SOI: 1.1/TAS DOI: 10.15863/TAS
International Scientific Journal
Theoretical & Applied Science

p-ISSN: 2308-4944 (print) **e-ISSN:** 2409-0085 (online)

Year: 2020 **Issue:** 01 **Volume:** 81

Published: 30.01.2020 **http://T-Science.org**

QR – Issue QR – Article

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NONLINEAR VIBRATIONS OF A CURRENT-CARRYING ANISOTROPIC CYLINDRICAL SHELL IN A MAGNETIC FIELD

Abstract: *A nonlinear two-dimensional model of magnetoelasticity of a current-carrying cylindrical shell is constructed in the paper with account for anisotropy of conductive properties. It is assumed that the main directions of the anisotropy of shell material properties coincide with the directions of the corresponding coordinate axes, and it is believed that an anisotropic body is linear with respect to magnetic and electrical properties. A coupled system of nonlinear differential equations is obtained that describes the stress-strain state of flexible current-carrying cylindrical shells with anisotropy of conductive properties, under unsteady-state power and electromagnetic loads; a boundary-value problem is formulated.*

Key words: *shell, magnetic field, magneto elasticity.*
Language: *English*
Citation: Indiaminov, R., Kholjigitov, S. M., & Narkulov, A. S. (2020). Nonlinear vibrations of a current-carrying anisotropic cylindrical shell in a magnetic field. *ISJ Theoretical & Applied Science, 01 (81),* 205-211.
Soi: http://s-o-i.org/1.1/TAS-01-81-38 ***Doi***: crossref https://dx.doi.org/10.15863/TAS.2020.01.81.38
Scopus ASCC: *2210.*

Introduction

The study of continuous medium motion with account for electromagnetic effects presents an important field in the mechanics of conjugate fields. Studies on the mechanics of coupled fields in deformable bodies have both fundamental and applied importance, which makes them especially relevant. These issues were studied in [1,2,3,5,7,8,11,18, 19,22,23,24,25,26]. In modern technology, structural materials are used that are anisotropic in the undeformed state, and the anisotropy of the properties of such materials arises as a result of application of various technological processes. The nature of the shell material anisotropy is not determined entirely by its behavior as an elastic body and the anisotropy of the material can manifest itself in relation to its other physical properties, for example magnetic and dielectric permeability and electrical conductivity. Some of the most important anisotropic materials have a crystalline structure. The most characteristic feature of crystals physical properties is their anisotropy and symmetry. Due to the periodicity, regularity, and symmetry of internal structure, a number of properties are discovered in crystals that are impossible to find in isotropic bodies. The anisotropic physical properties of crystals are extremely sensitive to external influences. Therefore, selecting and combining these effects, we may create the materials with unique, unusual properties that are used in modern technology.

Problems interaction between electro-magnetic field and deformed bodies are frequent in advanced technology.

I. MAGNETOELASTIC EQUATIONS.

Let the body be in a magnetic field generated by an electric current in the body itself and by a source located far from the body. Assume that the body serves as a conductor of electric current (current-carrying body), which is supplied to the ends of the body from an external source. It is assumed that an external electric current in an unperturbed state is uniformly distributed over the body (the current density does not depend on the coordinates). The body has finite anisotropic electrical conductivity and does not possess the property of unauthorized polarization and magnetization. Determine the values and write down the equations that characterize the properties of electromagnetic fields. Let the electromagnetic field of the body in the Eulerian coordinate system be characterized by electric field vector $\vec{e}$, magnetic field vector $\vec{h}$, electric induction vector $\vec{d}$ and magnetic induction vector $\vec{b}$, and in the Lagrangian coordinate system by $\vec{E}, \vec{H}, \vec{D}$ and $\vec{B}$, respectively. The analysis of electromagnetic effects is possible on the basis of the Maxwell system of equations, together with constitutive equations connecting the vectors $\vec{d}$ and $\vec{e}$, $\vec{b}$ and $\vec{h}$, $\vec{j}$ and $\vec{e}$, which in the case of linear isotropic medium have the form [12]:

$$\vec{d} = \varepsilon_\alpha \vec{e}, \quad \vec{b} = \mu_\alpha \vec{h}, \quad \vec{j} = \sigma \vec{e}$$

where ε_α, μ_α - are called electric and magnetic permeabilities, respectively, σ - electrical conductivity of the medium.

