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SOME VALUES OF THE ZETA FUNCTION

Abstract: *This paper investigates some values of the Zeta function along the critical line.*

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Introduction

Harmonic series are a special case of a more General type of function called the Zeta function ζ(s).

The real Zeta function is given for two real numbers r and n [1-2]:

$$\zeta(n)=\sum_{r=1}^{\infty}\frac{1}{r^n}=1+\frac{1}{2^n}+\frac{1}{3^n}+\frac{1}{4^n}+\frac{1}{5^n}+...+\frac{1}{(r-1)^n}+\frac{1}{r^n}+...$$

The Zeta-function

If we substitute n = 1, we get a harmonic series that diverges. However, for all values n > 1, the series converges, that is, the sum with increasing r tends to a certain number, and does not go to infinity. [3]

Euler product formula

The first connection between Zeta functions and primes was established by Euler when he showed that for two natural (integer and greater than zero) numbers n and p, where is a Prime, the following holds true [4-6]:

$$\sum_n\frac{1}{n^s}=\prod_p\frac{1}{1-p^{-s}}$$

The Euler product for two numbers n and p, where both are greater than zero and p is Prime.

This expression first appeared in the article 1737 under the title Variae observationes circa series infinitas. It follows from the expression that the sum of the Zeta function is equal to the product of quantities, the inverse of one, minus the inverse of Prime numbers to the power of s. This relationship laid the Foundation of modern Prime number theory, in which since then the Zeta function ζ (s) has been used as a way of studying Prime numbers [1, 7-9].

Proof of Euler product formula

Euler starts with the General Zeta function

$$\zeta(s)=1+\frac{1}{2^s}+\frac{1}{3^s}+\frac{1}{4^s}+\frac{1}{5^s}+\frac{1}{6^s}+...$$

The Zeta-function

First it multiplies both parts by the second term:

$$\frac{1}{2^s}\times\zeta(s)=\frac{1}{2^s}+\frac{1}{4^s}+\frac{1}{6^s}+\frac{1}{8^s}+\frac{1}{10^s}+\dots$$

Zeta function multiplied by $\frac{1}{2^s}$

It then subtracts the resulting expression from the Zeta function:

$$\left(1-\frac{1}{2^s}\right)\zeta(s)=1+\frac{1}{3^s}+\frac{1}{5^s}+\frac{1}{7^s}+\frac{1}{9^s}+\dots$$

The Zeta function minus $\frac{1}{2^s}$, multiplied by the Zeta-function

It repeats this process, further multiplying both sides by the third term

$$\frac{1}{3^s}\times\zeta(s)\left(1-\frac{1}{2^s}\right)=\frac{1}{3^s}+\frac{1}{9^s}+\frac{1}{15^s}+\frac{1}{21^s}+\dots$$

Zeta function minus $\frac{1}{2^s}$ multiplied by Zeta function multiplied by $\frac{1}{3^s}$

And then subtracts the resulting expression from the Zeta function

$$\left(1-\frac{1}{3^s}\right)\left(1-\frac{1}{2^s}\right)\zeta(s)=1+\frac{1}{5^s}+\frac{1}{7^s}+\frac{1}{11^s}+\frac{1}{13^s}+\frac{1}{17^s}+\frac{1}{19^s}+\dots$$

Zeta function minus $\frac{1}{2^s}$ multiplied by Zeta function minus $\frac{1}{3^s}$ multiplied by Zeta function

If we repeat this process indefinitely, eventually we will have an expression:

$$\dots\left(1-\frac{1}{13^s}\right)\left(1-\frac{1}{11^s}\right)\left(1-\frac{1}{7^s}\right)\left(1-\frac{1}{5^s}\right)\left(1-\frac{1}{3^s}\right)\left(1-\frac{1}{2^s}\right)\zeta(s)=1$$

1 minus all the value reverse to the ordinary numbers multiplied by the Zeta-function

If this process is familiar to you, it is because Euler essentially created a sieve very similar to the sieve of Eratosthenes. It filters out non-Prime numbers from the Zeta function.

