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Optimizing maintenance decisions using linear programming

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Given that the competition between the industrial companies is increasingly tough, the problem of setting up by them of the spare parts manufacturing programs with own resources has become of a great importance. Determining an optimal manufacturing program means setting up a program to ensure, with the material and human resources of the unit, an optimization criterion such as maximum profit, minimal expenses, minimum costs, etc. Next, we present the case where the "profit" optimization criterion is chosen using linear programming.

Keywords: *maintenance, repair, simplex*

1. Introduction

Maintenance represents a set of technical and organizational measures and actions designed to extend the life of certain equipment. In establishing the maintenance policy, we follow the principle that there is no generally valid maintenance method, but a particular maintenance policy is adopted for each industrial equipment, which consists in achieving a technical and economic optimization compromise, taking into account the general objectives of the enterprise and the objectives of industrial maintenance.

In selecting the maintenance method, the classification of the equipment will have to be taken into account in terms of its criticality, reliability, direct maintenance costs and indirect costs of unavailability of equipment in the event of accidental damage.

The criteria on which to choose the maintenance method are: the age of the equipment, its interdependence in production, the cost of the equipment, the complexity of the equipment, its origin, the robustness of the equipment, the working regime, the consequence of a fall on production and the execution time.

The mathematical modeling of the maintenance systems allows both the provision of resources for system maintenance, system missions and moments of initialization, as well as the evaluation of system activity durations in different calendar stages with the help of availability.

2. The mathematical model of a linear programming problem

A linear programming problem can be written down as follows:
To be determined the numbers x_1, x_2,..,x_n which maximizes a linear function with the form:

$$f= c_1x_1 + c_2x_2 + ...+ c_n x_n \quad (1)$$

and to satisfy the conditions:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 +...+ a_{1n}x_n &\leq b_1 \\ a_{21}x_1 + a_{22}x_2 +...+ a_{2n}x_n &\leq b_2 \quad (2) \\ &\cdots\cdots \\ a_{m1}x_1 + a_{m2}x_2 +...+ a_{mn}x_n &\leq b_m \end{aligned}$$

$$x_1 \geq 0,\ x_2 \geq 0, ..., x_m \geq 0, \quad m < n \quad (3)$$

a_{ij}, b_i, c_j being real given numbers.

The function (1) whose maximum is required is called an objective function. The system of inequities (2) is the restriction of the problem, and (3) are called non-negativity conditions. [4]

The relations (1), (2), (3), which represent the mathematical model of a linear programming problem, can also be written in the abbreviated form:

$$\begin{aligned} &\max \textstyle\sum_{j=1}^{n} c_j x_j \\ &\textstyle\sum_{j-1}^{n} a_{ij} x_j \leq b_i \text{ , i= 1,2,3,..., m} \quad (4) \\ &x_j \geq 0, \text{ j=1,2,3,..., n.} \end{aligned}$$

f the minimum function f is required, the restrictions must be met with the sign ≥ . The linear program will be:

$$\begin{aligned} &\min \textstyle\sum_{j=1}^{n} c_j x_i \\ &\textstyle\sum_{j=1}^{n} a_{ij} x_j \geq b_i \text{ , i= 1,2,3,..., m} \quad (5) \\ &x_j \geq 0, \text{ j=1,2,3,..., n..} \end{aligned}$$

Classification of solutions:

- A system of n real numbers xj that satisfy relations (2), (3) is called admissible solution.
- If among the n real numbers that satisfy (2) we have xj < 0 conditions (3) are not satisfied, the solution is called inadmissible.
- A system of m values $x_j > 0$, and n-m values $x_j = 0$, which checks the system (2), is called admissible base solution.
- An admissible base solution containing m values $x_j > 0$ and n-m values $x_j = 0$ is called non-generated admissible base solution.

• An admissible solution containing m-r values $x_j > 0$ is called degenerate admissible base solution.

• An admissible base solution for which the objective function becomes maximum (or minimal) is called the optimal solution [4].

The system of inequalities can be transformed into a system of equations by means of compensation variables.

• For problem (4) where $b_i > 0$ is assumed, we compile a compensation variable $x_i^e \geq 0$ in the left member of the inequality i and the system of inequalities becomes:

$$a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{in}x_n + x_i^e = b_i \quad i = 1,2,\ldots,m$$
$$x_i^e \geq 0, \; i=1,2,\ldots,m, \; j = 1,2,\ldots,n$$

• For problem (5) we deduce a compensation variable in the left-hand side of each inequality and obtain the equation system:

$$a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{in}x_n - x_i^e = b_i \quad i = 1,2,\ldots,m$$
$$x_j, x_i^e \geq 0, \; i=1,2, \ldots, m, \quad j = 1,2,\ldots,n$$

