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## RESEARCHES OF JOINT WORK OF BEAMS AND SOIL BASES

**Abstract:** Basic principles for calculating contact tasks of a structure interacting with a soil base. The main objectives of the theory of contact interaction tasks (lying or embedded in an array of soil building structures loaded with external or gravitational loads) of structure and soil foundation: a) Structures interacting with the ground foundation perceive external  $q(x)$  and reactive stress  $P(x)$  soil pressure, from the difference of which in the body structure bindings  $W(x)$ , bending moments  $M(x)$  and crossing stress  $Q(x)$ . In this case, the calculation of the displacements and stress of the beam must satisfy two basic requirements: when calculating the deflection of a beam, the static requirements must be satisfied  $q(x) + P(x) = 0$ . b) The deflections of the beam must be calculated taking into account the elastic-plastic work of the soil foundation corresponding to the moment of stabilization of its sediment. Depending on the type of loading, at a certain distance  $l$  from the axis of loading will have restrictions on displacements  $0 \leq W(x) \leq f_{max}$ . The precipitation of the foundation system as a whole is carried out taking into account the basic laws of soil mechanics, in particular, according to elastic-plastic models. c) The deflection of building structures and (deformation) of the soil in the contact area meets the requirements of continuity, i.e.  $0 \leq P(x) \leq 1, 2R$ , where  $R$ -reactive stresses do not exceed the quasilinear region of soil deformation. The functional of the reactive pressure  $P(x)$  depends on the type of loading and the type of soil located in the contact area of the beam thickness  $-H_s$ . d) The total or total stiffness of the structure ( $EI$ ) lying in or embedded in the ground and the quasilinear deformable soil base  $t$  (bending stiffness of the soil) forms the deflection function of the structure  $0 \leq W(x) \leq f_{max}$  and the function of the radius of curvature  $\rho(x) = W'(x)$ . The function of the soil reactive pressure on the structure  $P(x)$ , together with external forces  $q(x)$  while maintaining the equilibrium condition, forms the function of internal forces and affects the magnitude of the bending moments  $M(x)$  and cutting forces  $Q(x)$ .

**Key words:** structures interacting; contact interaction; buckling; reactive pressure.

**Language:** English

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### Introduction

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It is known from classical mechanics that the calculated deformations and the internal forces of a loaded beam (slab), the stiffness  $EI$  lying on an elastic foundation, depend on the adopted contact model. In contact models, the reactive pressure  $p(x)$  is assumed constant as linearly dependent on the stiffness of the soil to local shear  $C_1$  [1] (proposed by Mousse,

Winkler - Zimmerman, Pasternak P.L.). The mathematical expression of this model is as follows

$$\frac{d^4 Y}{dx^4} = \frac{qb}{EI} - \frac{4}{s^4} Y. \quad (1)$$

Differential equalization (1) characterizes the bending of a beam loaded with external  $q(x)$  and reactive loads  $P(x)$ . The function of reactive pressure is considered unknown, for this reason its value is selected in accordance with the accepted models or as a functional. In particular, in expression (1), its value is in accordance with the Winkler model. In this case,

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according to the statics law, the beam at the edges is conventionally fixed on the fixed articulated supports. If  $\frac{dy}{dx} = 0$  deflection  $\frac{d^4y}{dx^4} = 0$  reactive pressure under the beam is uniform, in this case the condition of the Winkler – Zimerman model is satisfied.

Given the similarity of the functions of the derivatives  $w^{IV}(x)$  и  $w^1(x)$  and to simplify the problem of determining the value of reactive pressure Pasternak PL [1] reduces the task to mind

$$P(x) = C_1 w(x) + C_2 \dot{w}(x). \quad (2)$$

The second part of equalization (2) allows you to take into account the deflection of the beam when a uniformly distributed load is applied to the beam. When describing expression (2), the authors assumed that the first part of the equation corresponds to the Winkler model and characterizes the reactive pressure  $P(x)$  proportional to the value of the vertical draft of the beam due to soil compression (draft) [1]. It can be represented as a model for immersing a wooden beam in an aquatic environment

$$P(x) = N/l = (\gamma_w - \gamma_c)z = Cw(x), \quad (3)$$

where  $C = (\gamma_w - \gamma_c)$  – is the difference liquid and solid density component.

