

## Impact Factor:

ISRA (India) = 4.971  
ISI (Dubai, UAE) = 0.829  
GIF (Australia) = 0.564  
JIF = 1.500

SIS (USA) = 0.912  
PIHII (Russia) = 0.126  
ESJI (KZ) = 8.716  
SJIF (Morocco) = 5.667

ICV (Poland) = 6.630  
PIF (India) = 1.940  
IBI (India) = 4.260  
OAJI (USA) = 0.350

SOI: [1.1/TAS](#) DOI: [10.15863/TAS](#)

### International Scientific Journal Theoretical & Applied Science

p-ISSN: 2308-4944 (print) e-ISSN: 2409-0085 (online)

Year: 2019 Issue: 12 Volume: 80

Published: 18.12.2019 <http://T-Science.org>

QR – Issue



QR – Article



**Ismoil Ibragimovich Safarov**

Institute of Chemistry and Technology

Doctor of Physical and Mathematical Sciences,

Professor to department of Advanced Mathematics, Tashkent, Uzbekistan

[Safarov54@mail.ru](mailto:Safarov54@mail.ru)

**Mansur Tulaganovich Ergashev**

Institute of Chemistry and Technology

Senior Lecturer to Department of Technology Engineering,

Tashkent, Republic of Uzbekistan

**Nurillo Raximovich Kulmurov**

Navoi State Mining Institute

Senior Lecturer to Department of Technology Engineering, Uzbekistan

[nurillo.Kulmurov.64@mail.ru](mailto:nurillo.Kulmurov.64@mail.ru)

## OSCILLATIONS OF SPHERICAL HETEROGENEITY IN A VISCOELASTIC MEDIUM

**Abstract:** This paper considers the problem of radial, torsional and spheroidal oscillations of a deformable spherical inclusion in an infinite viscoelastic medium. Based on the methods of special functions of mathematical physics, the Muller method and the Gauss method, an algorithm is developed for solving electronic computing machine problem. Based on the constructed complex dispersion equations with complex output parameters, numerical results are obtained and analysis is made.

**Key words:** inclusions, cavity, viscoelastic medium, dispersion equation, oscillations, natural frequency, equations of motion.

**Language:** English

**Citation:** Safarov, I. I., Ergashev, M. T., & Kulmurov, N. R. (2019). Oscillations of spherical heterogeneity in a viscoelastic medium. *ISJ Theoretical & Applied Science*, 12 (80), 244-249.

**Soi:** <http://s-o-i.org/1.1/TAS-12-80-49> **Doi:**  <https://dx.doi.org/10.15863/TAS.2019.12.80.49>

**Scopus ASCC:** 2210.

### Introduction

Due to the increased interest in the development of new approaches to solving urgent modern problems of non-destructive remote control of materials, medical diagnostics, geoaoustic problems associated with seismic sounding of underground engineering structures and several others, it becomes necessary to formulate and analyze some model problems, the solution of which will give the key to the development of new highly effective methods for actively sensing in homogeneities. Any heterogeneity, together with its surrounding medium, must possess, like any elastic mechanical system, some spectrum of natural frequencies. Since the oscillations of the inclusion and

the surrounding medium, are interconnected, the damping of the oscillations due to the emission of elastic waves will occur and, therefore, the natural frequencies will be complex. Thus, the tasks associated with the identification of heterogeneities, with the determination of their size and physical characteristics, are very important and relevant. Since geophysicists use different approaches for these purposes, such as gravity exploration, electromagnetic methods, the study of electrical conductivity, etc., the seismic method is perhaps the most direct and, when interpreted, gives the least doubtful results. In this case, it is natural to expect that when the frequency of the incident wave is close to the real component of the

## Impact Factor:

ISRA (India)	= 4.971	SIS (USA)	= 0.912	ICV (Poland)	= 6.630
ISI (Dubai, UAE)	= 0.829	PIHHI (Russia)	= 0.126	PIF (India)	= 1.940
GIF (Australia)	= 0.564	ESJI (KZ)	= 8.716	IBI (India)	= 4.260
JIF	= 1.500	SJIF (Morocco)	= 5.667	OAJI (USA)	= 0.350