Programs (4) and (5) will be written in the form:

$$\max \sum_{j=1}^{n} c_j x_j$$
$$\sum_{j=1}^{n} a_{ij} x_j + x_i^e = b_i \; ; i = 1,2, \ldots, m \qquad (4')$$
$$x_j, + x_i^e \geq 0 \; ; i = 1,2,\ldots, m \; ; j = 1,2,\ldots, n$$

$$\min \sum_{j=1}^{n} c_j x_j$$
$$\sum_{j=1}^{n} a_{ij} x_j - x_i^e = b_i, \quad i = 1,2,\ldots, n \qquad (5')$$
$$x_j, + x_i^e \geq 0, \; i = 1,2, \ldots, m \quad j = 1,2,\ldots, n;$$

Programs (4) and (5) are also called linear canonical programs and (4') and (5') standard linear programs. A linear canonical program can be brought to the standard form if a zero-compensation compensation variable xie 0 is dropped, respectively, of the limitation of the problem.

The compensation variables are sometimes marked with another letter $y_1, y_2, \ldots, y_m$ or with the same letter, the indices being numbered further: $x_{n+1}, x_{n+2, \ldots,} x_{n+m}$.
E.g.:

$$A=\begin{pmatrix} a_{11} & a_{12} & \cdots\cdots & a_{1n} \\ a_{21} & a_{22} & \cdots\cdots & a_{2n} \\ \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots \\ a_{m1} & a_{m2} & \cdots\cdots & a_{mn} \end{pmatrix}$$

The matrix of the coefficients a_{ij}, of the variables x_j in the system of inequities (2) and:

$$X=\begin{pmatrix} x_1 \\ x_2 \\ \dots\dots \\ x_n \end{pmatrix};\quad C=\begin{pmatrix} c_1 \\ c_2 \\ \dots\dots \\ c_n \end{pmatrix};\quad B=\begin{pmatrix} b_1 \\ b_2 \\ \dots\dots \\ b_n \end{pmatrix};\quad X^e=\begin{pmatrix} x_1^e \\ x_2^e \\ \dots\dots \\ x_m^e \end{pmatrix},$$

Column matrices whose elements are:

$$x_j,\ c_j,\ \ j=1,2,\dots,n,\qquad b_i,\ x_i^e,\ \ i=1,2,\dots,m$$

The canonical form (4) and (5) of a linear program can be written as a matrix:

$$\text{Max } C'X;\ \ AX\leq B;\ \ X\geq 0$$
$$\text{Min } C'X;\ \ AX\geq B;\ \ X'\geq 0$$

And the standard form (4 ') and (5'):

$$\text{Max } C'X;\ \ AX+X'=B;\ X\geq 0;\ X'\geq 0;$$
$$\text{Min } C'X;\ AX-X'=B;\ ;\ X\geq 0;\ X'\geq 0;$$

where C 'represents the transposed matrix C.

We'll assume the linear program brought to the standard form. Restrictions of the problem form in this case a system of n unknowns (m<n):

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n &= b_2 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n &= b_m \end{aligned} \tag{6}$$

As m < n, the system (6) generally admits an infinity of solutions. Suppose now that A = m; this means that there are non-zero determinants of m, which can be formed with the elements of matrix A.

E.g.:

$$|A| = \begin{vmatrix} a_{11} & a_{12} & \dots\dots\dots & a_{1m} \\ a_{21} & a_{22} & \dots\dots\dots & a_{2m} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots\dots & a_{mn} \end{vmatrix} \neq 0$$

such a determinant. The system (6) can be written in the form:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n &= b_1-(a_{1,m+1}x_{m+1}+\dots+a_{1n},x_n) \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n &= b_2-(a_{2,m+1}x_{m+1}+\dots+a_{2n},x_n) \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n &= b_m-(a_{m,m+1}x_{m+1}+\dots+a_{mn},x_n) \end{aligned} \tag{6'}$$

in which $x_1, x_2, ..., x_m$ are the main unknowns, and $x_{m+1}, ..., x_n$ the secondary unknowns of the system.

To obtain a basic system solution (6 ') we will cancel out the unknown, that is, we will assume:

$$x_{m+1} = x_{m+2} = ... = x_n = 0.$$

Thus we obtain the system:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + ... + a_{1m}x_m &= b_1 \\ a_{21}x_1 + a_{22}x_2 + ... + a_{2m}x_m &= b_2 \\ a_{m1}x_1 + a_{m2}x_2 + ... + a_{mm}x_m &= b_m \end{aligned} \tag{6''}$$

which is compatible determined as we assumed $|A| \neq 0$. If the solution of this system $\bar{x}_j$; j=1,2, ..., m will contain only values of $\bar{x}_j \geq 0$, it will be an acceptable basic solution.

This solution is:

- Non-generated if it contains m values of $\bar{x}_j > 0$;
- Degenerated if it contains m-r values of $\bar{x}_j > 0$, the others being null ($r \geq 1$).

If all of the m determinants that can form with the n columns of the matrix A are different from zero, maximum C min solutions will be obtained (admissible or not).