The second part describes the magnitude of the reactive pressure  $P(x)$  proportional to the radius of curvature, i.e. due to the resistance of the soil shear arising when bending  $w(x)$ . If we assume that the beam is affected by a uniformly distributed load with intensity  $q$  then with its symmetric bend [8]

$$P(x) = C_2 \frac{f}{l} = C_2 \varepsilon, \quad (4)$$

the coefficient characterizing the stiffness of the structure bending

$$C_2 \cong 4 \frac{EI}{l^3}, \quad (5)$$

where  $\varepsilon = \frac{f}{l}$  - relative deflection of the beam.

In accordance with (2 and 3) this is possible only when  $q(x) - P(x) = const$  for a beam by contact, which gives rise to additional shear stresses directed from the edges to its center. The deflection established inside the aquatic environment of the beam can also be modeled by restricting the movement to connections established along its edges. In both cases, from a uniformly distributed load, deflection deformations will occur in the beam body. And so, the expressions of P.L. Pasternak [1], in contrast to the Winkler model, additionally characterizes the bending of the structure under uniform load. The main disadvantage of this expression is that the reactive pressure function  $P(x)$  artificially rises from a constant value to a power proportional to the coefficient  $C_2$ . The second part of equalization (2) can distort the actual deflection of the beam under other types of loads, since the deflection of the beam will

occur both due to uneven compression and due to the soil shear under the beam.

In work Simvulidi I.A. [9] the reaction of the soil in the contact area is replaced by the function of reactive pressure in the form of a power series. The forces and displacements in the beam are determined by integrating the differential equation, and the unknown parameters of the series are determined by the boundary conditions.

Thus, the solution of the differential equation (1) taking into account the boundary conditions is reduced to the definition of the mixing function  $w(x)$  or reactive function  $P(x)$  from reactive resistance environment. If we assume that the deflection function of a beam from various types of loads is well studied, then equalization (1) is preferably characterized as part of the beam deflection as from external loads and given reactive forces  $P(x)$ . In this case, the deflection of the beam will depend only on its structural rigidity.

The main task of solving equations is a mathematical method for selecting the most appropriate universal displacement function  $w(x)$  or reactive pressures and the determination of the numerical values of the parameters of the equation, taking into account the boundary conditions.

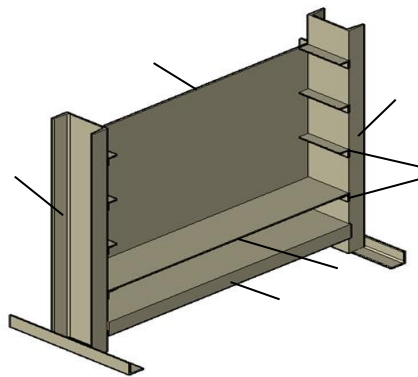
To determine the effort (Q and M), deflection and displacement ( $\rho, w$ ) the accepted mathematical expression in the form of a polynomial expression integrates and the displacement function  $w(x)$  vice versa differentiate. In this case, the initial functions in both cases are assumed to be different [2,3,4]. Unknown parameters of these equations are determined on the basis of boundary and contour conditions. The mathematical formulation of the initial equations in the particular case can satisfy the boundary conditions, but also manifest undesirable side effects in the inter-boundary conditions, which can contradict the actual physical processes. Thus, the ways to solve the problem are reduced to the methods of building mechanics, the results of which determine the main efforts in the beam. The solution is to define the function.  $w(x)$  or  $P(x)$  depending on its rigidity  $E_c I_c$ .

