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Contributions to determining the trajectory of a point on the average fiber of the flexible wheel of a double harmonic transmission

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Research on the harmonic transmissions has shown that their functional performances are mainly influenced by the kinematic and dynamic behavior of the flexible wheels in these transmissions. The paper presents a study of the particular aspects of kinematics within a double harmonic transmission. This study aims to present the mathematical algorithm for determining and constructing the trajectories of points located on the average fibers in the two front surfaces of the flexible wheel from a double harmonic transmission. In order to realize the graphical constructions of the trajectories of these points of the flexible wheel, an original calculation program was developed and run, written in the language Visual Basic.

Keywords: *transmission, wheel, movement, trajectory, coordinates*

1. Introduction

The intensification of the automation of the industrial processes and of the services has led to the increasing use and imposition of the harmonic gear transmissions in almost all the fields of the current technique (construction of cosmic ships and rockets, airplanes, helicopters, atomic reactors, industrial robots, radar antennas, naval mechanisms, servo-mechanisms, motors - reducers, machine tools, shareholders in hermetic spaces, etc. [2, 4, 12].

In the class of harmonic gear transmissions are included the double harmonic transmissions, which are able to provide very high positioning kinematic accuracy, at the lowest dimensions and weights.

Figure 1 shows the scheme of the double harmonic transmission studied [6, 7], which has the following active elements: the wave generator (1) is input element, the flexible toothed wheel (2) is intermediate element, the rigid toothed wheel (3) is fixed element and the rigid mobile toothed wheel (4) which is the output element.

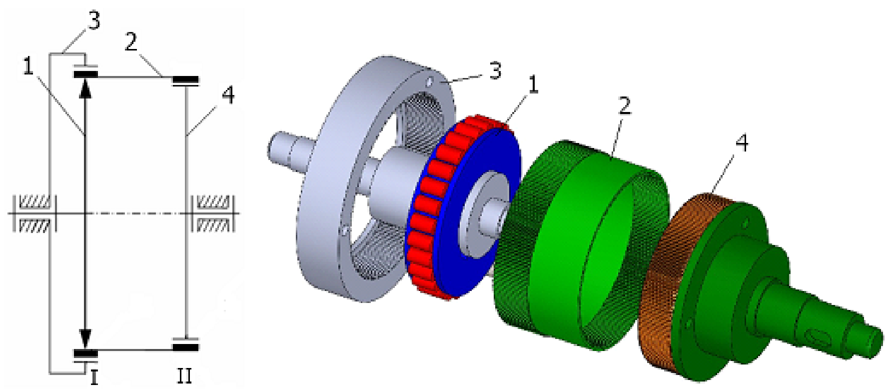

Figure 1. The scheme of the double harmonic transmission

The study of the conditions of harmonic gear in the two steps (I and II) of the double harmonic transmission (first step: flexible wheel (2) / fixed rigid wheel (3), respectively the second step: flexible wheel (2) / mobile rigid wheel (4)) it does by tracking the movements of certain points of the flexible wheel.

The paper presents the mathematical algorithm for determining the trajectories of points located on the middle fibers of the flexible wheel, in the two front surfaces corresponding to the two gear steps of the double harmonic transmission.

2. Mathematical algorithm for determining trajectories of points

For the study of the movement of a certain point M on the medium fiber of the deformed flexible wheel, the following reference systems were considered (Figure 2): $S_1(XOY)$ - mobile, connected to the wave generator, $S_2(x''O_2y'')$ - mobile, connected to the flexible wheel and $S_3(xOy)$ - fixed, connected to the rigid wheel.

By forced mounting of the wave generator (1) inside the flexible wheel (2), it deforms and in cross-section will have the shape of an ellipse. The instantaneous position of a certain point M located on the average fiber from a front surface (I) of the flexible wheel (in report to the origin O of the fixed reference system S_3), considered when passing the wheel from the unformed state to the deformed state, is given by the position vector $\vec{r}_M$, [5, 8, 9]:

$$\vec{r}_M(\varphi) = \vec{r}_{02}(\varphi_2) + \vec{w}(\varphi_2) + \vec{v}(\varphi_2) = \vec{r}_{02}(\varphi_2) + \vec{s}(\varphi_2) \tag{1}$$

where: $\vec{r}_{02}(\varphi_2)$ is the position vector of point M (confused with point O_2) for the non-deformed state of the flexible wheel, $\vec{s}(\varphi_2)$, $\vec{w}(\varphi_2)$ şi $\vec{v}(\varphi_2)$ - are the displacements (total, radial and tangential) of the point M in the mobile system S_2, when passing the flexible wheel from unformed state to the deformed state.