natural frequency, the heterogeneity will begin to radiate energy in the resonant mode. Therefore, for practical purposes, identifying possible resonance peaks on the spectral curve and establishing their relationship with the corresponding heterogeneities, it is very important to know the natural oscillation frequencies of elastic inclusions in an infinite elastic medium [2,3]. On the basis that numerical analysis of the behavior of the scattering cross section, the authors of the monograph [4] suggested that the extremal values of the cross section are observed when the frequency of the incident wave is close to the resonance frequencies of the medium – scattering center system, but no such consideration was performed. This gap was substantially filled after the publication of a number of articles [5–7], in which, using the analysis of the echo signal, the resonance properties of liquid cylinders and spheres placed in an infinite elastic medium are studied. A similar approach can be used to study the tectonic physical phenomenon - the behavior of the earthquake source. In the seismic field recorded by the seismic receiver, the heterogeneity will leave a mark in the form of some distortions of the original seismic wave. In spectral analysis, these distortions should appear in the form of some resonance peaks in the spectral curve, as is the case when determining the scattering cross section. Recently, experimental work has been intensively conducted on the propagation of ultrasonic waves in static models of elastic medium containing foreign inclusions and fractured zones [8]. An ideal elastic body has no losses [9,10]. Even if the equation is linear with respect to stress and strain, the presence of temporary derivatives is always associated with dissipation. As a result, under alternating voltage, a hysteresis effect occurs. This means that in the frequency range in which the attenuation is noticeable, the deformation will lag behind the stress. Such a connection, firstly, leads to the interaction of the considered elastic wave with other waves (for example, with thermal oscillations) and as a result there is a redistribution of energy between the waves. Secondly, the wave under consideration will generate higher harmonics, transmitting its energy to them. In both cases, the interaction depends on the strain amplitude. The nonlinear relationship between stress and strain in the presence of time derivatives also leads to damping, depending on the strain amplitude. In addition, the study of heterogeneity is of great interest for the study of an important tectonic physical phenomenon - the behavior of the focus of the impending earthquake. Now, among seismologists, the idea of the zone of preparation of seismic shocks is widely accepted as an area with elastic-density characteristics changing as a result of tectonic movements. From a mechanical point of view, this corresponds to heterogeneity with longitudinal and transverse wave velocities slightly changed relative to the external elastic medium, as well as, possibly, with density. Any heterogeneity, together with its surrounding medium, must possess, like any

elastic mechanical system, some spectrum of natural frequencies. Since the oscillations of the inclusion and the surrounding medium are interconnected, the damping of the oscillations due to the emission of elastic waves will occur and, therefore, the natural frequencies will be complex [11]. From a physical point of view, attenuation in an ideal elastic medium is explained by energy radiations excited by natural oscillations due to diverging elastic waves. The behavior of complex self-frequencies depending on the geometric and physical-mechanical parameters of the system is investigated. The interest in studying the self-frequencies of the system elastic inclusion - medium is also due to the following circumstance. When a heterogeneity is detected by seismic waves either from weak earthquakes or from pulsed artificial sources such as pneumatic emitters, the scattering problem must be solved in an unsteady setting. Such a body is characterized by a linear unambiguous relationship between stress and strain over the entire period of alternating stress. It follows that stress and strain are always in phase. The energy dissipation of the elastic wave will occur if the stress and strain are not uniquely related during the period of oscillation. The absence of such an unambiguous relationship between stress and strain arises when time derivatives appear in the equation connecting them [12].

As is well-known, in this case, for calculating the wave field, the stationary solution should be integrated over the frequency along with the spectrum of the given incident pulse. The resulting integral can, in general, be calculated by any direct numerical method. In some cases, however, preference should be given to the method of integration using theory of residues in the form of an expansion at the poles of the integrand, since it is this method that can reveal a number of useful physical features of the diffraction process. We note that the poles of interest to us coincide with the roots of the self-frequency equation and, therefore, in order to be able to deal with the problems of unsteady diffraction of elastic waves in the future, a thorough study of the behavior of the roots of the frequency equations depending on the ratio of the elastic-density parameters of the medium and inclusion is necessary [13]. This article discusses the oscillations of spherical bodies in a deformable medium. The main attention in the work will be given to the study of low-contrast heterogeneity. In this case, we were guided by the following considerations. The physical nature of such heterogeneities is closely related to convective currents in the earth's interior, as well as to various areas of faults and fragmentations. Such inclusions are very common and, therefore, have a significant effect on the scattering of seismic waves.

