ANALELE UNIVERSITĂȚII
"EFTIMIE MURGU" REȘIȚA
ANUL XXVI, NR.1, 2019, ISSN 1453 - 7397

Improving the precision of the frequencies evaluated with an interpolation method based on maximizer obtained after iterative signal truncation

Gilbert-Rainer Gillich, Vasile Iancu, Nicoleta Gillich, David Lupu, Andrea Amalia Minda

Estimating the harmonic components of a signal is based on calculating the signal energy on certain spectral bins that position is defined by the signal time length. Because of this mode of calculation, a difference between the true frequency and the estimate is usually found. In this paper, we present an advanced frequency estimation method, which searches first for the time length for which an entire number of cycles is contained in the signal and afterward calculates the Discrete Fourier Transform (DFT). The results have been found significantly closer to the true frequency as those obtained by a standard DFT applied to the original signal, but it still depends on the number of cycles involved. To further improve the estimation results we propose a methodology to make a correction depending on the number of cycles and sampling frequency.

Keywords: *frequency estimation, interpolation, signal truncation, sampling frequency*

1. Introduction

Standard frequency estimation methods, as the DFT or its derivatives, are not able to indicate small frequency changes because the results are in direct relation with the acquisition time [1]. The shorter the acquisition time, i.e. the length of the acquired signal, the bigger the distance between two spectral bins and coarser the frequency estimate is. Still, the accuracy of the frequency estimates is essential in many engineering applications [2-4]. Techniques to improve frequency estimation are based on determining a frequency correction term. This is smaller than the distance between two spectral lines and places the estimated frequency in an inter-bin position [5].

The simplest techniques involve interpolation to obtain the correction term. Interpolation is made for two or three DFT samples in the frequency range of interest in the spectrum [6-11]. We found that the results accuracy increases, but still depend on the acquired signal length [12].

A different approach to precisely identifying the frequency components of a signal, developed by the authors, is to truncate the original signal with a given number of samples by iteration and to calculate the DFT for each of the resulted signals. The achieved spectra are overlapped and a dense spectrum results. Now, the biggest maximizer is identified the frequency range of interest. Its position in the spectrum indicates the true frequency [13-16]. In this way we eliminates the dependence of the frequency estimate on the acquisition time and the resulted errors are very small compared to those achieved by using the actual techniques. However, applying this method we observed that the maximizer take positions after a pseudo-sinc function [17], which is asymmetric and is the cause of the small errors. In this paper, we propose a method to correct the interpolation results by considering the sampling frequency and the number of cycles contained in the finally truncated signal.

2. The pseudo-sinc function

We developed an algorithm that involves iterative truncation of the original signal having the time length $t_S\,[s]$, which presumes applying rectangular windows with different time lengths $t_{Wi}\,[s]$. This means reducing the number of samples N_S contained in the original signal with a certain number of samples by iteration. We reduce usually two samples by iteration, so after i iterations the number of samples cropped from the original signal is $2i$, remaining $N_{Si} = N_S - 2i$ samples. Obviously, the index i stay for the number of the applied window.

For each resulted signal the maximizer is found by calculating the DFT. The aim is to find the biggest maximizer and identify the spectral bin on which it is located. It indicates that the truncated signal for which this maximizer is found contains an entire number of cycles for the targeted frequency component. For this signal length the frequency resolution produces such a distribution of the spectral bins that one corresponds to the true frequency. Therefore, the measured frequency is extremely close to the true frequency, the errors being negligible.

Because the spectral bins of the overlapped spectrum are not equidistantly distributed, the envelope of the overlapped spectrum is not a sinc function but a pseud-sinc function [17]. This makes the envelope asymmetric. Due to the energy contained at each spectral bin, dependent on the width of the frequency resolution, the amplitudes are distorted. This made us considering that the frequency estimation can be further improved.

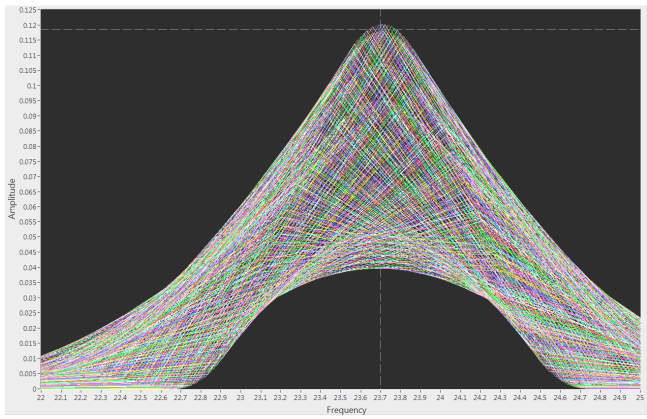

Figure 1. The overlapped spectrum obtained after many signal truncation

A characteristic representation of a dense overlapped spectrum that contains many maximizer is presented in Fig. 1. Each maximizer shown in this figure belongs to a spectrum obtained for a particular analysis time t_{Wi}. We find the frequency either on the spectral bin where the biggest maximizer is located, or finer, by interpolation made for the biggest three maximizer.