The properties of the media are characterized by parameters ε_α, μ_α and σ. Depending on the properties of parameters ε_α, μ_α and σ, the following media are distinguished: linear one, in which parameters ε_α, μ_α and σ do not depend on the magnitude of electric and magnetic fields, and nonlinear one, in which the parameters ε_α, μ_α and σ (or at least one of them) depend on the magnitude of electric or magnetic field. All real media are in essence non-linear ones. However, in weak fields, in many cases, the dependence ε_α, μ_α and σ on the magnitude of electric and magnetic fields can be neglected and the medium can be taken as a linear one. In turn, linear media are divided into homogeneous and inhomogeneous, isotropic and anisotropic ones.

The media are homogeneous when their parameters ε_α, μ_α and σ are independent of coordinates, i.e. the properties of the medium are similar at all points. Media in which at least one of the parameters ε_α, μ_α and σ is a function of coordinates is called inhomogeneous. If the properties of the medium are the same in different directions, then the medium is called isotropic. Accordingly, media whose properties are different in different directions are called anisotropic. In isotropic media, vectors $\vec{d}$ and $\vec{e}$, and $\vec{b}$ and $\vec{h}$ are parallel, in anisotropic media, they may not be parallel. In isotropic media, parameters ε_α, μ_α and σ - are the scalar quantities. In anisotropic media, at least one of these parameters is a tensor. Note that the determination of the relationships between the quantities $\vec{e}$ and $\vec{d}$, and $\vec{h}$ and $\vec{b}$ concretizes the model of the medium. Elastic media are the media in which initial relative positions of the particles affect the internal forces (both mechanical and magnetic) everywhere in the body at later times. Therefore, in the study of such objects, it is convenient to use the initial coordinates of each particle, i.e. Lagrangian coordinates. The transition from the Eulerian coordinate system to the Lagrangian one is done using the dependencies [3,11]:

$$\varepsilon_{ijk} \frac{\partial h_k}{\partial \xi_p} \frac{\partial \xi_p}{\partial x_j} = j_i + \frac{\partial d_i}{\partial t};$$

$$\varepsilon_{ijk} \frac{\partial e_k}{\partial \xi_p} \frac{\partial \xi_p}{\partial x_j} = -\frac{\partial b_i}{\partial t};$$

$$\frac{\partial b_i}{\partial \xi_p} \frac{\partial \xi_p}{\partial x_i} = 0; \quad \frac{\partial d_i}{\partial \xi_p} \frac{\partial \xi_p}{\partial x_i} = \rho_e \qquad (1)$$

where ρ_e – is the volume density of electric charges. Omitting the intermediate transforms, Maxwell's equations in Lagrangian variables take the form:

$$\varepsilon_{ijm} \frac{\partial H_m}{\partial \xi_p} = J_p + \frac{\partial D_r}{\partial t};$$

$$\varepsilon_{ijm} \frac{\partial E_m}{\partial \xi_p} = -\frac{\partial B_p}{\partial t};$$

$$\frac{\partial B_p}{\partial \xi_p} = 0; \quad \frac{\partial D_p}{\partial \xi_p} = R_e, \qquad (2)$$

where $H_m = h_k \frac{\partial x_k}{\partial \xi_m}; E_m = e_k \frac{\partial x_k}{\partial \xi_m};$

$$B_p = \Gamma b_i \frac{\partial \xi_p}{\partial x_i}; D_r = \Gamma d_i \frac{\partial \xi_r}{\partial x_i}; \qquad (3)$$

$$J_r = \Gamma j_i \frac{\partial \xi_r}{\partial x_i}; \quad R_e = \Gamma \rho_e; \quad \Gamma = \det\left|\frac{\partial x_i}{\partial \xi_j}\right|.$$

