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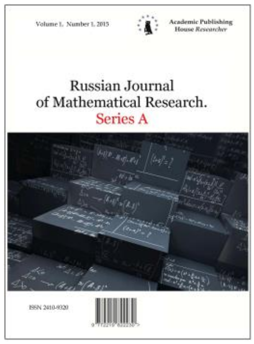

Does The Gravitational Weight Vary With the Area of a Flat-Sheet? A Comparative Study of Area Measurements: With Special References to Simpson's Rule and Bhattacharya's Theorem

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Abstract

A few methods are known for the measurements of bended flat areas. But the problems arise when the areas are bended with many bends and deflections. In this situation we have to depend on Simpson's rule. If we follow this rule however, we can measure areas with certain limitations. Therefore accurate results are not always achieved. Recently a theorem has been reported which is based on conversion of gravitational weight of a map occupied by the area of the flat surface. This procedure appears to be simple, prompt and accurate. However a comparative study of area measurements is therefore needful to determine the validity of this theorem.

Keywords: Simpson's rule, Uniform density, $I=W10^2$, Plotted plan, Parabola.

1. Introduction

The curiosity for the measurements of multiple bended flat surfaces was initially originated in the mind of men since the time of third century B.C. (1). We recall the work of Archimedes (287B.C.-212 B.C.) The work of Archimedes on geometrical construction and the measurement of flat and spherical surfaces(2). Many well-known scientists, such as Isaac Newton(1643), (3). Pythagoras (6 century B.C.), Euclid(3rd century B.C.), Ptolemy (2nd century A.D.) , Heron of Alexandria (dates not known) etc gifted their contributions in the technology of surface measurements. In fact, many scientists were then involved in this investigation, but one accurate method was never been reported. However various improved methods are now available, but these methods can be used only in the measurement of simple bended flat areas. (4,5). But actual objective of this investigation was to develop an easy procedure which will correctly measure all kinds of flat areas irrespective of sizes and geometric forms. (6).

Although a perfect method of such measurements is not yet known to us, but a few ongoing procedures which are somehow in practical use have attracted our attention. These are computations of areas from the mid-ordinate rule (3) and the application of Simpson's rule. (7) Surprisingly, a recent report of a theorem $I=W10^2$ claims that accurate measurements of areas surrounded by many bends and deflections are possible. (6) This theorem is based on the findings that gravitational weight (GW) of a flat sheet (FS) is directly proportional with the area of the FS which of course should be of uniform density. Now we feel it is necessary to compare the claims of this theorem with the other existing methods. Also, validity of this theorem and it's importance in the area measurements are required for careful verification.

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2. Materials and methods

The measurements of unknown areas bended by many bends and deflections may be done somehow with the help of some known methods.

A reduced size map of a water-land surrounded by a border line with numerous bends and deflections was marked for the determination of its area (Figure 1). A small size bended land was also present in the center of the large bended land. So the determination of the area of the water-land was done by subtracting the area of the central land from the area of the total land. Initially we selected eight different methods (Table 1) for these measurements. But actually we were able to work with four different methods only. These were: graphical method, mid-ordinate rule, Simpson's rule and Bhattacharya's theorem. We were unable to use the rest of the methods, such as: Average ordinate rule, Trapezoidal rule, Field notes and Plotted plan because the entire area can not be divided into regions of convenient geometrical shapes. Another reason was the ends of the ordinates drawn inside the map can not be assumed as straight lines.