### Basic relationships and equations.

The basic equations of motion of deformable (elastic or viscoelastic) spherical inclusions in an

infinite viscoelastic medium with linear self-oscillations have the form

$$(\tilde{\lambda}_j + 2\tilde{\mu}_j)graddiv\vec{u} - \tilde{\mu}_j rotrot\vec{u} = \rho_j \frac{\partial^2 \vec{u}}{\partial t^2}, \quad j=1,2 \quad (1)$$

Where,

$$\tilde{\lambda}_j \phi(t) = \lambda_{0j} \left[ \phi(t) - \int_0^t R_{\lambda_j}(t-\tau) \phi(\tau) d\tau \right];$$

$$\tilde{\mu}_j \phi(t) = \mu_{0j} \left[ \phi(t) - \int_0^t R_{\mu_j}(t-\tau) \phi(\tau) d\tau \right]. \quad (2)$$

$\tilde{\lambda}$  and  $\tilde{\mu}$  -operators modulus of elasticity [14,15],  $\phi(t)$ —arbitrary function of time;  $\rho_j$ —density,  $R_{\lambda}(t-\tau)$  and  $R_{\mu}(t-\tau)$  —relaxation core and  $\lambda_{01}, \mu_{01}$ —instant modulus of elasticity. We take the integral terms in (2) small, then the function  $\phi(t) = \psi(t)e^{-i\omega_R t}$ , where  $\psi(t)$ —slowly changing function of time,  $\omega_R$ —real constant. Further, applying the freezing procedure [16], we replace relations (2) with approximate forms

$$\tilde{\lambda}\phi = \lambda_{01} [1 - \Gamma_{\lambda}^C(\omega_R) - i\Gamma_{\lambda}^S(\omega_R)];$$

$$\tilde{\mu}\phi = \mu_{01} [1 - \Gamma_{\mu}^C(\omega_R) - i\Gamma_{\mu}^S(\omega_R)]\phi, \quad (3)$$

where

$$\Gamma_{\lambda}^C(\omega_R) = \int_0^{\infty} R_{\lambda}(\tau) \cos \omega_R \tau d\tau; \Gamma_{\lambda}^S(\omega_R) = \int_0^{\infty} R_{\lambda}(\tau) \sin \omega_R \tau d\tau,$$

$$\Gamma_{\mu}^C(\omega_R) = \int_0^{\infty} R_{\mu}(\tau) \cos \omega_R \tau d\tau, \Gamma_{\mu}^S(\omega_R) = \int_0^{\infty} R_{\mu}(\tau) \sin \omega_R \tau d\tau,$$

—the cosine and sine Fourier images of the core relaxation material, respectively. As an example of a viscoelastic material, we take three parametric relaxation nuclei  $R_{\lambda}(t) = R_{\mu}(t) = Ae^{-\beta t}/t^{1-\alpha}$ . On the function of influence  $R(t-\tau)$  imposed the usual inerrability requirement, continuity (except  $t = \tau$ ), sign-definiteness and monotony:

$$R)0, \frac{dR(t)}{dt} \leq 0, 0 < \int_0^{\infty} R(t) dt < 1.$$

Our task is to study periodic processes in a continuous elastic medium with a spherical inclusion that differs in its elastic-density and rheological characteristics from the corresponding characteristics of the enclosing medium. Therefore, we take the time dependence in the form  $\vec{u} = \vec{U}(r, \theta, \varphi)e^{i\omega t}$ . Spatial coordinate function  $\vec{U}(r, \theta, \varphi)$  can be represented as the sum of potential  $\vec{U}_p = grad\phi$  and solenoid  $\vec{U}_s = rot\vec{\psi}$  parts:  $\vec{U} = \vec{U}_p + \vec{U}_s$ , which satisfy the following equations

$$(\Delta + k_p^2)\vec{U}_p = 0; (\Delta + k_s^2)\vec{U}_s = 0,$$

$$div\vec{U}_p = 0; div\vec{U}_s = 0 \quad (4)$$

Where

$$k_p^2 = \omega^2/\Gamma_{pk}c_p^2; k_s^2 = \omega^2/\Gamma_{sk}c_s^2,$$

$$\Gamma_{pk} = 1 - \Gamma_{pk}^C(\omega_R) - i\Gamma_{pk}^S(\omega_R);$$

$$\Gamma_{sk} = 1 - \Gamma_{sk}^C(\omega_R) - i\Gamma_{sk}^S(\omega_R), c_p^2 = \frac{\lambda+2\mu}{\rho};$$

$$c_s^2 = \mu/\rho$$

— the propagation velocity of longitudinal and transverse waves in an elastic body.

