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SOI: [1.1/TAS](https://doi.org/10.15863/TAS) DOI: [10.15863/TAS](https://doi.org/10.15863/TAS)

International Scientific Journal Theoretical & Applied Science

p-ISSN: 2308-4944 (print) e-ISSN: 2409-0085 (online)

Year: 2019 Issue: 11 Volume: 79

Published: 14.11.2019 <http://T-Science.org>

QR – Issue



QR – Article



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SOME CONTROL CHARTS BASED ON THE CONSENT CRITERIA

Abstract: In the article, skewness control charts (a -charts), excess (γ -charts) and p -charts based on the consent criteria in the form of statistical assertions are constructed. They can, for example, be used in the first stage of statistical regulation or before a control check of technological processes.

Key words: skewness, excess, control, power function, hypothesis, quantile, charts.

Language: English

Citation: Ablazova, K. S. (2019). Some control charts based on the consent criteria. *ISJ Theoretical & Applied Science*, 11 (79), 23-28.

Soi: <http://s-o-i.org/1.1/TAS-11-79-6> **Doi:**  <https://dx.doi.org/10.15863/TAS.2019.11.79.6>

Scopus ASCC: 2613.

Introduction

Let the distribution of the measurable X quality characteristic in the general population have a normal distribution, namely, $F(x) = P(X < x) = \Phi(x; \mu, \sigma^2)$ and sampling X_1, X_2, \dots, X_n taken from X . We denote by $\bar{X}_n = \frac{1}{n} \sum_{k=1}^n X_k, s_n^2 = \frac{1}{n-1} \sum_{k=1}^n (X_k - \bar{X}_n)^2$ assessments μ and σ^2 respectively.

In practice, the assumption made above should be checked. Since the process under study can be released from the stable state from ordinary (random) and special (nonrandom) causes. These reasons strongly influence the distribution of the process being studied, namely, the expected $F(x) = \Phi(x; \mu, \sigma^2)$ - normality.

If one can not specify at least one parameter μ and σ^2 , then the assumption of normality of the general population is verified using a complex hypothesis, with a significance level α :

H_0 : The theoretical distribution function is normal;

H_1 : The theoretical distribution function is not normal;

To test hypotheses, criteria for concordance have now been developed on a practical and theoretical plans. (see, for example, [1], [2]). In production, the

normality check is carried out using nomograms, so-called "Probabilistic Paper", "Histogram", "Box Diagram", "Coefficient of Correlation", etc. ([3], [4]).

Theoretically, using more powerful criteria (Kolmogorov, Pierson, ω^2 and so on), and, also, on the basis of the comparison of the empirical and theoretical sampling moments, for example, "by the absolute central moment of the first order" [5], "by the sample coefficient of skewness: $a_n = \frac{1}{n s_n^3} \sum_{k=1}^n (X_k - \mu)^3$ ", "by the sample coefficient of excess: $\gamma_n = \frac{1}{n s_n^4} \sum_{k=1}^n (X_k - \mu)^4$ " and so on.

In this article, we present control charts (CCs) (see, for example, [1], [6] and [7]) based on a_n, γ_n and Kolmogorov test. And also based on the compiled programs on the computer, the principles of using these CCs in production are shown. Earlier reports of these results without proof were published in papers. ([8], [9] and [10])

CC is a statistical tool for statistical control of the process and visualize the progress of the production process on the diagrams. On the basis of this, to regulate it and thereby prevent the contamination of products by defective.

The CC technique in this article is used to re-test the statistical hypothesis. At the same time, this hypothesis is "adapted" for practical use.

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Since the restriction on the acceptance or rejection of the hypotheses advanced can only be expressed with a certain probability, therefore the assertions presented in this paper are statistical.

Before we carry out the main results, we note that our results are used in the initial period of statistical regulation of processes. If it is determined that the process is stable and is able to meet the requirements at the moment, further studies are performed, looking at the important problems of the enterprise other CCs are constructed, long-term reproducibility values, etc. are calculated.

