropic media. To solve a particular physically nonlinear problem, it is necessary to have an appropriate solution to the linearly elastic problem. However, the accuracy of the approximate solution in the case of nontrivial boundary value problems strongly depends on the magnitude of small parameters deflecting the nonlinear medium from the linear medium.

The study was carried out with the financial support of RFBR and the Lipetsk Region as part of the research project No. 19-41-480003 "p_a".

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