The equations of motion of a material body, which describe their interaction with an electromagnetic field, have the form [11]:

$$\frac{\partial}{\partial \xi_k}\left(\Gamma t_{ij}\frac{\partial \xi_k}{\partial x_j}\right)+\rho_0\left(F_i+F_i^{\wedge}\right)=\rho_0\frac{\partial^2 u_i}{\partial t^2}, \quad (4)$$

where t_{ij} – are the components of the Euler stress tensor; $\rho_0=\Gamma\rho$ – is the density of the material in an undeformed state. Using the Lagrange stress tensor

$$T_{ik}^0=\Gamma t_{ji}\frac{\partial \xi_k}{\partial x_j}, \quad (5)$$

the equations of motion is written in the form

$$\frac{\partial T_{ik}^0}{\partial \xi_k}+\rho_0\left(F_i+F_i^{\wedge}\right)=\rho_0\frac{\partial^2 u_i}{\partial t^2}. \quad (6)$$

Later, equation (10) can be presented as

$$S_{ij,j}+\rho_0\left(F_i+F_i^{\wedge}\right)=\rho_0\frac{\partial^2 u_i}{\partial t^2}. \quad (7)$$

where $\hat{S}$ – is the stress tensor introduced by V.V. Novozhilov [2,3]. In a vector form, the equations of magnetoelasticity have the form [2,3,5,11]:

$$div\,\hat{S}+\rho_0\left(\vec{F}+\vec{F}^{\wedge}\right)=\rho_0\frac{\partial^2\vec{u}}{\partial t^2},$$

$$rot\,\vec{E}=-\frac{\partial\vec{B}}{\partial t},\ rot\,\vec{H}=\vec{J}-\frac{\partial\vec{D}}{\partial t}; \quad (8)$$

$$div\,\vec{B}=0;\ div\,\vec{D}=R_e$$

Relations (3) are written in the vector form as follows:

$$\vec{H}=\vec{h}F^T;\ \vec{E}=\vec{e}F^T;\ \vec{B}=\Gamma\vec{b}F^{-1};$$
$$\vec{D}=\Gamma\vec{d}F^{-1};\ \vec{J}=\Gamma\vec{j}F^{-1}, \quad (9)$$

where $F=\frac{\partial x_i}{\partial \xi_j}\quad (i,\ j=1,2,3)$.

Ohm's generalized law is

$$\vec{J}=\sigma\Gamma F^T F^{-1}\left[\vec{E}+\vec{V}\times\vec{B}\right]+R_e\vec{V}, \quad (10)$$

and the expression for the ponderomotive Lorentz force in constitutive variables is written as

$$\rho_0\vec{F}^{\wedge}=\sigma\Gamma^{-1}F^{-1}\left[(\vec{E}+\vec{V}\times\vec{B})\times\vec{B}\right]+\Gamma^{-1}R_e\vec{V}. \quad (11)$$

Thus, in the final form, the equations of magnetoelasticity are written as follows:

$$div\,\hat{S}+\rho_0\left(\vec{F}+\vec{F}^{\wedge}\right)=\rho_0\frac{\partial^2\vec{u}}{\partial t^2},$$

$$rot\,\vec{E}=-\frac{\partial\vec{B}}{\partial t},\ rot\,\vec{H}=\vec{J},$$

$$div\,\vec{B}=0;\ div\,\vec{D}=0; \quad (12)$$

$$\rho\vec{F}^{\wedge}=\sigma\Gamma^{-1}F^{-1}\left[(\vec{E}+\vec{V}\times\vec{B})\times\vec{B}\right];$$
$$\vec{J}=\sigma\Gamma F^T F^{-1}\left[\vec{E}+\vec{V}\times\vec{B}\right]$$

The system of equations (12) should be completed with initial conditions, boundary conditions, and conditions at infinity [1,3].

Note that div and rot - are the divergence operators of the rotor in relation to the fixed Cartesian basis; σ is the electrical conductivity.

The system of equations of magnetoelasticity must be closed by relations connecting the intensity and induction vectors of electromagnetic field, and by Ohm's laws, which determine the density of the conduction current in a moving medium. If the anisotropic body is linear with respect to magnetic and electrical properties, then the governing equations for the electromagnetic characteristics of the field and the kinematic equations for electrical conductivity, and the expressions for the Lorentz force, taking into account the external current $\vec{J}_{cm}$ in the Lagrange variables, are written in the form [3, 11]:

$$\vec{B}=\mu_{ij}\vec{H},\quad \vec{D}=\varepsilon_{ij}\vec{E},$$

$$\vec{J}=\sigma_{ij}\Gamma F^T F^{-1}\left[\vec{J}_{cm}+\vec{E}+\vec{\upsilon}\times\vec{B}\right], \quad (13)$$

$$\rho\vec{F}^{\wedge}=\Gamma^{-1}F^{-1}\left[\vec{J}_{cm}\times\vec{B}+\sigma_{ij}\left(\vec{E}+\upsilon\times\vec{B}\right)\times\vec{B}\right].$$