Models are known when the system “building - foundation” is taken as a whole; at the same time, the three-dimensional soil base under the building is considered as an elastic, inseparable medium [6,4]. In this case, the distribution capacity of the elastic among the considered infinite, hence the large design deflections of the beams, which contradicts the actual observed in practice.

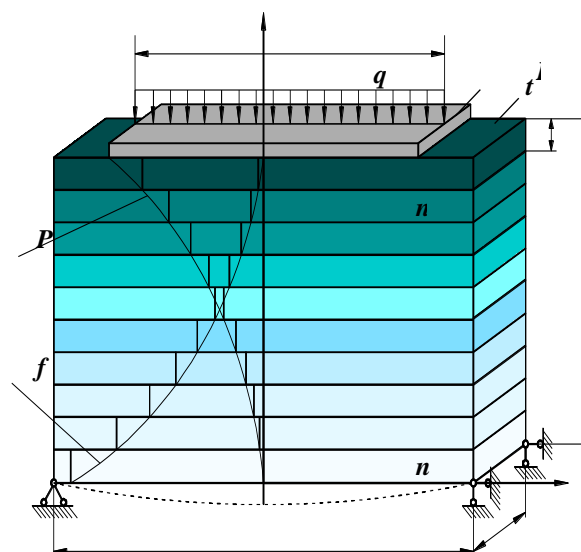
**Models of the soil base and ways to solve the contact problem.**

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**Fig.1. The design of the experimental tray to determine the deflection of the beam**



**Fig. 2. Mechanical model for determining the bending stiffness of the soil and used in the calculation of beams on a soil base**

This article attempts to solve this complex task by experimental determination of the beam deflection  $w(x)$ , lying on a soil base from various external loads. In this case, the integral value of the flexural rigidity is taken as the sum of the structural rigidity of the beam  $E_c I_c$  and bending stiffness of the soil foundation -  $t$ . The stiffness of a beam whose movement is limited at the ends is determined by methods known in mechanics. As for the flexural soil stiffness, then it is determined experimentally in a special flat tray. Unlike traditional testing methods, a metal beam (strip) is installed at the bottom of the tray and rests on two hinged supports. The experiment is carried out with the measurement of the maximum displacement (deflection) of the beam (strip)  $w(L/2)$  or in its absence with the measurement of reactive force in the center of the beam  $P(L/2)$  (Fig-1) in the process of filling the soil layer. The method for determining the flexural stiffness of the soil and the maximum deflection of the beam by the authors was considered

in [7,8]. In accordance with these studies, it was found that the magnitude of the reduced stiffness of the beam  $E_c I_c$  and soil foundation -  $t$  is taken as one and is determined by the expression:

$$EI = (E_c I_c + t) = E_c I_c + k E_0 \frac{b H_s^3}{12} \quad (6)$$

where  $H_s \cong t g \varphi (l/2)$  – the thickness of the active layer of soil bending;  $k$ -correction factor. The model of a composite beam (strip) taking into account the flexural rigidity of the soil  $t$  is taken as the joint operation of the elastic beam rigidity  $E_c I_c$  the span  $L$  and soil layer thick  $H_s$ , with width  $b$ . The maximum deflection of the composite beam, from the condition of equality of external (including gravitational) and reactive forces depends on the type of loading (Fig. 3-5):

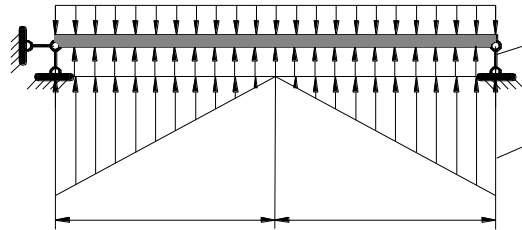
**I-distributed load intensity  $q$  (Fig. 3).** It is believed that with an external uniformly distributed load  $q$ , a reactive voltage appears under the beam of