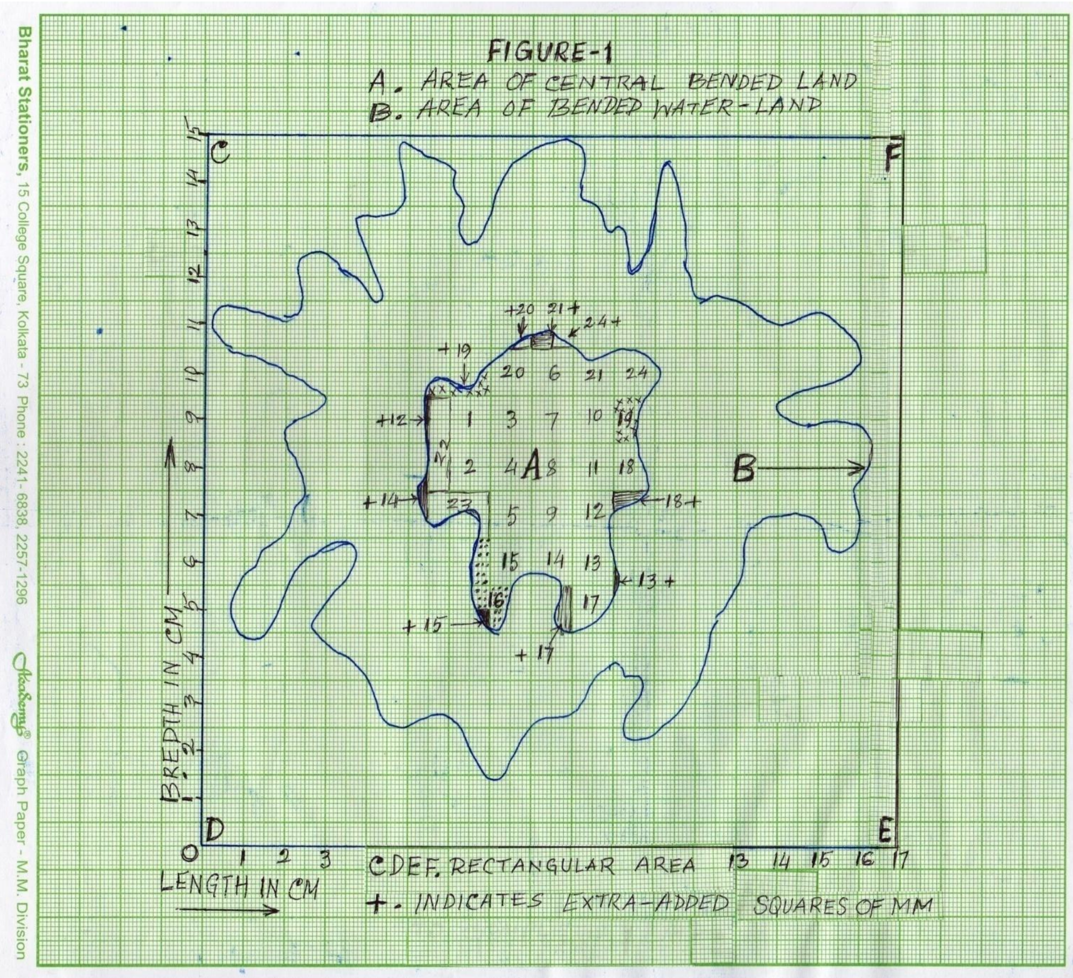

Fig. 1. Maps of the central high-land surrounded by the water-land showing the border lines with many bends and deflections