On specified two bodies, the conditions of continuity (hard contact) of displacements and stresses are set at  $r = R$ :

$$\sigma_{rr1} = \sigma_{rr2}; \quad \sigma_{r\theta 1} = \sigma_{r\theta 2}; \quad \sigma_{r\varphi 1} = \sigma_{r\varphi 2};$$

$$u_{r1} = u_{r2}; \quad u_{\theta 1} = u_{\theta 2}; \quad u_{\varphi 1} = u_{\varphi 2}, \quad (5)$$

where index 1 refers to a decision within the inside sphere and 2—interfering medium. It is known [17] that a vector displacement field can be decomposed into three vector fields in spherical coordinates, each of which is determined by only one scalar function  $\vec{U} = \vec{U}_p + \vec{U}'_s + \vec{U}''_s$ . In this view  $\vec{U}_p$ —longitudinal,  $\vec{U}'_s$  and  $\vec{U}''_s$ —transverse parts of the solution. Their expressions through scalar functions have the form

$$\vec{U}_p = \frac{1}{k_p} grad\psi_0; \vec{U}'_s = rot(\vec{r}\psi_1);$$

$$\vec{U}''_s = \frac{1}{k_s} rotrot(\vec{r}\psi_2),$$

So each of the scalar functions  $\psi_i$  ( $i=0,1,2$ ) satisfies the equation

$$(\Delta + k_i^2)\psi_i = 0; k_i = \{k_p, i = 0; k_s, i = 1,2\}. \quad (6)$$

Where  $\Delta$  - second order operator in spherical coordinates. The solution of the scalar Helmholtz equation (6) for each of the functions  $\psi_i(r, \theta, \varphi)$  has the form

$$\psi_{0l} = \sum_{n=0}^{\infty} \sum_{m=n}^n A_{mnl} h_n(k_{pl}r) P_n^m(\cos \theta) \exp(im\varphi);$$

$$\psi_{1l} = \sum_{n=0}^{\infty} \sum_{m=n}^n B_{mnl} h_n(k_{sl}r) P_n^m(\cos \theta) \exp(im\varphi);$$

$$\psi_{2l} = \sum_{n=0}^{\infty} \sum_{m=n}^n C_{mnl} h_n(k_{sl}r) P_n^m(\cos \theta) \exp(im\varphi),$$

where  $h_n(z)$  - Bessel spherical function;  $l=1,2$ ;  $l=1$ —refers to the medium, and  $l=2$  to the spherical body;  $P_n^m(\cos \theta)$  – the adjoint Legendre function of the first kind of the  $n$ -th degree and  $m$ -th order. In calculating the Legendre function  $n \gg 1$ , we used the asymptotic formulas from [18]

$$P_{n-\frac{1}{2}}(\cos \theta) = \left( \frac{2}{\pi n \sin \theta} \right)^{\frac{1}{2}} \left[ \cos \left( n\Delta - \frac{\pi}{4} \right) + \frac{ctg\theta}{8n} \sin \left( n\theta - \frac{\pi}{4} \right) + \theta(1/n^2) \right].$$

For an external problem (fluctuations in the medium,  $l=1$ ) we will take as  $h_n(z)$  Hankel function of the second kind

$$h_n(z) = \sqrt{\frac{\pi}{2z}} H_{n+1/2}^{(2)}(z),$$

Which sets off at infinity ( $r \rightarrow \infty$ ) diverging waves. For an internal problem (switching oscillations,  $l=2$ ) we will take as  $h_n(z)$  Bessel function of the first kind

$$h_n(z) = \sqrt{\frac{\pi}{2z}} J_{n+1/2}(z) = j_n(z),$$

which satisfies the condition of roundedness at zero. As a result, we obtain the following expression for the bias

## Impact Factor:

ISRA (India) = 4.971  
 ISI (Dubai, UAE) = 0.829  
 GIF (Australia) = 0.564  
 JIF = 1.500

SIS (USA) = 0.912  
 PIIHJ (Russia) = 0.126  
 ESJI (KZ) = 8.716  
 SJIF (Morocco) = 5.667

ICV (Poland) = 6.630  
 PIF (India) = 1.940  
 IBI (India) = 4.260  
 OAJI (USA) = 0.350

$\vec{u}$ , describing the oscillatory process in the enclosing medium and spherical heterogeneity:

$$u_{r,l} = \sum_{n=0}^{\infty} \sum_{m=-n}^n \left[ \frac{A_{mnl}}{k_{pl}r} D_1(k_{pl}r) + \frac{B_{mnl}}{k_{sl}r} n(n+1)h_n(k_{sl}r) \right] \Phi_n^m$$

$$u_{\theta,l} = \sum_{n=0}^{\infty} \sum_{m=-n}^n \left\{ \left[ \frac{A_{mnl}}{k_{pl}r} h_n(k_{pl}r) + \frac{B_{mnl}}{k_{sl}r} D_2(k_{sl}r) \right] \frac{\partial \Phi_n^m}{\partial \theta} + C_{mnl} h_n(k_{sl}r) \frac{\partial \Phi_n^m}{\partial \phi} / \sin \theta \right\} \quad (7)$$

$$u_{\phi,l} = \sum_{n=0}^{\infty} \sum_{m=-n}^n \left\{ \left[ \frac{A_{mnl}}{k_{pl}r} h_n(k_{pl}r) + \frac{B_{mnl}}{k_{sl}r} D_2(k_{sl}r) \right] \frac{\partial \Phi_n^m}{\partial \phi} \frac{1}{\sin \theta} - C_{mnl} h_n(k_{sl}r) \frac{\partial \Phi_n^m}{\partial \theta} \right\}$$