Note that in the Maxwell equations the bias currents, the vector of electric induction and the volume density of electric charges (quasistatic field) are neglected; $\sigma_{ij},\varepsilon_{ij},\mu_{ij}$ are the tensors of electrical conductivity, dielectric and magnetic permeability, respectively. For homogeneous anisotropic media, they are symmetric tensors of the second rank [1,4,6,21].

II. NONLINEAR MODEL OF MAGNETO-ELASTICITY OF THE CURRENT-CARRYING SHELLS.

When constructing two-dimensional equations of the internal problem of magnetoelasticity of anisotropic shells in a geometrically nonlinear statement, the Kirchhoff–Love hypothesis and the electromagnetic hypotheses adequate to it are used [1,2,3,5].

The proposed two-dimensional model of magnetoelasticity for problems of the theory of anisotropic shells is constructed in the quadratic approximation; cubic nonlinearity is taken into account in the terms for the Lorentz forces.

This is due to the fact that in this statement, the interaction of the electromagnetic field with the strain field is carried out due to these forces only.

When studying the electrodynamic equations of motion of the theory of anisotropic shells, a system of curvilinear coordinates $\theta^i\,(i=1,2,3)$ in relative configuration of the body is used.

Considering a finite three-dimensional anisotropic shell in three-dimensional Euclidean space, we assign coordinates θ^i to each material point of the body.

Let $\vec{r}$ be the radius vector of a characteristic particle in a given configuration of the body at t, and similarly, denote by $\vec{R}$ the radius vector in a fixed relative configuration, which is the initial one.

In this case, in the framework of three-dimensional theory, kinematic relations are represented as follows:

$$\vec{R} = \vec{R}(\theta^i); \quad \vec{r} = \vec{r}(\theta^i, t);$$
$$\vec{G} = \frac{\partial \vec{R}}{\partial \theta^2}; \vec{g}_i = \frac{\partial \vec{r}}{\partial \theta^i}; \vec{G}^i \cdot \vec{G}^j = \delta^i_j;$$
$$\vec{g}^i \cdot \vec{g}_j = \delta^i_j;$$
$$G_{ij} = \vec{G}_i \cdot \vec{G}_j; G^{ij} = \vec{G}^i \cdot \vec{G}^j;$$
$$g_{ij} = \vec{g}_i \cdot \vec{g}_j; g^{ij} = \vec{g}^i \cdot \vec{g}^j; F = \vec{g}_i \otimes \vec{G}^i;$$
$$\vec{g}_i = F\vec{G}_i; \vec{G}^i = F^T \vec{g}^i;$$
$$\Gamma = \det F = \sqrt{g/G}; \tag{14}$$
$$\sqrt{g} = \vec{g}_1 \cdot (\vec{g}_2 \times \vec{g}_3);$$
$$\sqrt{G} = \vec{G}_1 \cdot (\vec{G}_2 \times \vec{G}_3),$$

where: $\vec{g}_i, \vec{g}^i$ – covariant and contravariant base vectors, respectively; g_{ij}, g^{ij} covariant and contravariant metric configuration tensors at a time t; $\vec{G}^i, \vec{G}_i, G_{ij}, G^{ij}$ – appropriate values for the initial configuration; δ_i^j - the Kronecker delta; symbol $\otimes$ - stands for tensor of the procedure. Einstein summation convention is not performed.

Considering (14), the vectors of electromagnetic quantities can be represented by the following relationships:

$$\vec{e} = e_i \vec{g}^i; \ \vec{h} = h_i \vec{g}^i; \ \vec{b} = b^i \vec{g}_i,$$
$$\vec{j} = j^i \vec{g}_i; \vec{E} = E_i \vec{G}^i; \ \vec{H} = H_i \vec{G}^i; \tag{15}$$
$$\vec{B} = B^i \vec{G}_i; \ \vec{j} = j^i \vec{G}_i$$

With (1) and (14), we obtain

$$E_i = e_i; \ H_i = h_i; G^{1/2} B^i = g^{1/2} b^i;$$
$$G^{1/2} j^i = g^{1/2} j^i, \ (i = 1,2,3). \tag{16}$$

Expressions (16) establish the relationship between electromagnetic quantities in the Lagrangian and Euler coordinate systems through the metric of the initial and current configurations.When constructing the two-dimensional theory of thin shells in a geometrically nonlinear statement, the equations and relations of the theory of flexible shells are used in quadratic approximation, using the classical model of undeformable normals and hypotheses in electrodynamics [1,2,3,5].