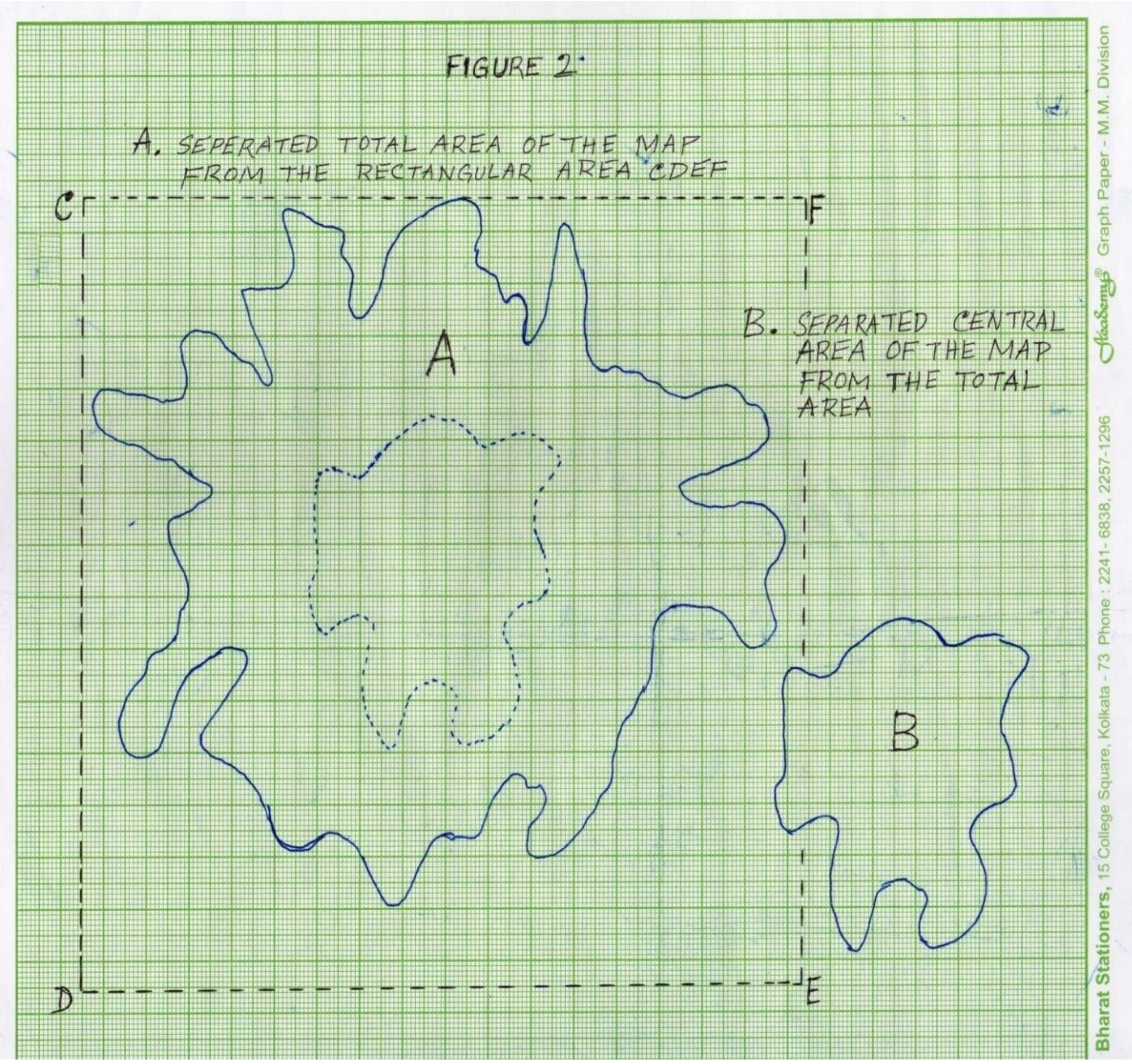

Fig. 2. The different areas of the total map were separated for the determination of gravitational weights

i. Graphical method:

The total map was drawn onto a graph paper by tracing the map after inserting a carbon paper between the map and the graph paper. The total number of squares in centimeters were counted easily. But we had the problems to count near the border line where the bends and deflections were present. However we followed the give and take principle (Figure 1) Moreover, some assumptions were required to complete the countings. In this method eventually correct measurements were not possible due to many bends and deflections in the boundary line. However approximately correct results were obtained in the measurements of central land. (Table 1)

ii. The Mid-Ordinate Rule:

We determined the area of the central land according to this rule Area=Common distance * sum of mid-ordinates.

