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CO-OPTIMIZATION OF PRODUCTION CAPACITY AND ADVERTISING

SHERSTENNIKOV Y. V.

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Sherstennikov Y. V.

Co-Optimization of Production Capacity and Advertising

The work deals with the development of one of the methods for simulating the logistics system (LS) of an enterprise. The aim of the article is using the approach developed by J. Forrester to build a simulation model, based on which it is possible to perform mathematical co-optimization of the production capacity and advertising of an enterprise which produces a single product. The paper formulates a system of equations describing the LS of the enterprise. The calculations of the time behavior of all rates of the logistics system (rate of production, rate of delivery), as well as the behavior of the inventory levels at the wholesale warehouse and retail are carried out. The optimization problem of determining the maximum economic efficiency is formulated and solved. In this case, production capacity and advertising costs are considered as variable parameters of the optimization problem. By numerical calculations, it is proved that there is a single point of maximum economic efficiency as a function of the planned capacity of the enterprise.

Keywords: *logistics system, production capacity, economic and mathematical model.*

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Fig.: *14.* ***Formulae:*** *18.* ***Bibl.:*** *8.*

Sherstennikov Yuriy V. – *Doctor of Sciences (Economics), Associate Professor, Professor of the Department of Economic Cybernetics, Oles Honchar Dnipro National University (72 Haharina Ave., Dnipro, 49010, Ukraine)*

E-mail: *hm001@ukr.net*

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Шерстенников Ю. В. Спільна оптимізація виробничої потужності та рекламної кампанії

Роботу присвячено розробці одного з методів імітаційного моделювання логістичної системи (ЛС) підприємства. Мета статті – використовуючи підхід Дж. Форрестера, розробити імітаційну модель, на підставі якої можна виконувати спільну модельну оптимізацію виробничої потужності та рекламної кампанії монопродуктового підприємства. В роботі сформульовано систему рівнянь, які описують ЛС підприємства. Виконано розрахунки тимчасової динаміки всіх темпів логістичної системи (темпу виробництва, темпів перевезень), а також динаміки рівнів товару на оптовому складі та в мережі роздрібної торгівлі. Сформульовано та розв'язано оптимізаційну задачу визначення максимуму економічної ефективності. При цьому виробнича потужність і витрати на рекламу розглядалися як варіаційні параметри оптимізаційної задачі. Числовими розрахунками доведено, що існує єдина точка максимуму економічної ефективності як функції планової потужності підприємства.

Ключові слова: *логістична система, виробнича потужність, економіко-математична модель.*

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Шерстенников Юрій Всеволодович – *доктор економічних наук, доцент, професор кафедри економічної кібернетики, Дніпровський національний університет імені Олеся Гончара (просп. Гагаріна, 72, Дніпро, 49010, Україна)*

E-mail: *hm001@ukr.net*

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Шерстенников Ю. В. Совместная оптимизация производственной мощности и рекламной кампании

Работа посвящена разработке одного из методов имитационного моделирования логистической системы (ЛС) предприятия. Цель статьи – используя подход Дж. Форрестера, разработать имитационную модель, на основании которой можно выполнять совместную модельную оптимизацию производственной мощности и рекламной кампании монопродуктового предприятия. В работе сформулирована система уравнений, которые описывают ЛС предприятия. Выполнены расчеты временной динамики всех темпов логистической системы (темпа производства, темпов перевозок), а также динамики уровней товара на оптовом складе и в сети розничной торговли. Сформулирована и решена оптимизационная задача определения максимума экономической эффективности. При этом производственная мощность и затраты на рекламу рассматривались в качестве вариационных параметров оптимизационной задачи. Числовыми расчетами доказано, что существует единственная точка максимума экономической эффективности как функции плановой мощности предприятия.

Ключевые слова: *логистическая система, производственная мощность, экономико-математическая модель.*

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Шерстенников Юрий Всеволодович – *доктор экономических наук, доцент, профессор кафедры экономической кибернетики, Днепровский национальный университет имени Олеся Гончара (просп. Гагарина, 72, Днепр, 49010, Украина)*

E-mail: *hm001@ukr.net*

Introduction and problem statement. The economic task of any enterprise is to optimally expand its market niche and bring the production capacities in line with the current demand for its products [1; 2]. In this case, an important role is played by marketing research of the current demand along with organizing an effective advertising campaign.

The aim of the advertising campaign is to make maximum use of the available production capacities, create conditions for their development and, in particular, their further buildup.

Analysis of recent researches and publications. The problem of planning operational activities of enterprises is highlighted in numerous papers of both domestic and foreign scholars, with many of its aspects being considered [3; 4]. At the same time, the mentioned scientific works do not sufficiently cover the quantitative relationship between the parameters of the logistics system (LS) of an enterprise and the current conditions in the consumer market, namely: potential demand for its products and their consumption rate. This shortcoming of the contemporary theory complicates studying the influence of an advertising campaign on economic performance of an enterprise. In publication [4], a model which fundamentally meets the formulated requirements is proposed. It allows for a detailed consideration of conditions in the market. However, the model has a significant drawback: its application leads to unsustainable solutions in a wide range of parameters. Publication [5] presents a method to eliminate this drawback, which is based on averaging sales rates (selected) over a short time interval.

To date, there are no efficient methodologies for planning an advertising campaign of an enterprise in real-time taking into account the enterprise's logistics and market demand for its products.

The aim of the article is developing an economic and mathematical model of production activities of an enterprise taking into account both its logistics and the market demand for its products; applying the developed model to conduct a co-optimization of an enterprise's production capacity and advertising of everyday goods produced by it, with the specifics of their disposal being taken into account.