Where

$$\Phi_n^m = P_n^m(\cos \theta) \exp(im\phi),$$

$$D_1(z) = nh_n(z) - zh_{n+1}(z),$$

$$D_2(z) = (n+1)h_n(z) - zh_{n+1}(z).$$

Stresses  $\sigma_{rr}, \sigma_{r\theta}, \sigma_{\theta\theta}$  are written out through displacements in spherical coordinates according to the following formulas:

$$\sigma_{rr} = \lambda \operatorname{div} \vec{u} + 2\mu \frac{\partial u_r}{\partial r}, \sigma_{r\theta} = \mu \left( \frac{1}{r} \frac{\partial u_r}{\partial \theta} + \frac{\partial u_{\theta}}{\partial r} - \frac{u_{\theta}}{r} \right),$$

$$\sigma_{r\phi} = \mu \left( \frac{1}{r \sin \theta} \frac{\partial u_r}{\partial \phi} + \frac{\partial u_{\phi}}{\partial r} - \frac{u_{\phi}}{r} \right). \quad (8)$$

Substituting (7) in (8), we obtain for stresses inside and outside the heterogeneity

$$\sigma_{rr,l} = \frac{2\tilde{\mu}}{r} \sum_{n=0}^{\infty} \sum_{m=-n}^n \left[ \frac{A_{mnl}}{k_{pl}r} D_3(k_{pl}r) + \frac{B_{mnl}}{k_{sl}r} n(n+1)D_4(k_{sl}r) \right] \Phi_n^m,$$

$$\sigma_{r\theta,l} = \frac{2\tilde{\mu}}{r} \sum_{n=0}^{\infty} \sum_{m=-n}^n \left\{ \left[ \frac{A_{mnl}}{k_{pl}r} D_4(k_{pl}r) + \frac{B_{mnl}}{k_{sl}r} D_5(k_{sl}r) \right] \frac{\partial \Phi_n^m}{\partial \theta} + 0.5 C_{mnl} \frac{D_4(k_{sl}r)}{\sin \theta} \frac{\partial \Phi_n^m}{\partial \phi} \right\}, \quad (9)$$

$$\sigma_{r\phi,l} = \frac{2\tilde{\mu}}{r} \sum_{n=0}^{\infty} \sum_{m=-n}^n \left\{ \left[ \frac{A_{mnl}}{k_{pl}r} D_4(k_{pl}r) + \frac{B_{mnl}}{k_{sl}r} D_5(k_{sl}r) \right] \frac{\partial \Phi_n^m}{\partial \phi} \frac{1}{\sin \theta} - 0.5 C_{mnl} D_4(k_{sl}r) \frac{\partial \Phi_n^m}{\partial \theta} \right\},$$

where

$$D_3(z) = (n^2 - n - \frac{1}{2}z^2) \cdot h_n(z) + 2zh_{n+1}(z), D_4(z) = (n-1) \cdot h_n(z) - zh_{n+1}(z),$$

$$D_5(z) = (n^2 - n - \frac{1}{2}z^2) \cdot h_n(z) + zh_{n+1}(z),$$

Coefficients  $A_{nm1}, B_{nm1}, C_{nm1}, A_{nm2}, B_{nm2}, C_{nm2}$  must be determined from the boundary conditions (6) on the surface of the sphere. This system leads to two important classes of natural oscillations: torsional and spheroidal.

### Radial oscillations.

Determine the complex frequency of radial oscillations of a spherical cavity in an unbounded viscoelastic medium for which  $c_p \gg c_s$ ,  $u_r = u = \partial \phi / \partial r$ . The oscillation equation (1) in the potentials (longitudinal waves) of displacements, taking into account the periodicity of the oscillations, reduces to the Helmholtz equation

$$\Delta \phi = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial \phi}{\partial r} \right) = -k_p^2 \Gamma_{pk}. \quad (10)$$