Main Results

Let now it is required to check statistically the normality of the general set. At specified times $t = 1, 2, \dots, m$ select instant samples $X_{1t}, X_{2t}, \dots, X_{nt}$ with a constant volume n . On the basis of them we determine the estimates μ, σ^2, a and γ respectively.

$$\bar{X}_{nt} = \frac{1}{n} \sum_{k=1}^n X_{kt} \quad S_{nt}^2 = \frac{1}{n-1} \sum_{k=1}^n (X_{kt} - \bar{X}_{nt})^2$$

$$\gamma_{nt} = \frac{1}{nS_{nt}^4} \sum_{k=1}^n (X_{kt} - \bar{X}_{nt})^4$$

$$a_{nt} = \frac{1}{nS_{nt}^3} \sum_{k=1}^n (X_{kt} - \bar{X}_{nt})^3$$

We denote by a_α and γ_α respectively quantiles of distributions a_n and γ_n at the level of significance α .

In production control, the criterion power function is used to estimate the CC

$$G(\theta) = P\{g(\bar{X}) \in CR|\theta\}$$

Where CR is region of the hypothesis deviation, θ is the value of the unknown distribution parameter $F(x)$, $g(\bar{X})$ assessment for θ where $\bar{X} = (X_1, X_2, \dots, X_n)$

$$\begin{aligned} G_a(\sigma_t) &= P(S_{nt}^2 \leq LCL|\sigma_t) = P\left(S_{nt}^2 \leq \sqrt[3]{\frac{\bar{\mu}_3^2}{a_{1-\alpha/2}^2}} \sigma_t\right) = \\ &= P\left(\frac{n-1}{\sigma_t^2} S_{nt}^2 \leq \frac{n-1}{\sigma_t^2} \sqrt[3]{\frac{\bar{\mu}_3^2}{a_{1-\alpha/2}^2}} \sigma_t\right) = Ch\left\{\frac{n-1}{\sigma_t^2} \sqrt[3]{\frac{\bar{\mu}_3^2}{a_{1-\alpha/2}^2}} | n-1\right\} \end{aligned}$$

Here σ_t - standard deviation X at the time t and the fact that $\frac{n-1}{\sigma_t^2} S_{nt}^2$ it has χ^2 distribution with $n-1$ degree of freedom. Theorem 1 is proved.

Theorem 1. At a significance level of α , the hypothesis H_0 is accepted if

$$S_{nt}^2 > LCL$$

where LCL (lower control limit) = $\sqrt[3]{\frac{\bar{\mu}_3^2}{a_{1-\alpha/2}^2}}$, $\bar{\mu}_3 = \frac{1}{m} \sum_{t=1}^m \mu_{3t}$, $\mu_{3t} = \frac{1}{n} \sum_{k=1}^n (X_{kt} - \bar{X}_{nt})^3$.

In this case, the power function of the a chart has the form:

$$G_a(\sigma_t) = Ch\left\{\frac{n-1}{\sigma_t^2} \sqrt[3]{\frac{\bar{\mu}_3^2}{a_{1-\alpha/2}^2}} | n-1\right\},$$

where $Ch\{*|n-1\}$ - χ^2 distribution with $n-1$ degree of freedom.

Proof. To construct the LCL a -charts assume that the process is stable in a certain period of time, namely, the expected normal law is preserved $F(x) = \Phi(x; \mu, \sigma^2)$

We select instantaneous samples with a constant volume n . Based on these samples, $t = 1, 2, \dots, m$ we find μ_{3t} and $\bar{\mu}_3$ (All this can be done artificially with the help of computer tools or we use more powerful statistical criteria). Using the following equivalent relations, we construct $(1-\alpha) \cdot 100\%$ confidence interval for a_{nt} .

$$-a_{1-\alpha/2} < a_{nt} < a_{1-\alpha/2}$$

$$\begin{aligned} S_{nt}^2 &> \sqrt[3]{\frac{1}{a_{1-\alpha/2}^2} \left(\frac{1}{n} \sum_{k=1}^n (X_{kt} - \bar{X}_{nt})^3\right)^2} \\ S_{nt}^2 &> \sqrt[3]{\frac{\bar{\mu}_3^2}{a_{1-\alpha/2}^2}} \end{aligned}$$

With constant n and α denoting $LCL = \sqrt[3]{\frac{\bar{\mu}_3^2}{a_{1-\alpha/2}^2}}$ we get a one-way a chart. We find the power function a chart.