For the case of quadratic nonlinearity [2,3,5] under consideration, the strains and rotation angles are small, but the latter are significantly superior to the former. The selected infinitely small volume element under strain changes its position in space (due to displacement and rotation) and, in addition, changes its size and form. An account for the elongation smallness and neglect of the shifts compared with the angles of rotation allows us to make no differences between the dimensions of the volume element before and after strain. This makes it possible to present the volume element after strain as equal to the volume element before strain with the only difference (geometrical) in its position in space. The above allows us to accept that

$$S_i^* / S_i \simeq 1 \ \text{и} \ V^* / V \simeq 1, \ (i = 1,2,3)$$

Here S_i - is the elementary areas with the normals $\vec{n}_i$ before strain, S_i^* - the same area after strain; V и V^* are the volumes of the elementary element before and after strain. This approach allows nonlinearity to be taken into account in relations for strains, curvatures and torsion. In this case, the shell metric remains practically undeformed, since the radii of curvature and the Lame parameters correspond to the undeformed state of the shell. Note that, based on these considerations, relations (6) are reduced to

$$E_i = e_i; \ H_i = h_i; \ B^i = b^i;$$
$$j^i = j^i, \ (i = 1,2,3) \tag{18}$$

Let us consider a thin current-carrying circular cylindrical shell under unsteady electromagnetic and mechanical loads, neglecting the effect of polarization and magnetization processes, and thermal stresses. Assume that an alternating electric current from an external source is supplied to the shell end.

The elastic properties of shell material are considered orthotropic, the main directions of elasticity coincide with the directions of the corresponding coordinate lines, the electromagnetic properties of the material are characterized by tensors of electrical conductivity σ_{ij}, magnetic permeability μ_{ij}, and dielectric permittivity ε_{ij} $(i, j = 1, 2, 3)$.

Based on the physics of crystals [4] and form [3,11, 21], it is assumed that the tensors σ_{ij}, μ_{ij}, ε_{ij} take a diagonal form for the considered class of conducting orthotropic media with a rhombic crystal structure.

According to the results of [8-17], and considering the geometry of the shell, a complete system of equations in a curvilinear orthogonal coordinate system is derived; it allows mathematical description of a nonlinear two-dimensional model of magnetoelasticity of orthotropic cylindrical shells; it consists of:

equations of motion

$$\frac{\partial}{\partial s}(RN_s)+\frac{\partial S}{\partial\theta}+R\left(p_s+\rho F_s^{\wedge}\right)=R\rho h\frac{\partial^2 u}{\partial t^2};$$

$$\frac{\partial N_\theta}{\partial\theta}+\frac{1}{R}\frac{\partial}{\partial s}\left(R^2S\right)+\frac{\partial H}{\partial s}+Q_\theta+$$

$$+\left(p_\theta+\rho F_\theta^{\wedge}\right)=R\rho h\frac{\partial^2 v}{\partial t^2} \qquad (19)$$

$$\frac{\partial}{\partial s}(RQ_s)+\frac{\partial Q_\theta}{\partial\theta}-N_\theta+R\left(p_z+\rho F_z^{\wedge}\right)=R\rho h\frac{\partial^2 w}{\partial t^2};$$

$$\frac{\partial H}{\partial\theta}+\frac{\partial}{\partial s}(RM_s)-RQ_s-R+$$

$$\left(N_s-\frac{M_\theta}{R}\right)\vartheta_s-RS\vartheta_\theta=0;$$

$$\frac{1}{R}\frac{\partial}{\partial s}\left(R^2H\right)+\frac{\partial M_\theta}{\partial\theta}-RQ_\theta-RN_\theta\vartheta_\theta-RS\vartheta_s=0;$$

$$\left(S=N_{\theta s}=N_{s\theta}-\frac{1}{R}M_{\theta s},H=M_{s\theta}=M_{\theta s}\right)$$