3. Tests performed to improve the method's accuracy

We make simulation for several cases consisting in the number of entire cycles contained in the original signal, see Table 1. For these cases we perform estimation and show the frequencies estimated with the standard DFT and the method that involves truncation of the signal until an entire number of cycles is achieved. We generated the analyzed signal with the frequency $f = 6.33\,\text{Hz}$ to have a clear benchmark. The number of cycles taken into consideration is 5, 6, 12 and 17, and the corresponding number of samples is calculated in respect to the sampling rate. Fig. 2 shows the inter-bin amplitude resulted from interpolation.

Table 1. Simulation cases

Number of samples	Sampling frequency	Generated frequency	No. of cycles	Standard DFT estimation	PyFEST estimation
N [-]	r [-]	f [Hz]	n [-]	f_{DFT} [Hz]	f_{PyFEST} [Hz]
360	1000	6.33	5	6.6852	6.3107
410	1000	6.33	6	5.8680	6.3197
790	1000	6.33	12	6.0837	6.3317
1090	1000	6.33	17	6.2443	6.3326

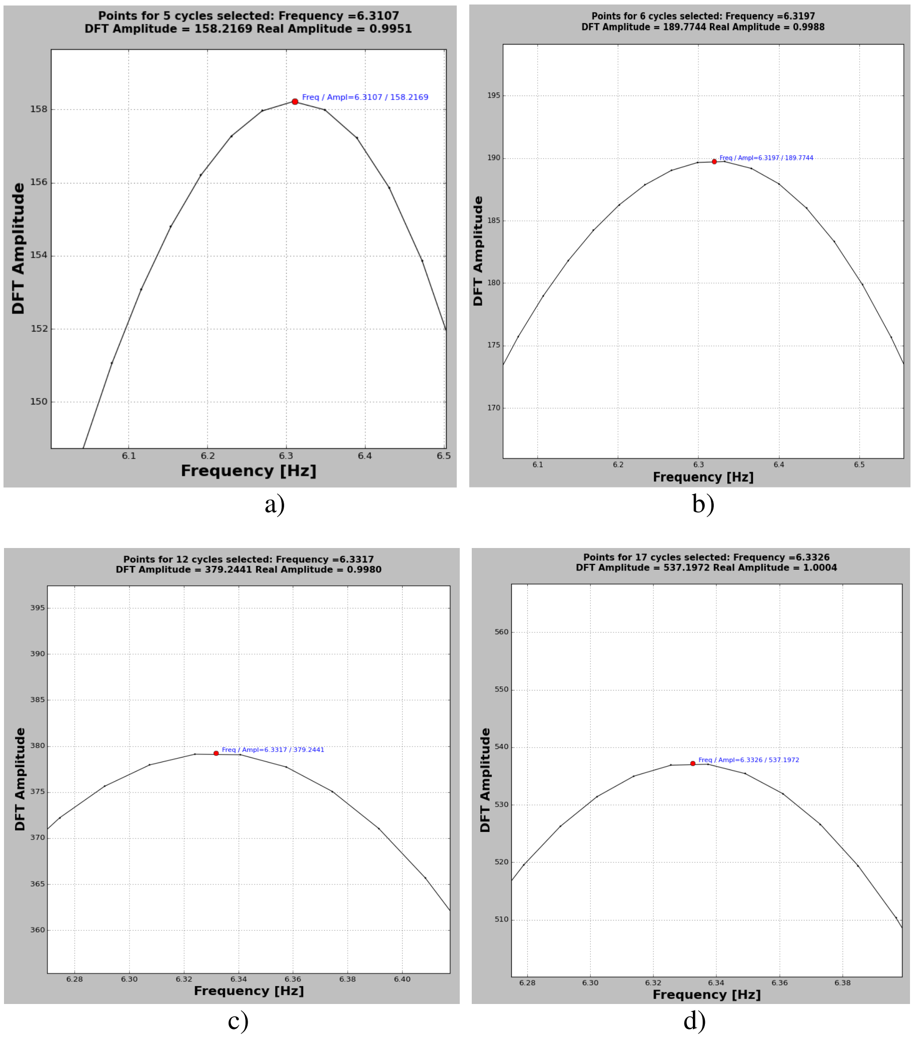

Figure 2. The overlapped spectra containing the DFT samples used for interpolation obtained after many signal truncation

It can be observed from Figure 2 the asymmetry of the overlapped spectrum. It is also obvious that this asymmetry loss on significance when the number of cycles contained in the original signal increase and consequently the time length increase. By lengthening the time, we get better results.