Theorem 2. At the significance level of α , the main hypothesis H_0 is accepted if

$$LCL < S_{nt}^2 < UCL$$

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where LCL (lower control limit) = $\sqrt{\frac{\bar{\mu}_4}{\gamma_{1-\alpha/2}}}$; UCL

(upper control limit) = $\sqrt{\frac{\bar{\mu}_4}{\gamma_{\alpha/2}}}$

$$\bar{\mu}_4 = \frac{1}{m} \sum_{t=1}^m \mu_{4t} \quad \mu_{4t} = \frac{1}{n} \sum_{k=1}^n (X_{kt} - \bar{X}_{nt})^4$$

In this case, the power function of γ - chart has the form:

$$G_a(\sigma_t) = 1 - Ch \left\{ \frac{n-1}{\sigma_t^2} \sqrt{\frac{\bar{\mu}_4}{\gamma_{\alpha/2}}} \right\} n - 1 \left. \right\} + Ch \left\{ \frac{n-1}{\sigma_t^2} \sqrt{\frac{\bar{\mu}_4}{\gamma_{1-\alpha/2}}} \right\} n - 1 \left. \right\}$$

Proof. $(1 - \alpha) \cdot 100\%$ confidence interval for γ_{nt} has the form:

$$\begin{aligned} G_\gamma(\sigma_t) &= P(S_{nt}^2 \leq LCL | \sigma_t) + P(S_{nt}^2 \geq UCL | \sigma_t) = \\ &= P\left(\frac{n-1}{\sigma_t^2} S_{nt}^2 \leq \frac{n-1}{\sigma_t^2} \sqrt{\frac{\bar{\mu}_4}{\gamma_{1-\alpha/2}}} \sigma_t\right) + P\left(\frac{n-1}{\sigma_t^2} S_{nt}^2 \geq \frac{n-1}{\sigma_t^2} \sqrt{\frac{\bar{\mu}_4}{\gamma_{\alpha/2}}} \sigma_t\right) = \\ &= 1 - Ch \left\{ \frac{n-1}{\sigma_t^2} \sqrt{\frac{\bar{\mu}_4}{\gamma_{\alpha/2}}} \right\} n - 1 \left. \right\} + Ch \left\{ \frac{n-1}{\sigma_t^2} \sqrt{\frac{\bar{\mu}_4}{\gamma_{1-\alpha/2}}} \right\} n - 1 \left. \right\} \end{aligned}$$

Theorem 2 is proved.

Remark. When $S_{nt}^2 < LCL$, the process meets all the requirements, so you should not fit into the process. Then the power function γ - chart, it is better to find by the formula:

$$G_\gamma(\sigma_t) \approx 1 - Ch \left\{ \frac{n-1}{\sigma_t^2} \sqrt{\frac{\bar{\mu}_4}{\gamma_{\alpha/2}}} \right\} n - 1 \left. \right\}$$

Now we define another CC based on Kolmogorov's consent criterion.

Let $X_1^*, X_2^*, \dots, X_n^*$ variation series X_1, X_2, \dots, X_n taken from the general set X .

With a significance level α one should check a simple hypothesis:

$$H_0: F(x) = \Phi(x; \bar{X}_n, S_n^2).$$

Kolmogorov's criterion prescribes to accept the hypothesis H_0 if

$$\rho < k_{1-\alpha}$$

$$\gamma_{\alpha/2} < \gamma_{nt} < \gamma_{1-\alpha/2} \text{ or } \sqrt{\frac{\bar{\mu}_4}{\gamma_{1-\alpha/2}}} < S_{nt}^2 < \sqrt{\frac{\bar{\mu}_4}{\gamma_{\alpha/2}}}$$

Hence, with constant n and α we denoted

$$\text{by } LCL = \sqrt{\frac{\bar{\mu}_4}{\gamma_{1-\alpha/2}}} \text{ and}$$

$$UCL = \sqrt{\frac{\bar{\mu}_4}{\gamma_{\alpha/2}}}$$

we get γ - chart.

We find power function of γ - chart we find as follows:

Where $k_{1-\alpha}$ quantile of distribution of statistics ρ :

$$\rho = \max_{1 \leq k \leq n} \left| \Phi(X_k^*; \bar{X}_n, S_n^2) - \frac{2k-1}{2n} \right| + \frac{1}{2n}$$

Here $\Phi(X_k^*; \bar{X}_n, S_n^2) = \Phi(Y_k^*)$, $Y_k^* = \frac{X_k^* - \bar{X}_n}{S_n}$, $\Phi(Y_k^*) \sim N(0,1)$ normal distribution.