Then divide the expression into all its terms, which are the inverse of primes, and we get:

$$\zeta(s)=\left(\frac{1}{1-\frac{1}{2^s}}\right)\times\left(\frac{1}{1-\frac{1}{3^s}}\right)\times\left(\frac{1}{1-\frac{1}{5^s}}\right)\times\left(\frac{1}{1-\frac{1}{7^s}}\right)\times\left(\frac{1}{1-\frac{1}{11^s}}\right)\times\dots.$$

Functional relationship of the Zeta function with primes for the first primes 2,3,5,7 and 11.

Simplifying the expression, we showed the following:

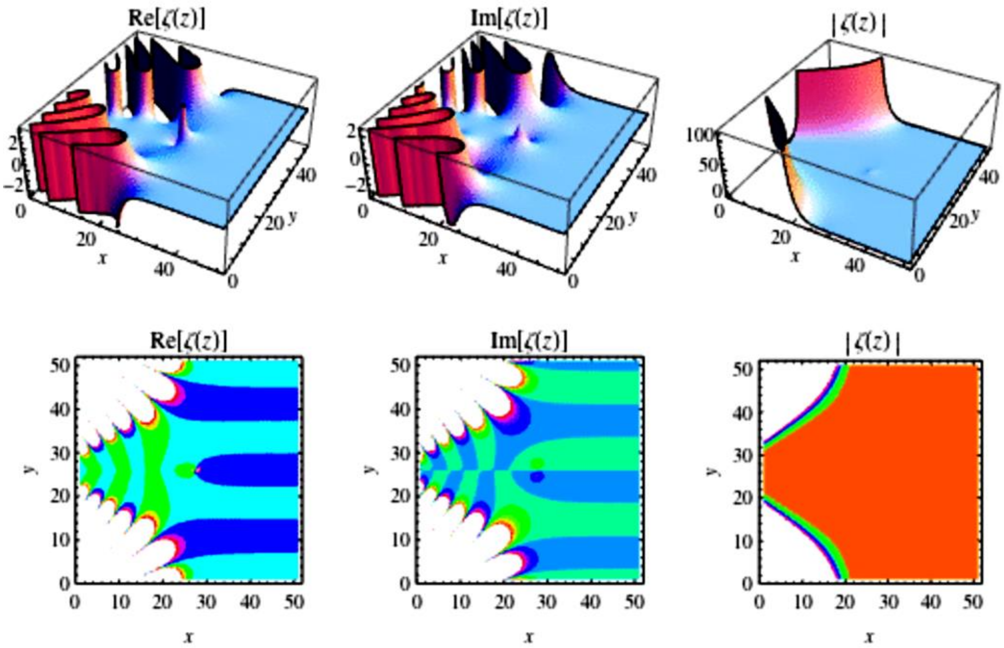
$$\sum_n\frac{1}{n^s}=\prod_p\frac{1}{1-p^{-s}}$$

The Euler product formula is an equality showing the relationship between primes and the Zeta function

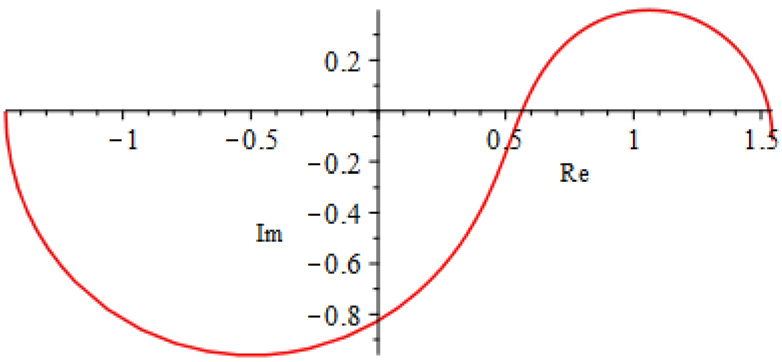
Substitute s = 1, and find an infinite harmonic series, re-proving the infinity of primes.