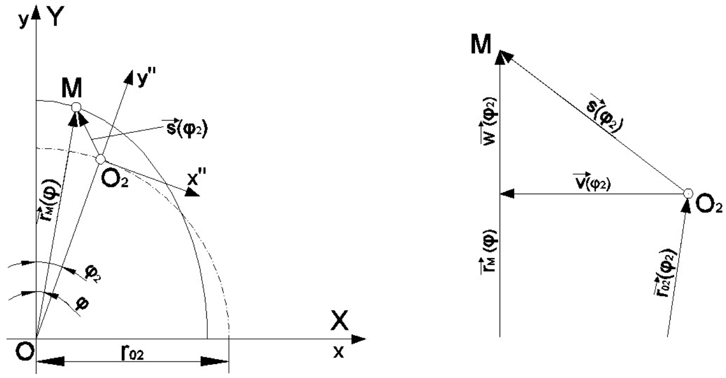

Figure 2. Positioning of a point M on the medium fiber of the deformed flexible wheel

For the calculation of radial and tangential displacements, a law of cosine-deformation of the flexible wheel was adopted [3, 11]:

$$\begin{cases} w = w_0 \cdot \cos 2\varphi_2 \\ v = -\left(w_0 / 2\right) \cdot \sin 2\varphi_2 \end{cases} \tag{2}$$

where: w_0 is the maximum radial deformation of the flexible wheel.

In the case of the dynamic regime of the double harmonic transmission, when the rotational motion is transmitted from the wave generator (1) to the mobile rigid wheel (4) through the flexible wheel (2), the mobile system S_2 (connected to the flexible wheel) will have a slow rotational motion with angular velocity ω_2 around the origin O of the fixed reference system S_3. Thus, the study of the movement of point M in relation to the fixed system S_3 will be reduced to the study of the relative motion of point M in relation to point O_2, which executes a transport movement.

For the determination of the trajectory (T_M) of the point M on the medium fiber of the flexible wheel, a somewhat intermediate position of the wave generator was considered (Figure 3), which rotates clockwise with the angular velocity ω_1 and travels an angle at the center φ_1. It is observed that when rotating the wave generator with an angle φ_1, the point M remains behind an angle φ with respect to the axis of the ordinates Oy, and the point O_2 corresponding to the point M (for the imaginary state, not deformed by the flexible wheel) will be positioned below an angle φ_2 to the axis Oy. The flexible wheel rotates in the opposite direction to the rotation of the wave generator.

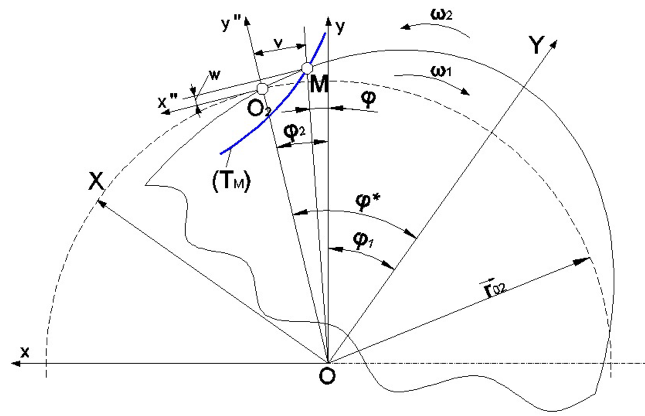

Figure 3. The trajectory of a point M on the average fiber of deformed flexible wheel

The trajectory (T_M) of the point M with respect to the fixed system S_3 will result from the transport movement of the system S_2 and the relative movement of the point M with respect to the mobile system S_2.