Theorem 3. At the significance level of α , the main hypothesis H_0 is accepted if

$$\rho < UCL$$

where UCL (upper control limit) = $\frac{k_{1-\alpha}}{\sqrt{n}}$.

The proof of the assertion follows from $(1 - \alpha) \cdot 100\%$ confidence interval for ρ : $\rho < \frac{k_{1-\alpha}}{\sqrt{n}}$ with constant n and α .

For example, at $n = 50$ and $\alpha = 0.05$ we find, $k_{1-\alpha} = 1,13$ then we have $\rho < 0.19$ Find the power function ρ - charts have not yet been successful. Therefore, in order to estimate ρ -chart, you can use the stability factor C_p and the share of defective products.

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That is,

$$C_p = \frac{UCL - LCL}{6\sigma}$$

where *UCL* - upper control limit, *LCL* - lower control limit.

The fraction defective of products is defined as the area of the tails of the expected normal law outside the standard.

Practical Part

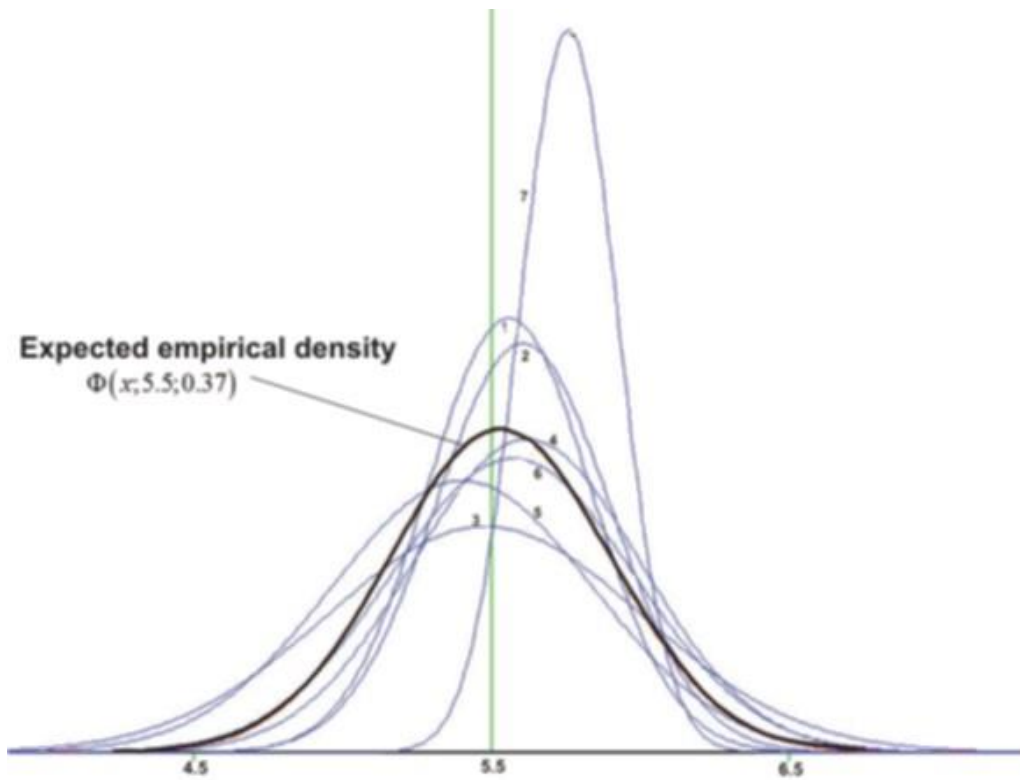
As a teaching and explanatory implementation of the above proved statements, we give a solution to one problem from the technological process. The problem arose from the welding department when welding part of the car model "DAMAS".

Name of the problem in the enterprise: 01 CLEARANCEFRT. DR * PNL. LH. Spec: 5.5 ± 1, namely, preservation of the technical tolerance when welding doors of a machine of this brand.

Taking the middle of the tolerance $\mu = \frac{6.5+4.5}{2} = 5.5$ for the average expected normal law $\Phi(x; 5.5, 0.37)$, where, σ^2 estimated using pre-study samples, testing a complex hypothesis with the help of α and γ a simple hypothesis with ρ - chart.