The total bended area was first measured by combining this method with graphical method, particularly at the regions of boundary line. Here we also used the give and take principle. However long time was required to complete one task. Even some unavoidable minor errors were present in counting. The data were recorded in Table-1. However approximate results were obtained in the measurement of central area. But we were unable to count correctly due to irregular shape of the border line.

iii. The Average Ordinate Rule:

We attempted to measure all the areas according to the formula:

Area = $\frac{\text{Sum of ordinates}}{\text{no. of ordinates}}$ * length of the base line. It appeared that this formula by itself only can not help in the measurements of our proposed areas; because the bended border line can not be assumed as the straight lines. Therefore the total area was measured with the help of average ordinate rule combining with the graphical method. But still the results were not found to be correct. More over extended long time was required to complete the calculations. Therefore this method was considered as an unsuitable one for our tasks.

iv. From Field Notes:

According to this procedure it was not possible to draw geometrical figures within the bended areas. So our attempts for the measurements were discontinued.

v. The Trapezoidal Rule:

We were unable to apply this rule for the measurements; because we could not assume boundaries between the ends of ordinates to be straight. So, the areas between the base line and the irregular boundary line can not be considered as trapezoid. Therefore we did not proceed further for the measurements.

vi. Simpson's Rule

To apply this rule we drew a base line in the middle of the total map. The ordinates were drawn from both sides of the base line. The boundaries between the ends of ordinates were assumed to form the arc of parabolas. The number of ordinates were odd numbers and divisions were even numbers. The areas were calculated according to the formula:

Area = $\frac{\text{Common distance}}{3}$ { first ordinate + last ordinate + 2 × (Sum of remaining odd ordinates) + 4 × (Sum of even ordinates) }. In this measurement we had to ignore some strips of areas near the arcs of parabolas. The data of Simpson's rule are written in Table 1. The countings were performed separately by five persons. But correct results were not obtained. In the measurements of the central land, it were easier tasks and the results were also approximately correct. This was due to simple bended structure of the central land.

vii. Bhattacharya's Theorem(6):

While using this theorem for the measurement of irregular areas four straight lines were drawn on the four sides of the map to form a rectangle (Figure 1). The lengths of these lines were measured and written in centimeters. The scale of the map shows the exact ratio between the size of the map and that of the original bended lands.

A piece of polyethylene sheet (PES) was used for taking a photographic image of the map onto it. The outline of the photographic picture of the map was drawn on the PES and then it was separated from the PES sheet by cutting it with an electric needle. The GW of this portion of PES was measured in an electronic balance(6).

3. Results

i. Typical procedure of calculations

Proportional weight of the unknown bended area of the PES:

The weight of PES (total map) = 950 mg which is marked as W_1 and the weight of the known rectangular area of PES = 1.875 gm. Which is marked as W_2. The ratio of the weights of total unknown and known rectangular areas will be $\frac{W1}{W2}$, say, this ratio = W

So W = $\frac{W1}{W2} = \frac{0.950}{1.875}$ = 0.506

The index (I) of the unknown bended total area of the map is,

I = $\frac{W1}{W2}$ * 100 = 0.506 * 100 = 50.6 %

ii. Area of the total map:

Area of the rectangle = 15 cm * 17 cm = 255^2 cm

GW of the PES (rectangle) = 1.875 gm (measured in a balance)

Therefore 1.875 gm of PES = 255^2 cm of area

Then 1 gm of PES = $\frac{255}{1.875}$ = 136^2 cm of area

GW of the PES (total map) = 0.950 gm (measured in a balance)

Hence 0.950 gm PES = 0.950 * 136 = 129.2^2 cm (area of the total map)

iii. Area of the Central land:

GW of the PES of the central land = 0.2 gm (measures in a balance)

1 gm of PES = 136^2 cm of area

0.2 gm of PES = $\frac{255}{1.875}$ * 0.2 = 27.2^2 cm (area of the central land)

iv. Area of The Water Land

Total land minus the central land

129.2 – 27.2 = 102^2 cm (area of the water land)

All experiments including calculations were separately performed by five different persons (one author and four surveyors). Each experiment was repeated five times and the average number was recorded in Table 1.