The successive positions of the point M in the fixed system S_3 are expressed analytically by means of the point transformation relation [1, 10], which defines the rotational translation of the mobile system S_2 with respect to the fixed system S_3:

$$\left(\vec{r}_M\right)=\left[M_\varphi\right]\cdot\left(\vec{r}''_M\right) \tag{3}$$

$$\left(\vec{r}_M\right)=\begin{bmatrix} X \\ Y \\ 1 \end{bmatrix};\ \left[M_\varphi\right]=\begin{bmatrix} \cos\varphi_2 & \sin\varphi_2 & r_{02}\cdot\sin\varphi_2 \\ -\sin\varphi_2 & \cos\varphi_2 & r_{02}\cdot\cos\varphi_2 \\ 0 & 0 & 1 \end{bmatrix};\ \left(\vec{r}''_M\right)=\begin{bmatrix} x'' \\ y'' \\ 1 \end{bmatrix} \tag{4}$$

where: $\left(\vec{r}_M\right)$ is the matrix of the position vector of the point M with respect to the system S_3; $\left(\vec{r}''_M\right)$ - the matrix of the position vector of the point M in relation to the system S_2; $[M_\varphi]$ - the transfer matrix from the mobile system S_2 to the system S_3.

The coordinates of point M in the mobile system S_2 can be expressed analytically by the relations (5), and in relation to the fixed system S_3 by the relations (6):

$$\begin{cases} x''=v\left(\varphi^*\right)=-\left(w_0/2\right)\cdot\sin 2\varphi^* \\ y''=w\left(\varphi^*\right)=w_0\cdot\cos 2\varphi^* \end{cases} \tag{5}$$

$$\begin{cases} X = r_{02} \cdot \sin\varphi_2 + x'' \cdot \cos\varphi_2 + y'' \cdot \sin\varphi_2 \\ Y = r_{02} \cdot \cos\varphi_2 - x'' \cdot \sin\varphi_2 + y'' \cdot \cos\varphi_2 \end{cases} \tag{6}$$

where: r_{02} is the radius of the average fiber of the non-deformed flexible wheel, w_0 – the maximum radial deformation of the deformed flexible wheel.

The trajectory of the point M in the fixed system S_1 is a curve very close to a hypocycloid, which can be graphically constructed by applying the mathematical algorithm presented.

3. Graphical construction of the trajectories

Based on the mathematical algorithm presented, a calculation program was developed, written in Visual Basic, which allowed the graphical construction of the trajectories of points M (in section I) and M' (in section II). Points M and M' are positioned on the same generatrix of the average cylindrical surface of the flexible wheel and belong to one front surface of the wheel.

For the case of double harmonic transmission with the cam wave generator, characterized by the following constructive and functional parameters: the transmission ratio, $i_{14}^{(3)}$ = 48,47; the radius of the middle surface of the non-deformed flexible wheel, r_{02} = 29,3 mm; the maximum radial deformation in the frontal section I of the flexible wheel, w_0 = 0,3 mm; the maximum radial deformation in the frontal section II of the flexible wheel, w'_0 = 0,27 mm; the angle, φ = (0...90)° and angular step, j = 10°, the results presented in table 1 - for the coordinates of point M and respectively in table 2 - for the coordinates of point M' were obtained.

Table 1. The coordinates of point M

Step I: r_{02} = 29,3 [mm]; w_0 = 0,3 [mm]								
Angle [°]		Displacement [mm]		Coordinates of point M [mm]				Radius [mm]
		radial	tangential	static regime		dynamic regime		
φ	φ_2	w	v	x''_M	y''_M	X_M	Y_M	r_M
0	0	0,300	0,000	0,000	0,300	0,000	29,600	29,600
10	0,1	0,282	-0,052	-0,052	0,282	0,000	29,582	29,582
20	0,2	0,228	-0,097	-0,097	0,228	0,006	29,529	29,529
30	0,3	0,147	-0,131	-0,131	0,147	0,024	29,448	29,448
40	0,4	0,048	-0,148	-0,148	0,048	0,057	29,348	29,348
50	0,5	-0,057	-0,147	-0,147	-0,057	0,108	29,243	29,243
60	0,6	-0,155	-0,128	-0,128	-0,155	0,177	29,144	29,145
70	0,7	-0,234	-0,094	-0,094	-0,234	0,262	29,065	29,066
80	0,8	-0,285	-0,047	-0,047	-0,285	0,358	29,013	29,015
90	0,9	-0,300	0,005	0,005	-0,300	0,460	28,996	29,000

Table 2. The coordinates of point M'