Conclusion

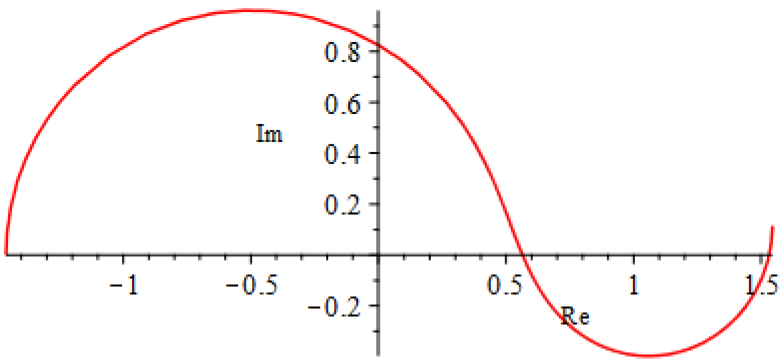
These algorithms make it possible to display the Zeta function function along the critical line (Pic. 2-8). The findings data can be investigated in further studies.



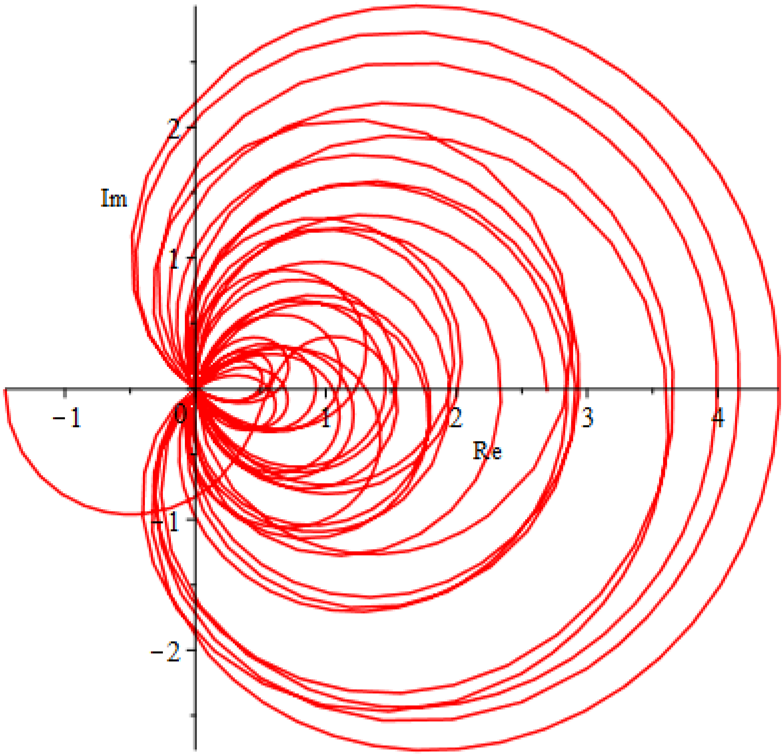