1. The Statistical Procedure

The standard deviation of the means and the t-tests for the

significance at the level P<0.5 were determined adopting the procedure described in SPSS computer program. (6)

Table 1. Measurements of Bended Areas (Results are expressed in square centimeters)

	Graphical Method 1.	Mid Ordinate rule 2. 5	Average Ordinate rule 2.	Trapezoidal rule 2.	Field notes 2.	Plotted plan 3.	Simpson's rule 1, 4, 5	Bhattacharya's theorem 6.
Total area	118.4 ± 2.35	131.2 ± 4.25	N.M.	N.M.	N.M.	N.M.	139.4 ± 5.75	129.2 ± 0.95
Central area	25.43 ± 1.8	24.6 ± 3.8	N.M.	N.M.	N.M.	N.M.	28.2 ± 4.55	27.2 ± 0.05
Water area	93.37 ± 2.85	106.6 ± 4.85	N.M.	N.M.	N.M.	N.M.	110.7 ± 5.25	102.0 ± 0.88

N.M. Not measurable

1. Takes long time to complete.

2. Many bends and deflections are present in the boundaries between the ends of ordinates; can not be assumed to be straight lines.

3. The entire area can not be divided into regions of convenient shapes.

4. The boundaries between the ends of ordinates can not be assumed to form an arc of parabolas.

5. Some difficult portions of bended areas were measured by graphical method.

6. Takes short time to complete. No problem for many bends and deflections.

4. Discussion

All the methods written in this paper for the measurements of irregularly bended flat areas were examined separately by five different experts. Our objective was to select just one dependable method. So far, we examined eight methods. Except one, all the remaining methods were noticed to be attached with one or more unsolved questions. Sometimes one or more requirement to design a specific structure created problems in the measurements. As an example: Computation of area from plotted plan. Here the entire area was required to divide into regions of convenient geometrical figures, such as: triangles, squares, trapeziums, parallel lines etc. In this case, the entire area was changed to a particular shape in order to make easy measurements. However the graphical procedure was noticed to be an easy method for the measurement of simple bended areas. But long time was required to complete just-one measurement. Some assumptions were frequently needed when countings were done near the bends and deflections of the areas. When careful calculations were done with graphical procedure nearly correct results were obtained (Table 1).

When the graphical method was combined with mid-ordinate rule the time required for calculations was greatly reduced and the results were approximately correct. But the mid-ordinate rule without combination of graphical method appeared to be difficult to complete the measurements.

Simpson's rule is a well-known method for the measurements of bended flat areas. In this rule the boundaries between the ends of ordinates were assumed to form arcs of parabola. This rule was applicable only when the number of divisions of an area was even, i.e. the number of ordinates were odd, it gave nearly accurate results as long as the bends were considered as parabolas. However up to certain limits Simpson's rule was applicable to obtain nearly accurate results. When the areas appeared to have many bends and deflections (Figure 1) Simpson's rule became difficult to apply. Additionally prolonged time was required to complete a measurement.

An alternative method has been reported recently which is based on a theorem $I = W10^2$. (6)

We examined this theorem and followed the method repeatedly on various types and sizes of multiple bended flat areas. We noticed that the procedure was prompt, simple, short and accurate. In this procedure we never had to do any complicated calculations or assumptions.

Now if we compare the results of eight experiments (Table 1) we clearly see the standard deviation (SD) of the means of all results except one are highly significant. Whereas insignificant SD of the means are noticed in all the results obtained by applying Bhattacharya's theorem indicate the accuracy of the findings obtained in the measurements.

So, we can write with confidence that the unique method based on Bhattacharya's theorem can be used for the accurate measurements of any flat areas with many bends and deflections.

5. Conclusion

Does the gravitational weight vary with the area of a flat sheet ? The answer is yes. Based on this report, we hope the old problem of two thousand years in area measurements should no longer exist.

6. Acknowledgements

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