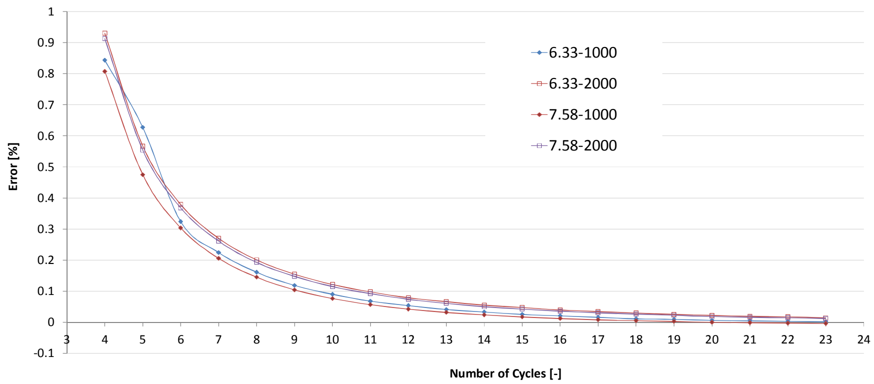

Figure 3. Error curves for different signals–interpolation made for DFT samples

Figure 3 shows the error curves plotted for four curves: two having the frequency 6.33 Hz and are generated with the sampling rate 1000 respectively 2000 and the other two having the frequency 7.58 Hz, generated again with the sampling rate 1000 respectively 2000. Because the signal is iteratively truncated and other energy is found in it, we propose herein to consider the Power Spectral Density (PSD), which normalize the energy of the spectral bin with the frequency resolution. In this way, the accuracy of the achieved results increases and the error is predictable.

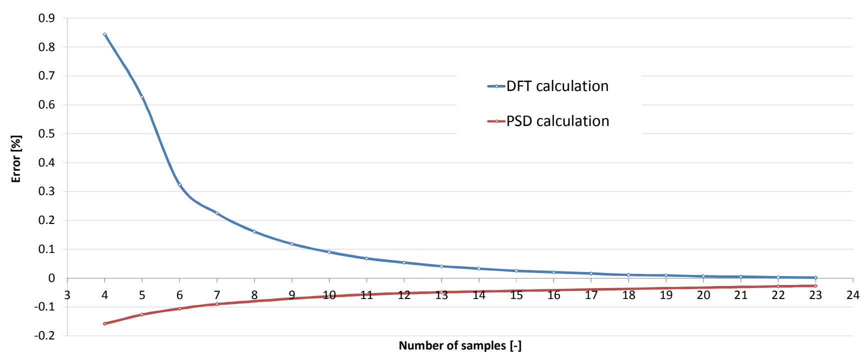

Figure 4. Error curves for interpolation made on DFT respectively PSD samples

4. Conclusion

We analyze an original interpolation method to estimate the frequency components of a signal using DFT samples belonging to spectra obtained for different time lengths. The method is proved reliable and errors less than 1% are obtained. The precision increases with the time signal's length becoming negligible when considering numerous cycles in the original signal.

The proposed method can be improved by taking for interpolation PSD samples. These present the advantage that, for the different cropped signals, the effect of the time length is suppressed by normalization. The results are significantly improved compared with the case when we use the DFT samples, especially for a small number of cycles.

References

[1] Gillich G.R., Mituletu I.C., Signal Post-processing for Accurate Evaluation of the Natural Frequencies, *Structural Health Monitoring. Smart Sensors, Measurement and Instrumentation*, 26, Springer Cham, 2017, pp. 13-37.

[2] Khatir S., Dekemele K., Loccufier M., Khatir T., Wahab M.A., Crack identification method in beam-like structures using changes in experimentally measured frequencies and Particle Swarm Optimization, *Comptes Rendus Mécanique*, 346(2), 2018, pp. 110-120.

[3] Gillich G.R., Furdui H., Wahab M.A., Korka Z.I.**,** A robust damage detection method based on multi-modal analysis in variable temperature conditions**,** *Mechanical Systems and Signal Processing***,** 115, 2019, pp. 361-379**.**

[4] Tufoi M., Hatiegan C., Vasile O., Gillich G.R., Dynamic Analysis of Thin Plates with Defects by Experimental and FEM Methods, *Romanian Journal of Acoustics and Vibration,*10 (2), 2013, pp. 83-88.

[5] Djukanović S., Popović T., Mitrović A., Precise sinusoid frequency estimation based on parabolic interpolation, *Proceedings of the 24th Telecommunications Forum TELFOR*, 2016, pp. 1-4.