Picture 1. Zeta function graph



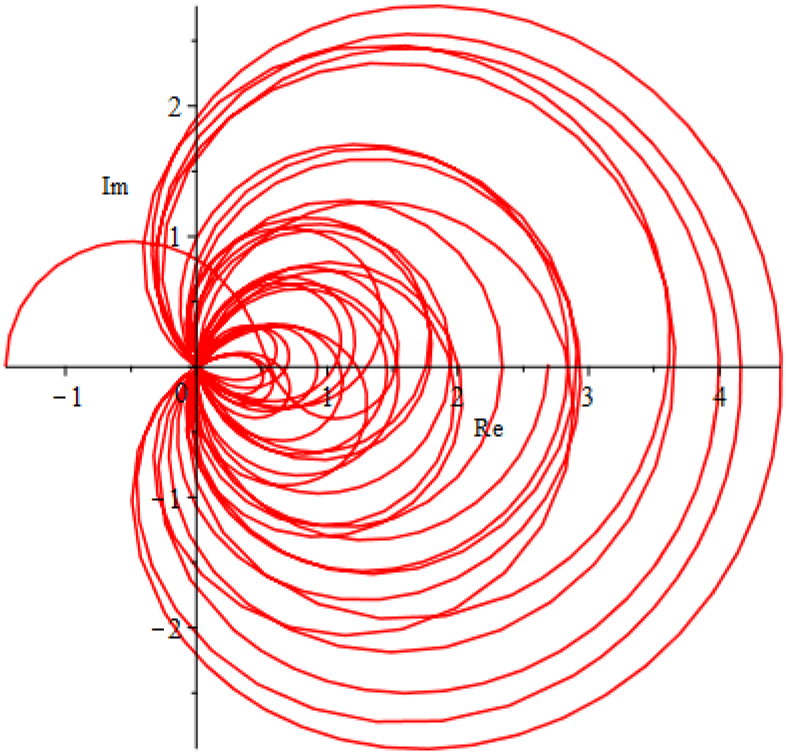
Picture 2. Plot of $\zeta\left(\frac{1}{2}+t\cdot i\right)$ at $t=0\ldots10$.



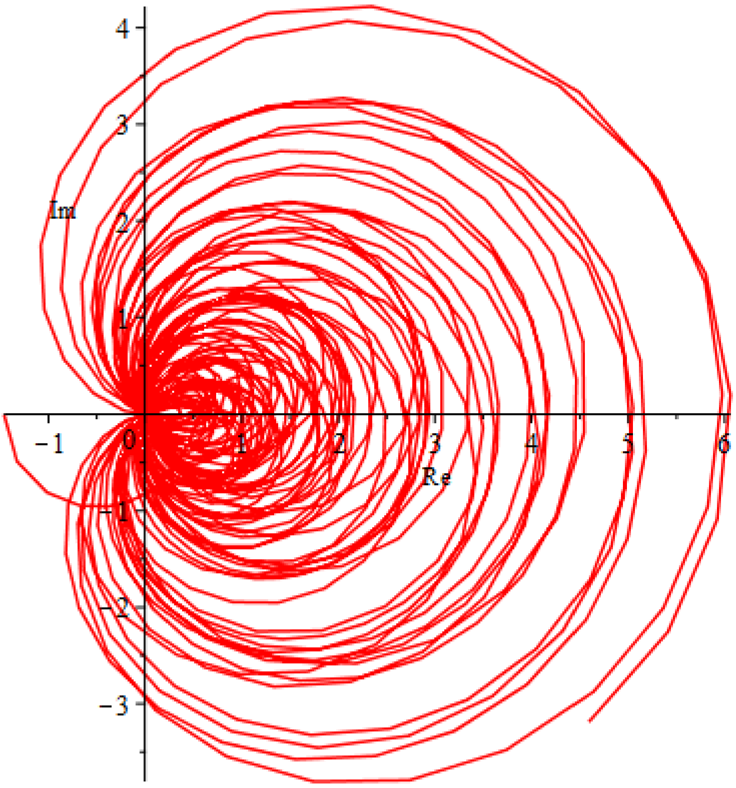
Picture 3. Plot of $\zeta\left(\frac{1}{2}+t\cdot i\right)$ at $t=\text{-}10\ldots0$