If the complex of assertions 1-3 is retained, according to preliminary instantaneous volume samples $n = 50$, $\alpha = 0.05$ at $m = 2$ borders were found α , γ and ρ - chart. In this case, the quantiles are taken from [11]. Charts are entered in the current monitoring and in the set time $t = 1, 2, \dots, 7$, the situation of the technological process is analyzed by computer means. The results are drawn in the form of graphs, diagrams (CCs) and tables.

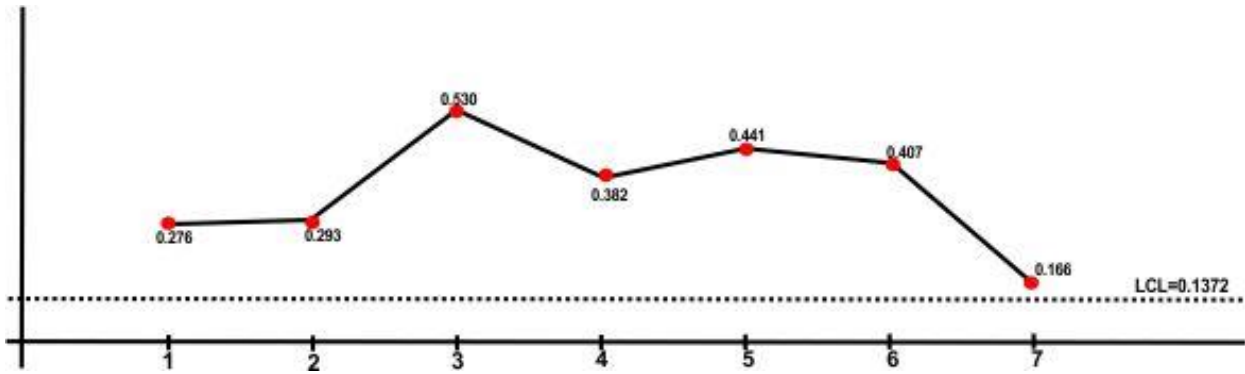
Behaviour of density functions in time



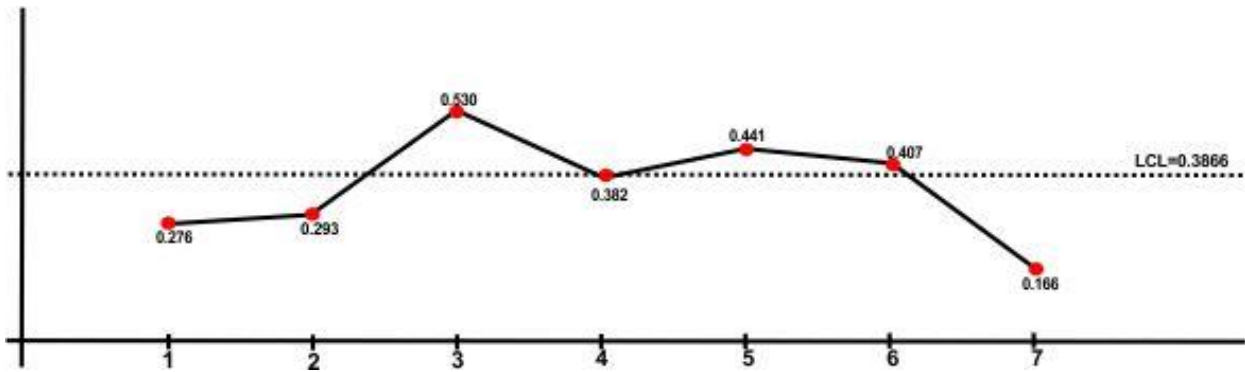
The situation of densities in time

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α - chart.