Maxwell equations

$$-\frac{\partial B_z}{\partial t}=\frac{1}{R}\left(\frac{\partial(RE_\theta)}{\partial s}-\frac{\partial E_s}{\partial\theta}\right);$$

$$\sigma_1\left[E_s-\frac{\partial v}{\partial t}B_z-0,5\frac{\partial w}{\partial t}\left(B_\theta^{+}+B_\theta^{-}\right)\right]=$$

$$=\frac{1}{R}\left(\frac{\partial H_z}{\partial\theta}-\frac{R\left(H_\theta^{+}-H_\theta^{-}\right)}{h}\right); \qquad (20)$$

$$\sigma_2\left[E_\theta-\frac{\partial u}{\partial t}B_z+0,5\frac{\partial w}{\partial t}\left(B_s^{+}+B_s^{-}\right)\right]=$$

$$=\left(-\frac{\partial H_z}{\partial s}+\frac{\left(H_s^{+}-H_s^{-}\right)}{h}\right),$$

strain relationships

$$\varepsilon_s=\frac{\partial u}{\partial s}+\frac{1}{2}\theta_s^{\,2};\ \varepsilon_\theta=\frac{1}{R}\frac{\partial v}{\partial\theta}+\frac{1}{R}w+\frac{1}{2}\theta_\theta^{\,2};$$

$$\varepsilon_{s\theta}=\frac{1}{R}\frac{\partial u}{\partial\theta}+R\frac{\partial}{\partial s}\left(\frac{v}{R}\right)+\vartheta_s\vartheta_\theta;$$

$$\chi_s=\frac{\partial\theta}{\partial s};\ \ \chi_\theta=\frac{1}{R}\frac{\partial\vartheta_\theta}{\partial\theta};$$

$$2\chi_{s\theta}=\frac{\partial\vartheta_\theta}{\partial s}+\frac{1}{R}\frac{\partial\vartheta_s}{\partial\theta}+\frac{1}{R}\frac{\partial v}{\partial s}; \qquad (21)$$

where

$$\vartheta_s=-\frac{\partial w}{\partial s};\vartheta_\theta=-\frac{1}{R}\frac{\partial w}{\partial\theta}+\frac{v}{R}. \qquad (22)$$

elasticity relationships

$$N_s=\frac{e_s h}{1-\nu_s\nu_\theta}\left(\varepsilon_s+\nu_\theta\varepsilon_\theta\right);$$

$$N_\theta=\frac{e_\theta h}{1-\nu_s\nu_\theta}\left(\varepsilon_\theta+\nu_s\varepsilon_s\right);\ S=g_{s\theta}h\varepsilon_{s\theta};$$

$$M_s=\frac{e_s h^3}{12(1-\nu_s\nu_\theta)}\left(\chi_s+\nu_\theta\chi_\theta\right); \qquad (23)$$

$$M_\theta=\frac{e_\theta h^3}{12(1-\nu_s\nu_\theta)}\left(\chi_\theta+\nu_s\chi_s\right);$$

$$H=g_{s\theta}\frac{h^3}{12}\chi_{s\theta}$$

Here $\nu_s=\nu_{\theta s}$; $\nu_\theta=\nu_{s\theta}$; $e_s\nu_\theta=e_\theta\nu_s$.

The components of the ponderomotive force are:

$$\rho F_s^{\wedge}=-hJ_{\theta cm}B_z+$$

$$\sigma h\left[E_\theta B_z-\frac{\partial u}{\partial t}B_z^2+0,5\frac{\partial w}{\partial t}\left(B_s^{+}+B_s^{-}\right)B_z\right]+$$

$$+\sigma h\frac{\partial v}{\partial t}(0,25\left(B_s^{+}+B_s^{-}\right)\left(B_\theta^{+}+B_\theta^{-}\right)+$$

$$++\frac{1}{12}\left(B_s^{+}-B_s^{-}\right)\left(B_\theta^{+}-B_\theta^{-}\right)-\frac{1}{12}\left(B_\theta^{+}+B_\theta^{-}\right)B_z\ )$$