[6] Grandke T., Interpolation Algorithms for Discrete Fourier Transforms of Weighted Signals, *IEEE Transactions on Instrumentation and Measurement*, 32, 1983, pp. 350-355.

[7] Quinn B.G., Estimating Frequency by Interpolation Using Fourier Coefficients, *IEEE Transactions on Signal Processing*, 42, 1994, pp. 1264-1268.

[8] Jain V.K., Collins W.L., Davis D.C., High-Accuracy Analog Measurements via Interpolated FFT, *IEEE Transactions on Instrumentation and Measurement*, 28, 1979, pp. 113-122.

[9] Ding K., Zheng C., Yang Z., Frequency Estimation Accuracy Analysis and Improvement of Energy Barycenter Correction Method for Discrete Spectrum, *Journal of Mechanical Engineering*, 46 (5), 2010, pp. 43-48.

[10] Voglewede P., Parabola approximation for peak determination, *Global DSP Magazine*, 3 (5), 2004, pp. 13-17.

[11] Jacobsen E., Kootsookos P., Fast, accurate frequency estimators, *IEEE Signal Processing Magazine*, 24 (3), 2007, pp. 123-125.

[12] Minda A.A., Gillich G.R., Sinc Function based Interpolation Method to Accurate Evaluate the Natural Frequencies, *Analele Universitatii „Eftimie Murgu" Resita, Fascicula de Inginerie,* 24(1), 2017, pp. 211-218.
[13] Minda A.A., Gillich N., Mituletu I.C., Ntakpe J.L., Manescu T., Negru I., Accurate frequency evaluation of vibration signals by multi-windowing analysis, *Applied Mechanics and Materials*, 801, 2015, pp. 328-332.
[14] Gillich G.R., Mituletu I.C., Praisach Z.I., Negru I., Tufoi M., Method to Enhance the Frequency Readability for Detecting Incipient Structural Damage, *Iranian Journal of Science and Technology, Transactions of Mechanical Engineering*, 41(3), 2017, pp. 233–242.
[15] Gillich G.R., Mituletu I.C., Negru I., Tufoi M., Iancu V., Muntean F., A Method to Enhance Frequency Readability for Early Damage Detection, *Journal of Vibration Engineering & Technologies*, 3(5), 2015, pp. 637-652.
[16] Ntakpe J.L., Gillich G.R., Mituletu I.C., Praisach Z.I., Gillich N., An Accurate Frequency Estimation Algorithm with Application in Modal Analysis, *Romanian Journal of Acoustics and Vibration*, 13(2), 2016, pp. 98-103.
[17] Gillich G.R., Minda A.A., Korka Z.I., Precise estimation of the resonant frequencies of mechanical structures involving a pseudo-sinc based technique, *Journal of Engineering Science and Innovation*, 2(4), 2017, pp.37-48.
[18] Minda P.F., Praisach Z.I., Gillich N., Minda A.A., Gillich G.R., On the Efficiency of Different Dissimilarity Estimators Used in Damage Detection, Romanian *Journal of Acoustics and Vibration,* 10(1), 2013, pp. 15-18.
[19] Gillich G.R., Minda P.F., Praisach Z.I., Minda A.A., Natural frequencies of damaged beams – a new approach*, Romanian Journal of Acoustics and Vibration*, 9(2), 2012, pp. 101-108.
[20] Gillich G.R., Maia N., Mituletu I.C., Tufoi M., Iancu V., Korka Z., A new approach for severity estimation of transversal cracks in multi-layered beams, *Latin American Journal of Solids and Structures*, 13(8), 2016, pp. 1526-1544.

Addresses:

- Prof. Dr. Eng. Gilbert-Rainer Gillich, "Eftimie Murgu" University of Reşiţa, Piaţa Traian Vuia, nr. 1-4, 320085, Reşiţa,gr.gillich@uem.ro
- Lect. Dr. Eng. Vasile Iancu, "Eftimie Murgu" University of Reşiţa, Piaţa Traian Vuia, nr. 1-4, 320085, Reşiţa, v.iancu@uem.ro
- Prof. Dr. Eng. Nicoleta Gillich, "Eftimie Murgu" University of Reşiţa, Piaţa Traian Vuia, nr. 1-4, 320085, Reşiţa, n.gillich@uem.ro
- PhD Stud. David Lupu, "Eftimie Murgu" University of Reşiţa, Piaţa Traian Vuia, nr. 1-4, 320085, Reşiţa, david.lupu@messer.ro
- Lect. Dr. Andrea Amalia Minda, "Eftimie Murgu" University of Reşiţa, Piaţa Traian Vuia, nr. 1-4, 320085, Reşiţa, a.minda@uem.ro