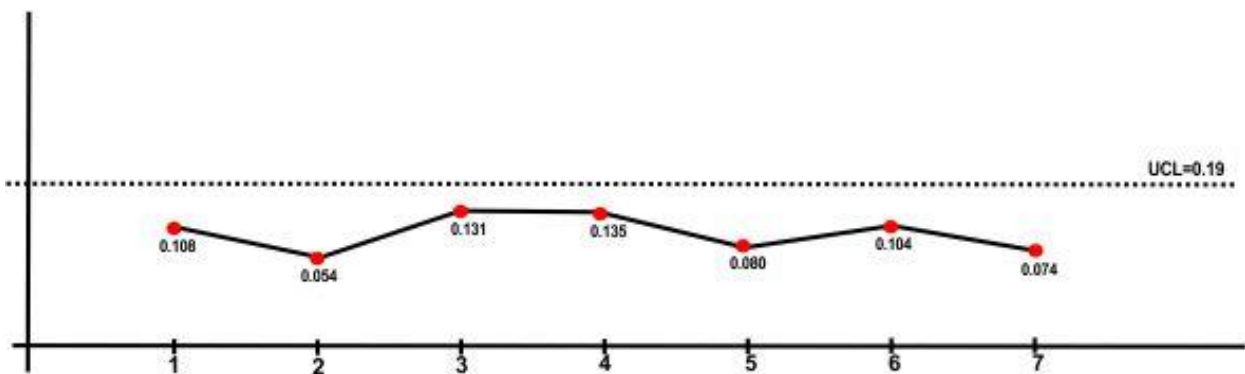


γ - chart.

Table 1: Percentage values of power functions for process intervention

<i>t</i>	1	2	3	4	5	6	7
$G_{\alpha}(\sigma_t)$	0,125%	0,056%	0,000%	0,001%	0,000%	0,000%	20,118%
$G_{\gamma}(\sigma_t)$	3,282%	6,548%	92,123%	45,128%	71,375%	57,388%	0,000%

ρ - chart.



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Table 2: Estimation of ρ -charts

t	1	2	3	4	5	6	7
C_p	0,634	0,616	0,458	0,539	0,502	0,522	0,818

In the end, we give some statistical conclusions about the studied process.

- At $t = 1$ and it is advisable to build charts of mean values and root-mean-square deviations ($\bar{X} - S$), then calculate the potential indicators of the technological process.
- At $t = 3,4,5$ and 6 it is not recommended to calculate the potential indicators of the process. First you need

to consult a specialist with a S card. After becoming a stable state, you can calculate these indicators.

- At $t = 7$ graphics and charts indicate that the potential of the technological process is improved in a positive way. In this case, constructing \bar{X} card together with specialists should find out the shift of the mean to the right. If there is improvement, then new boundaries of control charts are found, and technical standards may change.

References:

1. Mittag, H.-J., & Rinne, H. (1995). *Statistical methods of quality assurance*. Moscow: Mashinostroyeniye.
2. Smirnov, N.V., & Dunin-Barkovskiy, I.V. (1969). *Course of probability theory and mathematical statistics, for technical applications*. Moscow: Nauka.
3. Axmedov A.S. (2005). Statistical management of processes. Methodical manual. Andijan.
4. Axmedov, A.S., Ablazova, K.S., Abdullayev, A., & Mamatohunova, Y. (2015). Control chart method of checking processes to normality. *Scientific notification. No4 May. Andijan 2015*, pp. 5-9.
5. Bocharov, P.P., & Pechinkin, A.B. (1998). *Probability theory and mathematical statistics*. Moscow.
6. Shtorm, R. (1970). *Probability theory mathematical statistics statistical quality control*. Moscow: Mir.
7. Adler, Y., Maksimova, O., & Shper, V. (2011). Shewhart charts in Russian and abroad: A brief review of the current state (statistical aspects). *Journal of Standards and quality, No. 8*.
8. Axmedov, A.S., Ablazova, K.S., Zaxidov, D., & Egamberdiyeva, B. (2014). About the power functions of the asymmetry control charts and the excess of the normal process. *Journal of scientific publications of graduate and scientific researchers. Rossiya. No 3, Marta, 2014*, pp 259-260
9. Akhmedov, S.A., & Ablazova, K.S. (2019). Some statistical tools at investigating the stability of technological process, *Reports of the academy of Sciences of the Republic of Uzbekistan, No3*, pp.16-19.
10. Akhmedov, S.A., & Ablazova, K.S. (2019). Some innovative statistical tools in the study of the stability of the production process, "Modern problems of probability theory and mathematical statistics", Tashkent, April 30-May 1, 2019, pp. 11-13.
11. Bol'shev, L., & Smirnov, N. (1983). *Mathematical statistics tables*. Moscow: Nauka.