$$\rho F_\theta^{\wedge}=hJ_{scm}B_z-$$

$$\sigma h\ (\frac{\mu}{\sigma R}\left(\frac{\partial B_z}{\partial\theta}-\frac{R\left(B_\theta^{+}-B_\theta^{-}\right)}{h}\right)-\frac{\partial v}{\partial t}B_z+$$

$$++0,5\frac{\partial w}{\partial t}\left(B_\theta^{+}+B_\theta^{-}\right))\,B_z+$$

$$(24)$$

$$+\sigma h0,5\frac{\partial w}{\partial t}\left(B_\theta^{+}+B_\theta^{-}\right)B_z-\sigma h\frac{\partial v}{\partial t}B_z^2-$$

$$\sigma h\frac{\partial v}{\partial t}\left(0,25\left(B_\theta^{+}+B_\theta^{-}\right)^2+\frac{1}{12}\left(B_\theta^{+}-B_\theta^{-}\right)^2-0,5\left(B_s^{+}+B_s^{-}\right)B_z\right)$$

$$\rho F_z^{\wedge}=0,5h\left[-J_{scm}\left(B_\theta^{+}+B_\theta^{-}\right)+J_{\theta cm}\left(B_s^{+}+B_s^{-}\right)\right]+$$

$$+0,5\sigma h\ (\frac{\mu}{\sigma R}\left(\frac{\partial B_z}{\partial\theta}-\frac{R\left(B_\theta^{+}-B_\theta^{-}\right)}{h}\right)-\frac{\partial v}{\partial t}B_z+$$

$$+0,5\frac{\partial w}{\partial t}\left(B_\theta^{+}+B_\theta^{-}\right))\ \left(B_\theta^{+}+B_\theta^{-}\right)-$$

$$-\sigma h0,5E_\theta\left(B_s^{+}+B_s^{-}\right)+\sigma h0,5\frac{\partial u}{\partial t}\left(B_s^{+}+B_s^{-}\right)B_z+$$

$$+\sigma h0,5\frac{\partial v}{\partial t}\left(B_\theta^{+}+B_\theta^{-}\right)B_z-$$

$$-\sigma h \frac{\partial w}{\partial t}\left[0{,}25\left(B_\theta^+ + B_\theta^-\right)^2 + 0{,}25\left(B_s^+ + B_s^-\right)^2 + \right.$$

$$\left. + \frac{1}{12}\left(B_\theta^+ - B_\theta^-\right)^2 + \frac{1}{12}\left(B_s^+ - B_s^-\right)^2\right]$$

Here N_s, N_θ are the normal tangential efforts; S - shear force; Q_s, Q_θ - transverse forces; M_s, M_θ, H - bending and torsional moments, respectively; u, v, w - components of displacements; E_s, E_θ - components of the electric field strength; B_z - normal component of magnetic induction; B_s^+, B_s^- - known components of magnetic induction from the shell surface. $J_{s\,cm}, J_{\theta\,cm}$ - components of the density of electric current from an external source; σ – electrical conductivity; $h = h(s,\theta)$ - shell thickness; E - Young's modulus; ν – Poisson's ratio. The above equations correspond to the quadratic theory of shells [2,3,5,20]. The component of Lorentz forces take into account the strain rate of the shell, the external magnetic field, the magnitude and intensity of the conduction current relative to external magnetic field. The inclusion of nonlinearity in the equations of motion causes nonlinearity in the ponderomotive force.

III. CONCLUSION.

Dielectric and magnetic properties of a rigid body change under both the change in density, and under strains (shears), when the density remains the same. As a result of strain, the dielectric and magnetic properties of the body become anisotropic, and the scalar dielectric and magnetic permeability is replaced by tensors of the second rank. Based on the equations obtained, using the proposed technique, we are able to take into account both the anisotropy of material and the anisotropy of internal electromagnetic field of the shell, as well as the effect of strains on the electromagnetic properties of the body.

Such problems of electromagnetoelasticity are very relevant from the point of view of their applications. In the case of thin anisotropic or isotropic bodies with anisotropic electrical conductivity, it is possible to pose and solve optimal problems of magnetoelasticity by varying physicomechanical parameters of the body material. In particular, under constant mechanical and geometric parameters of the problem, by changing the anisotropic electrodynamic parameters, one can obtain structural elements with a qualitatively new mechanical behavior.

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