Step II: r_{02} = 29,3 [mm]; w'_0 = 0,27 [mm]								
Angle [°]		Displacement [mm]		Coordinates of point M' [mm]				Radius [mm]
		radial	tangential	static regime		dynamic regime		
φ	φ_2	w	v	$x''_{M'}$	$y''_{M'}$	$X_{M'}$	$Y_{M'}$	$r_{M'}$
0	0	0,270	0,000	0,000	0,270	0,000	29,570	29,570
10	0,1	0,253	-0,047	-0,047	0,253	0,005	29,553	29,553
20	0,2	0,206	-0,087	-0,087	0,206	0,015	29,506	29,506
30	0,3	0,133	-0,118	-0,118	0,133	0,036	29,433	29,433
40	0,4	0,043	-0,133	-0,133	0,043	0,072	29,343	29,343
50	0,5	-0,052	-0,133	-0,133	-0,052	0,123	29,249	29,249
60	0,6	-0,140	-0,115	-0,115	-0,140	0,190	29,160	29,160
70	0,7	-0,211	-0,084	-0,084	-0,211	0,271	29,088	29,089
80	0,8	-0,256	-0,043	-0,043	-0,256	0,363	29,042	29,044
90	0,9	-0,270	0,004	0,004	-0,270	0,460	29,026	29,030

Figure 4 graphically shows the trajectories of points M (from the front section I of the flexible wheel) and M' (from the front section II of the flexible wheel) obtained for rotating the wave generator with an angle $\varphi \in [0^\circ, 90^\circ]$. The trajectories obtained for the two points considered are two hypocycloid arcs.

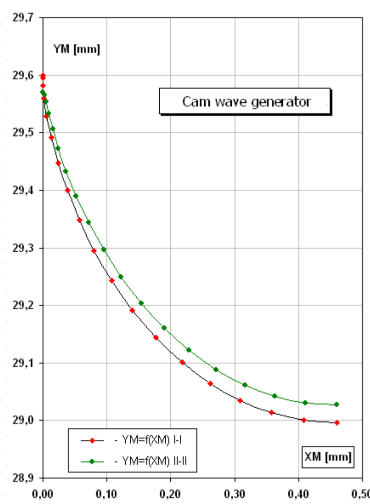

Figure 4. The trajectories of points M and M'

The correctness and veracity of the results (analytical and graphical) obtained after running the calculation program, designed on the basis of the presented mathematical algorithm, were also confirmed by using the specialized program Catia 5.0 (Figure 5).

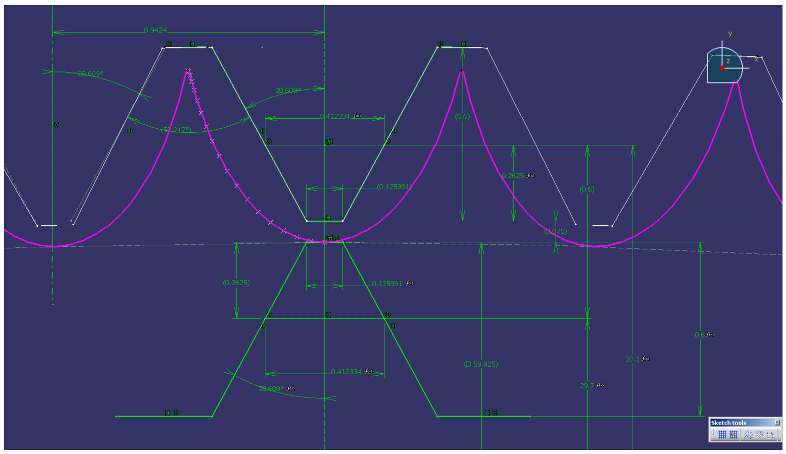

Figure 5. Trajectory of a point in the front section I of the flexible wheel

It is observed that the uniform rotation of the cam wave generator results in the rotation of the flexible wheel in the opposite direction. For a point in the front section I of the flexible wheel, positioned on the axis of symmetry of the tooth at the intersection with the addendum circle of the wheel, a curve very close to a hypocycloid is also obtained.

4. Conclusion

The paper presents a study of the kinematics of the points, positioned in the front sections of the flexible wheel from a double harmonic transmission.

In this context, a mathematical algorithm was presented for the graphical determination of the trajectories of the points located in the front sections of the flexible wheel, which was the basis for the elaboration of an original computer program, written in the Visual Basic language.

The correctness and veracity of the results obtained from the use of the elaborated computer program were verified and confirmed by comparing these results with those obtained after running the specialized program Catia 5.0.

The graphical constructions (Figure 4 and Figure 5) highlight the peculiarities of the harmonic gears from the two step of the double harmonic transmission. Also, these graphical constructions can be used to choose the basic geometric parameters of the flexible wheel tooth (tooth pitch, tooth height, clearance, profile angle, etc.) so as to avoid interference of the teeth.

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