We are looking for a solution to equation (10) in the form of a diverging spherical wave

$$\phi = A e^{ik_p r} / r.$$

Radial stresses

$$\sigma_{rr} = \rho [(c_p^2 - 2c_s^2) \Delta \phi + 2c_s^2 \phi''_{rr}]$$

or using the equation (10):

$$\frac{1}{\rho} \sigma_{rr} = -\omega^2 \phi - 4c_s^2 \phi' / r.$$

Boundary condition  $\sigma_{rr}(R) = 0$  leads to the equation

$$\left( k_p R \frac{c_p}{c_s \sqrt{\Gamma_s}} \right)^2 = 4(1 - ik_p R / \sqrt{\Gamma_p}). \quad (11)$$

Hence, for

$$R_{\lambda} = R_{\mu} = 0, c_p \gg c_s: \omega = \frac{2c_s}{R} (1 - i \frac{c_s}{c_p}).$$

For Poisson's medium

$$c_p / c_s = \sqrt{3} \text{ then } \omega = \frac{2c_s}{R} (1 - i\sqrt{3}).$$

It can be seen that for the Poisson medium, the wave is attenuated intensely. Real part  $\omega$  gives its own oscillation frequency, and the imaginary part is the attenuation coefficient. In an incompressible medium ( $c_p \rightarrow \infty$ ) attenuation would naturally be absent. As the radius of the hole increases, the corresponding frequencies and attenuation coefficients decrease with a hyperbolic law. In the study of the radial oscillations of a spherical inclusion in a viscoelastic medium, is necessary  $n = 0$ , then

$$u_r = A_{00} h_1(k_p r),$$

$$\sigma_{rr} = \frac{\mu}{r} A_{00} \left[ 4h_1(k_p r) - \frac{k_s}{k} (k_s r) h_0(k_p r) \right]$$

Using the boundary conditions  $\sigma_{rr1} = \sigma_{rr2}$ ,  $u_{r1} = u_{r2}$  for  $r = R$ , we arrive at the frequency equations of radial oscillations of a spherical inclusion

$$z_p z_{sp} z_{\omega} \operatorname{ctg}(z_p z_{sp} z_{\omega}) = 1 - \frac{z_{\omega}}{z_p z_{\omega}^2 + 4(\frac{1}{z_{\mu}} - 1)(1 + iz_{sp} z_{\omega})} \quad (12)$$

Here, similarly, there are two limiting cases: the radial oscillations of the full sphere ( $z_{\mu} \rightarrow 0$ ) and cavities ( $z_{\mu} \rightarrow \infty$ ).

Accordingly, we obtain

$$z_{\omega} \operatorname{ctg}(z_{\omega}) = 1 - \frac{z_{\omega}^2}{4z_{sp}^2}, z_{\omega}^2 - 4iz_{\omega} - 4 = 0.$$

### Torsional oscillations.

They are characterized by the vanishing of the radial component ( $u_r = 0$ ) displacement vectors as well as dilatations  $\operatorname{div} \vec{u}$ . It is easy to see that in the general solution (7) they correspond to the part including the  $C_{mn}$  coefficients. Substitution of this part in the boundary conditions (5) leads to the following system of equations for determining the  $C_{mn1}, C_{mn2}$  coefficients:

$$C_{mn1} h_n(k_{s1} R) = C_{mn2} j_n(k_{s2} R)$$

$$\mu_1 C_{mn1} [(n-1)h_n(k_{s1} R) - (k_{s1} R)h_{n+1}(k_{s1} R)] = \mu_2 C_{mn2} [(n-1)j_n(k_{s2} R) - (k_{s2} R)j_{n+1}(k_{s2} R)]$$



## Impact Factor:

ISRA (India) = 4.971  
 ISI (Dubai, UAE) = 0.829  
 GIF (Australia) = 0.564  
 JIF = 1.500

SIS (USA) = 0.912  
 PIIHII (Russia) = 0.126  
 ESJI (KZ) = 8.716  
 SJIF (Morocco) = 5.667

ICV (Poland) = 6.630  
 PIF (India) = 1.940  
 IBI (India) = 4.260  
 OAJI (USA) = 0.350

From equating the determinant of the system to zero, we obtain the transcendental equation for the natural frequencies of torsional oscillations of a spherical inclusion:

$$[n - 1 - G_i(z_s)] - z_\mu [n - 1 - G_h(z_\omega)] = 0, \quad (13)$$

Where

$$G_i(t) = t j_{n+1}(t)/j_n(t), G_h(t) = t h_{n+1}(t)/h_n(t),$$

$z_\omega = \omega R/c_{s2}$  - dimensionless frequency reduced to transverse velocity in a medium,