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length  $L = 2l$  as a sum of uniform intensity  $P_1 = kq$  and uneven, consisting of two inverted triangles with a maximum at the edge part  $P_2 = 2q(1 - k)$ . Coefficient  $k$  – makes it possible to transform the plot of reactive pressures: at  $k = 1$ , task fits Winkler

model, and at  $k = 0$  the model characterizes the full dispensing ability of a magnet base. Taking into account the accepted assumptions given in the beginning of the article, we define the possible maximum deflection of the beam using expressions known in mechanics (Fig. 3):



**Fig. 3. Model beams and soil base. Uniformly distributed load q.**

$$f_{max} = \frac{5ql^4}{24EI}(1 - k) - \frac{27ql^4}{180EI}(1 - k) = \frac{ql^4}{17EI}(1 - k) \quad (7)$$

At  $k = 0$

$$f_{max} = \frac{ql^4}{17EI}$$

At  $k = 1$  Winkler model condition holds

$$f_{max} = 0.$$

Based on the difference of external  $q$  and reactive, inverted triangular diagram of reactive

pressure  $P_2$  and evenly distributed  $P_1$  determine the forces from the distributed forces to the beam

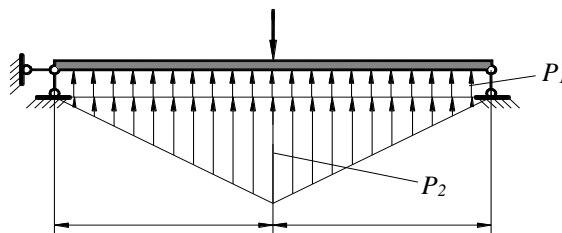
$$M_x = \frac{qx^2}{6l} [(k - 1)(3l - 2x)] \text{ at}$$

$$x = l \quad M_{max} = \frac{ql^2}{6}(k - 1);$$

$$Q_x = \frac{qx}{l} [(k - 1)(l - x)] \text{ at}$$

$$x = 0, l \quad Q_{x=l} = 0 \quad (8)$$

## II-concentrated load N (symmetric problem) (Fig. 4).



**Fig. 4. Model beams and soil base. Symmetric task. Focused load N**

It is believed that when an external point load  $N$  is set at the center of a beam of length  $L = 2l$ , the reactive voltage appears as a sum of uniform intensity  $P_1 = k \frac{N}{2l}$  and uneven, consisting of one inverted triangle with a maximum in the central part  $P_2 = \frac{N}{l}(1 - k)$ . Let us determine the possible maximum deflection of the beam using expressions known in mechanics (Fig.4):

$$f_x = \frac{Nx}{240EJl^2} (10kl^4 + 10l^4 + 2kx^4 - 2x^4 - 5lkx^3) \quad (9)$$

(fig 4.). At  $k = 0$

$$f_{max} = \frac{N(10l^4 - 2x^4)}{240EJl}$$

at  $k = 1$  Winkler model condition holds.

$$f_{max} = \frac{N(4l^3 - x^3)}{48EJ}$$

Based on the assumption of a triangular plot of reactive pressure  $P(x)$  = we determine the efforts from the concentrated force

$$M_x = \frac{Nx^2}{12l^2} (2x + 3lk - 2kx) \text{ at}$$

$$k = 0; \quad x = 0, l \quad M_{max} = 0, M_x = \frac{Nl}{6};$$

$$\text{at } k = 1 \quad x = 0, l \quad M_{max} = 0, M_x = \frac{Nl}{4};$$

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$$Q_x = \frac{Nx}{2l^2} [x + lk - kx]$$

at  $k = 0, 1$   $Q_{x=0} = 0, Q_{x=l} = \frac{N}{2}$  (10)

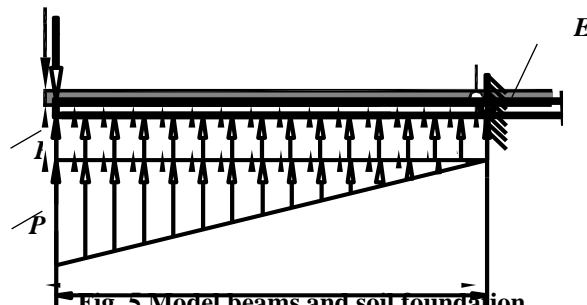


Fig. 5 Model beams and soil foundation. asymmetric task. Concentrated load N. Installed on its edge.