It follows that the maximum number of basic solutions is C_n^m and are obtained from (6) by annulling n-m from n unknowns.

If we determine all of the basic solutions and the admissible ones, we will introduce them into the purpose function, we will know which one is optimal (maximum or minimum) [4].

3. Case study

The object of the study was the optimization of the manufacturing process of the spare parts produced in the maintenance department of S.C. "METALICA" S.A. Oradea for the following machines:

1. In the PAI-25 press used for stamping the sheet with a thickness of less than 4 mm, frequent falls are recorded due to the wear of the bolt and the ball head seat, part of the drive rammer. The piece changes after 6,000 hours of operation of the press.
2. The SN 400x750 lathe has a 24x4 mm trapezoidal threaded bolt as a low reliability piece, from the transverse luge and is made with the corresponding nut. The screw thread length is 250 mm.
3. The maintenance program also includes making the socket table bolt and the corresponding nut for the universal milling machine FUS 32 with its own means.

The materials used to make the three parts - each with two components - are as follows:

• For making the spherical head screw, trapezoidal screw and luge table bolt, are used OLC45, steel rods purchased at the price of 552 € / ton plus VAT.

• The ball seat of the PAI 25 press and the nuts for the two latched screws are made of bronze bars BZ 14 which the unit purchases at the price of 21.5 lei / kg plus VAT.

The data collection was done by studying the technical book and the machinery sheet, analyzing the costs and the estimated benefit, taking into account the costs of the materials, the overhead costs and the fact that the average maintenance service fee is 14.58 lei / hour; these data are included in Table 1. The mathematical model used to solve this linear programming problem is the simplex method.

Table 1.

PART / MATERIAL	Screw and spherical head seat	Threaded screw on transverse luge	Luge table screw	Available raw material [kg]
Full bar made of steel OLC 45	10,2	3,1	15,3	3200
Full bar made of bronze BZ 14	6,1	1,3	2,2	1000
Unitary profit [hundreds of lei]	1,2	0,5	1,4	

The decision variables of the model are:
x_1 = number of spherical head screws and seats at the PAI - 25 press
x_2 = number of threaded screws to the transverse luge of the SN 400 × 750 lathe
x_3 = number of luge table screws at the SN 400 × 750 lathe

Objective function. In the linear programming model, the maximum total profit is obtained by maximizing the function:

$$(\max)\ z(x) = 1,2 \cdot x_1 + 0,5 \cdot x_2 + 1,4 \cdot x_3$$

Table 2. Microsoft Excel Report

Microsoft Excel 10.0 Answer Report
Worksheet: [OPTIM INT.xls]Sheet1
Report Created: 20.05.2005 19:42:23

Target Cell (Max)

Cell	Name	Original Value	Final Value
$I$16	OPTIM FC OBJ	0	429,4

Adjustable Cells

Cell	Name	Original Value	Final Value
$B$16	OPTIM X1	0	0
$C$16	OPTIM X2	0	632
$D$16	OPTIM X3	0	81

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$F$8	RESTRICȚII	3198,5	$F$8<=$H$8	Not Binding	1,5
$F$9	RESTRICȚII	999,8	$F$9<=$H$9	Not Binding	0,2
$F$10	RESTRICȚII	0	$F$10>=$H$10	Binding	0
$F$11	RESTRICȚII	632	$F$11>=$H$11	Not Binding	632
$F$12	RESTRICȚII	81	$F$12>=$H$12	Not Binding	81
$B$16	OPTIM X1	0	$B$16=integer	Binding	0
$C$16	OPTIM X2	632	$C$16=integer	Binding	0
$D$16	OPTIM X3	81	$D$16=integer	Binding	0

The model restrictions are:

- For full bar made of steel OLC 45:

$$10{,}2 \cdot x_1 + 3{,}1 \cdot x_2 + 15{,}3 \cdot x_3 \leq 3200$$

- For full bar made of bronze BZ 14:

$$6{,}1 \cdot x_1 + 1{,}3 \cdot x_2 + 2{,}2 \cdot x_3 \leq 1000$$

The negativity restrictions are:

$$x_i \geq 0, \quad x_i \in N \qquad i = 1,2,3$$

The data processing was done using the SOLVER software from the Microsoft EXCEL library and led to the results in Table 2.

Results: $x_1 = 0$, $x_2 = 632$ şi $x_3 = 81$

The analysis of the obtained data leads to the conclusion that the maximum profit in the amount of 429.4 hundreds lei is obtained under the conditions of manufacturing 632 threaded screwed to the transversal luge of the SN 400 × 750 lathe and 81 luge table screws on the SN 400 × 750 lathe.

4. Conclusions

The mathematical programming is a branch of applied mathematics, used in important decision-making issues such as production scheduling, alloy problems, transport problems, industrial maintenance, and so on. A problem of mathematical programming is to maximize or minimize a particular expression, called an objective function, whose defining variables satisfy certain restrictions.

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