$z_s = ((c_{s2}\sqrt{\Gamma_{sk2}})/(\sqrt{\Gamma_{sk1}c_{s1}}))$ ,  $z_\mu = \bar{\mu}_2/\bar{\mu}_1 = \rho z_s$  - the ratio of the shear moduli of the enclosing medium and the inclusion,  $\rho = \rho_2/\rho_1$  - density ratio. It is easy to see that equation (13) has a solution of a set of complex frequencies  $z_\omega = z_{\omega R} + iz_{\omega I}$ . The real part  $z_{\omega R}$  determines the natural frequency and the imaginary part  $z_{\omega I}$  corresponding attenuation coefficient. If  $z_\mu \rightarrow 0$ , then we naturally come to the real frequency equation of torsional oscillations of the full sphere

$$n - 1 - G_i(z_s) = 0.$$

Since there is no radiation, this equation defines the real discrete spectrum  $z_\omega^{(k)}$ . For  $z_\mu \rightarrow \infty$  we obtain a complex equation for determining the natural frequencies of the damping coefficient of torsional oscillations of a spherical cavity.

### Spheroidal oscillations.

This class of oscillations is characterized by the vanishing of the radial components  $rot\vec{u}$ . In the general solution (7), a part corresponds to this class, includes  $A_{mn}$ ,  $B_{mn}$  coefficients. Substitution of this part in the boundary conditions (5) gives a homogeneous algebraic equation for determining the  $A_{mn1}$ ,  $B_{mn1}$ ,  $A_{mn2}$ ,

$$\begin{vmatrix} n^2 - n - \frac{1}{2}(z_\omega)^2 + 2G_i(z_{spi}z_\omega) & n(n+1)[n-1-G_i(z_\omega)] \\ n-1-G_i(z_{spi}z_\omega) & n^2 - n - \frac{1}{2}(z_\omega)^2 + 2G_i(z_\omega) \end{vmatrix} = 0$$

where

$$z_{spi} = ((c_{s2i}\sqrt{\Gamma_{sk2i}})/(\sqrt{\Gamma_{pk2i}c_{p2i}})).$$

$$\begin{vmatrix} n^2 - n - \frac{1}{2}(z_\omega)^2 + 2G_h(z_{sph}z_\omega) & n(n+1)[n-1-G_h(z_\omega)] \\ n-1-G_h(z_{sph}z_\omega) & n^2 - n - \frac{1}{2}(z_\omega)^2 + 2G_h(z_\omega) \end{vmatrix} = 0.$$

### Summation of series.

When calculating the scattering coefficient by the formula (14), the summation is carried out until the ratio of the current term to the current particular sum becomes less  $10^{-10}$ , convergence of series (14) shows numerical experiments. For complex roots, the Müller method of I. Burstow simplifies calculations and provides faster convergence than the Newton method and Burstow simplifies calculations and provides more if the roots are close to each other. Table 1 shows

and  $B_{mn2}$  coefficients. The fact that the determinant of the system is equal to zero leads to a transcendental equation for the self-frequencies of spheroidal oscillations

$$\begin{vmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & c_{34} \\ c_{41} & c_{42} & c_{43} & c_{44} \end{vmatrix} = 0, \quad (14)$$

Where elements  $c_{ij}$  ( $i=1,2,3,4; j=1,2,3,4$ ):

$$c_{11} = n - G_i(z_p z_{sp} z_\omega), c_{12} = n(n+1),$$

$$c_{13} = n - G_h(z_{sp} z_\omega), c_{14} = n(n+1),$$

$$c_{21} = 1, c_{22} = n+1 - G_i(z_s z_\omega),$$

$$c_{23} = 1, c_{24} = n+1 - G_h(z_\omega),$$

$$c_{31} = n^2 - n - \frac{1}{2}(z_s z_\omega)^2 + 2G_i(z_p z_{sp} z_\omega), c_{32} = n(n+1)[n-1-G_i(z_s z_\omega)],$$

$$c_{33} = n^2 - n - \frac{1}{2}z_\omega^2 + 2G_h(z_{sp} z_\omega), c_{34} = n(n+1)[n-1-G_h(z_\omega)],$$

$$c_{41} = n-1 - G_i(z_p z_{sp} z_\omega) c_{42} = n^2 - 1 - \frac{1}{2}(z_s z_\omega)^2 + G_i(z_s z_\omega),$$

$$c_{43} = n-1 - G_h(z_{sp} z_\omega) c_{44} = n^2 - 1 - \frac{1}{2}z_\omega^2 +$$

$G_h(z_\omega)$  Here,  $z_p = ((c_{p2}\sqrt{\Gamma_{pk2}})/(\sqrt{\Gamma_{pk1}c_{p1}}))$  - ratio of longitudinal velocities outside and inside the sphere,  $z_{sp} = ((c_{s2}\sqrt{\Gamma_{sk2}})/(\sqrt{\Gamma_{pk2}c_{p2}}))$  - the ratio of transverse and longitudinal speeds for the enclosing medium, and the remaining notation has the same meaning as when considering torsional oscillations. Transcendental equations (13) for  $z_\mu \rightarrow 0$ , goes over into the real equation of the spheroidal oscillations of the full sphere