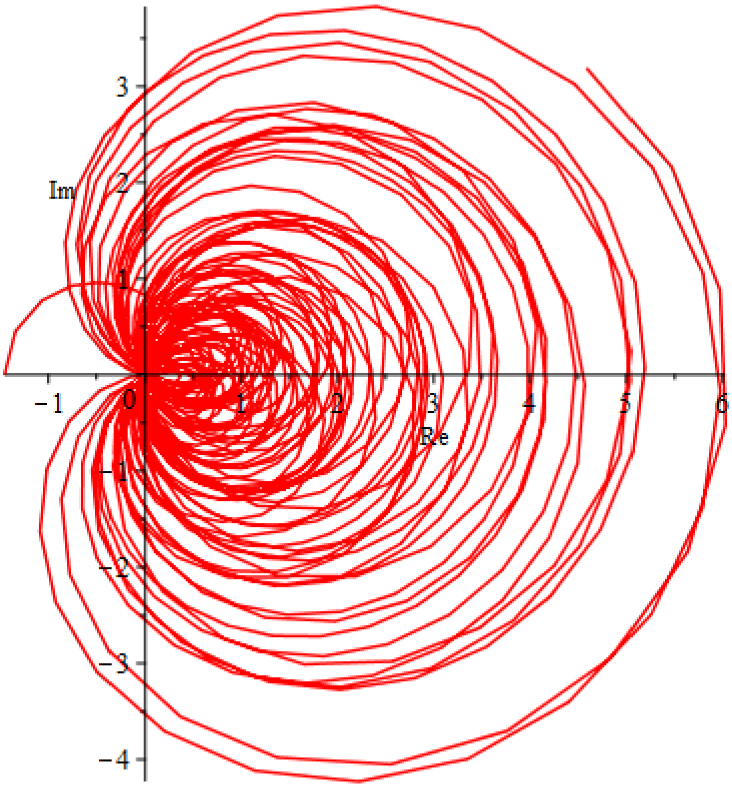
Picture 4. Plot of $\zeta\left(\frac{1}{2}+t\cdot i\right)$ at $t=0\ldots100$.



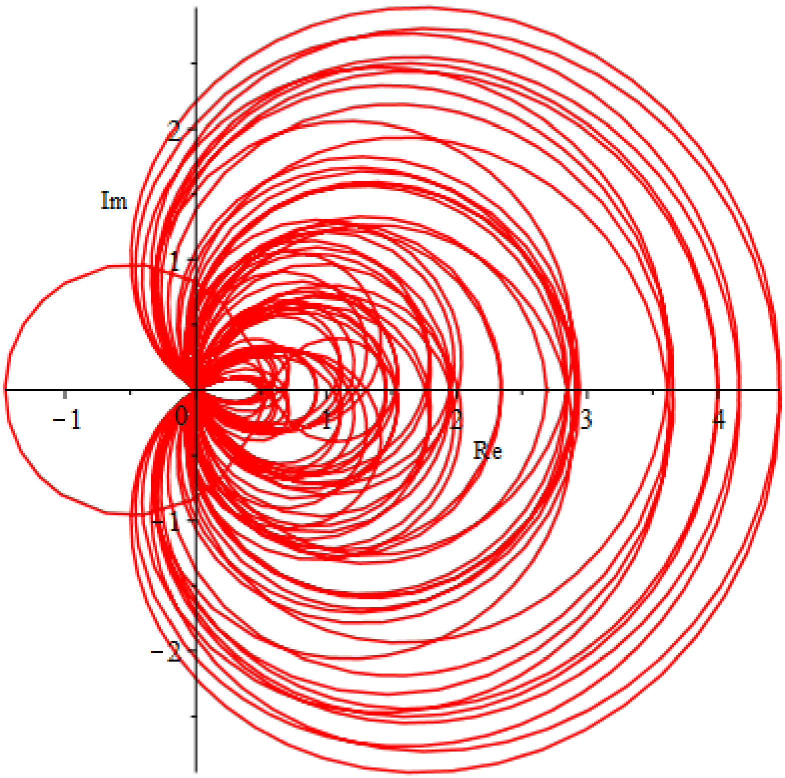
Picture 5. Plot of $\zeta\left(\frac{1}{2}+t\cdot i\right)$ at $t=$-100…0.



Picture 6. Plot of $\zeta\left(\frac{1}{2}+t\cdot i\right)$ at $t=0\ldots200$.



Picture 7. Plot of $\zeta\left(\frac{1}{2}+t\cdot i\right)$ at $t=$-200…0.



Picture 8. Plot of $\zeta\left(\frac{1}{2}+t\cdot i\right)$ at t =-100..100.

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