**III- concentrated load N, installed on the edge of the beam.** In this case, taking into account the non-symmetry of the problem being solved, the function of reactive pressure can be represented as a triangular diagram with a maximum at the edge of the beam. In the particular case, it can be taken as linear with a maximum at the beginning of the beam  $P_2(x = 0) > 0$  и  $P_2(x = l) = 0$ . For the case when the condition  $k = 0$  is satisfied over the entire length of the beam, the maximum deflection of the beam can be determined by the expression:

$$f_x = \frac{N(L-x)^3}{240EJL^2} (3kL^2 + 14L^2 + 3kx^2 - 6x^2 + 12Lx + 4kxL) \quad (11)$$

For the case when  $k > 0$  the maximum values of the plot of reactive pressures will be determined from the condition  $M(x = l) = 0$  (Fig 5):

$$P_1 = \frac{kN}{l} \quad P_2 = \frac{3N(1-\frac{k}{2})}{l} \quad (12)$$

$$M_x = \frac{Nx(L-x)}{4L^2} (4L - 2x + kx)$$

$$Q_x = \frac{N}{4L^2} (4L^2 - 3kx^2 + 6x^2 - 12Lx + 2Lkx) \quad (13)$$

Determine the magnitude of the moments and shear forces along the length of the beam.

Comparing expressions (7), (9) and (11), we can state the fact that the greatest flexural rigidity of the soil is manifested when the load  $q$  is intense and the minimum is under the action of a concentrated load N set along the beam edge. With symmetrical point loading of the beam, it is an intermediate value.

Movement function  $W(x)$  for various types of loading is determined experimentally and mathematically approximated as a series in the form of a polynomial of the sixth degree  $f_{max}$ . Other unknowns (angle of rotation  $W'(x)$ , moment  $M(x)$ , crossing  $Q(x)$  and reactive  $P(x)$  stresses are determined using the methods of numerical or analytical differentiation of the displacement function  $W(x)$ .

## Conclusion.

1. Contact one-parameter elastic models with constant stiffness more reliably characterize the vertical linear displacements of the base for sufficiently rigid beams (lanes). This model does not describe the deflection of a beam with a uniformly distributed load, since it is believed that the reactive pressure  $P(x)$  along the beam does not change.

2. Contact two-parameter models with constant bending stiffness  $C_2$  corresponding to the elastic laws, although formally take into account the distribution properties of the base, nevertheless, the numerical value of the parameter  $C_2$  is considered not known.

3. The widespread model of elastic half-space, unlike the above-discussed contact, is different in that the stresses spread throughout the entire volume. In this case, the distribution capacity of the soil is assumed as for a continuous medium. Comparing the results of calculations with the results of experiments it is easy to see that in the first case the beam deformation loaded with a distributed load is expected to be large, far exceeding the actual deflections of the beam in the ground.

4. It is known that when calculating a beam, a slab on a soil foundation, structural rigidity is used. Flexural (shear) stiffness of the soil base is completely ignored in the calculations. The proposed calculated expression of the bending stiffness of the soil allows you to more accurately calculate the displacement function and the maximum deflection of the foundation structure.

5. The model of the soil foundation presented by the authors makes it possible to more realistically assume the function of the reactive pressure of the soil. Moreover, depending on the chosen coefficient  $k$ , the model is transformed between the Winkler model and the model proposed by the authors, which characterizes the reactive pressure.

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