For  $z_\mu \rightarrow \infty$  we arrive at the complex transcendental equation for the complex self-frequencies of the spheroidal oscillations of the cavity:

examples of errors determined by the formula and the required number of members of the series. It is seen that to calculate the attenuation coefficient and natural frequencies, it is necessary to take 11-16 members of the series. In this case, the rounding error is up to 1% ( $\rho_m/\rho_0 = 0,02; \bar{C} = 0,5; a=1; 2. \rho_m/\rho_0 = 50; \bar{C} = 0,5; a=1$ ).

**Table 1. Error in determining the frequency and damping coefficients for different numbers of rows.**

Inclusions in the medium	$\Omega$	$\eta$	Error, %	Number of member
Germanium to Aluminum	1.0	0,46321	1	4
	5.0	2,86653	0,5	11
	10.0	3,51241	0,5	16
Aluminum to Germany	1.0	0,21235	1,8	4
	5.0	1,24673	0,7	11
	10.0	2,323573	0,5	16

**Conclusion.**

1. The theory and methods for calculating the complex self-frequencies of oscillations of an elastic spherical heterogeneity in an elastic medium are constructed. Such oscillations are classified into radial, torsional and spheroidal. Problems come down to finding those  $\Omega = \Omega_R + i\Omega_i$  ( $\Omega_R$  - real and  $\Omega_i$  - imaginary parts of complex natural frequencies), in which the system of equations of motion and shortened radiation conditions have a nonzero solution in the class of infinitely differentiable functions. It is shown that the problem has a discrete spectrum.
2. Detailed numerical calculations of self-frequencies and Q factors were performed for the radial and first several oscillation numbers of the torsion and

spheroidal classes. The case was considered when the viscoelastic and elastic characteristics of the inclusion and the host medium differ (not too much and too much).

3. It was found that at some values of viscoelastic density parameters, low-frequency self-oscillations arise. These oscillations are essentially some aperiodic motion, since the imaginary part of the natural frequency is large.
4. The obtained numerical results for plane mechanical systems in a particular case are compared with known values. In short waves ( $h/\lambda > 0,5$ ) results differ up to 10-15%, and in long waves ( $h/\lambda > 0,5$ ) up to 25%.

**References:**

1. Baz, A. I., Zeldovich, Ya. B., & Perelomov, A. M. (1971). *Scattering, reactions, and decays in nonrelativistic quantum mechanics.* (p.554). Moscow: Nauka.
2. Watson, G. N. (1949). *Theory of Bessel functions.* (p.798). Moscow: Publishing house of foreign countries.
3. Gradshteyn, I. S., & Ryzhik, I. M. (1971). *Tables of integrals, sums, series and products.* (p.1108). Moscow: Nauka.
4. Guz, A. N., Kubenko, V. D., & Chnerenko, M. A. (1978). *Diffraction of elastic waves.* (p.308). Kiev: Naukova Dumka.
5. Jacob, Dc. (1979). *Earth core.* (p.305). Moscow: Mir.
6. Dubrovsky, V. A. (1977). Seismic oscillations of inclusions. in the book. *Problems of oscillational transillumination of the Earth.* Moscow: Nauka, pp. 71-75.
7. Dubrovsky, V. A., & Morochnik, B. C. (1981). Natural oscillations of spherical inhomogeneity in an elastic medium. *Izv. USSR Academy of Sciences. Earth Physics,* 7, pp. 29-41.
8. Avliyakov, N. N., & Safarov, I. I. (2007). *Modern tasks of statics and dynamics of underground turbine pipelines.* (p.306). Toshkent.
9. Bozorov, M. B., Safarov, I. I., & Shokin, Yu. I. (1966). *Numerical simulation of oscillations of dissipatively homogeneous and heterogeneous mechanical systems.* (p.188). SB RAS, Novosibirsk.
10. Safarov, I. I. (1992). *Oscillations and waves in dissipatively inhomogeneous media and structures.* (p.250). Tashkent: Fan.
11. Safarov, I. I., & Kayumov, S. S. (2002). *Propagation and diffraction of waves in dissipatively inhomogeneous cylindrical deformable mechanical systems.* (p.214). Tashkent: FAN.
12. Gontkevich, B. C. (1964). *Natural oscillations of shells and liquids.* (p.103). Kiev: Science Dumka.