Presentation of basic material of the research. The research examines an enterprise which logistics corresponds to the scheme shown in Figure 1.

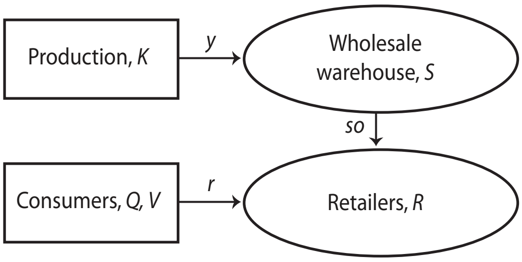

Fig. 1. Scheme of the enterprise's logistics

Working in a competitive market requires an enterprise's management to pay attention to expanding the market niche of the enterprise or at least maintaining it at an appropriate level. One of the effective means to achieve this task is conducting a periodic or continuous advertising campaign.

Therefore, the construction of the model begins with mathematical description of the effect of advertising on the potential demand Q.

Let us assume that the impact of advertising on the current potential demand $Q(t)$ corresponds to the additive contribution

$$Q(t) = Qn + Qr(t),$$

where Qn is the value of the potential demand in the absence of advertising, $Qr(t)$ is the contribution of advertising to the potential demand.

We suggest that the potential demand Q due to advertising is proportional to the number of consumers who have already got acquainted with the advertising of the enterprise's goods. Then the increase in the potential demand ΔQr will be proportional to the product of the number of consumers who have not yet got acquainted with the advertising of the goods ($Qm - Qr$) is the maximum possible number of potential consumers of the item) by the advertising costs ΔZ:

$$\Delta Qr = \frac{1}{tz}(Qm - Qr) \cdot \Delta Z,$$

where $\frac{1}{tz}$ is the proportionality coefficient ($tz$is a constant depending on the market conditions and the item considered).

Proceeding to differentials, we obtain the following equation:

$$\frac{dQr}{dz} = \frac{(Qm - Qr)}{tz}. \quad (1)$$

Since with zero advertising costs the potential demand Qr due to advertising has also the zero value $Qr = 0$, equation (1) needs to be solved under the initial condition $Qr(0) = 0$. Then equation (1) has the following solution:

$$Qr(Z) = Qm \cdot (1 - \exp\{-Ztz\}). \quad (2)$$

Equation (2) means that with the cost of advertising Z the value of potential demand due to the advertising campaign $Qr(Z)$ will be achieved.

Equation (2) with Qm =1000 and tz = 30results in the dependence of the maximum value of potential demand Qr on the costs for the advertising campaign Zr as shown in Figure 2.

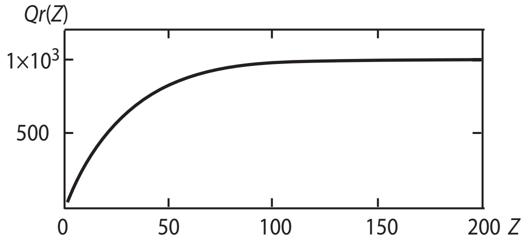

Fig. 2. Dependence of the maximum potential demand Qr on the advertising costs Z (UAH per period of time)

Figure 2 shows the curve of the maximum potential demand Qr which will be achieved with constant advertising costs Z (UAH per period of time). The maximum potential demand Qr cannot exceed a certain maximum value of Qm with any costs. However, we will be interested not only in the maximum value achieved with the costs Z but also in the behavior of the potential demand.

We will consider time as a discrete variable i (i = 0,1 ,2, … , T). Let us study the project of the T length (the planning horizon). The behavior of potential demand due to advertising campaign is described in the discrete-time model by the vector

Q_i. Applying to the vector Q_i the same reasoning which brought to formula (2), we get the following equation:

$$Q_{i+1} = Qr(Z)\cdot(1-\exp\{-i/tr\}). \tag{3}$$

Formula (3) means that the contribution to the potential demand due to the advertising campaign Q_i is described by the first-order lag model [7; 8].

Let us confine ourselves to considering the advertising campaign which is defined by equations (1) – (3).

We will formulate a system of equations which describe the logistics system of the enterprise presented in Figure 1. We assume that the enterprise is fully provided with working capital.

1. The change in the demand Q_i for products on the market is an input impact for the enterprise whose task is to align its output with the demand

$$r_{i+1} = rR_i\cdot(Q_i - V_i), \tag{4}$$

where r_i is the sales rate (unit per period of time) in the i^{th} period; n is the parameter which is determined by the average sales level for the previous quarter (or year); R_i is the inventory level at the retail in the i^{th} period; V_i is the inventory level in the hands of consumers (not consumed yet).

2. The inventory level at the retail is determined by the recurrence formula:

$$R_{i+1} = R_i + Td\cdot(so_i - r_i), \tag{5}$$

where so_i is the rate of delivery (units per week) from the wholesale warehouse to retailers; Td is the period of discretization of the model, the time interval between decisions.

3. The level R_i should be within the limits of $0 \le R_i \le R_m$, where R_m is the maximum possible inventory level at the retail. The requirement is described by the following formula for the rate of delivery from the wholesale warehouse to retailers:

$$so_{i+1} = min\left[r_i\cdot\left(1+\frac{R_m-R_i}{R_m}\right), \frac{R_m-R_i}{Td}, \frac{S_i}{Td}\right], \tag{6}$$

where S_i is the inventory level at the wholesale warehouse.

Publication [5] substantiates the need for averaging when performing the calculations with the proposed model:

$$s'o_i = \langle so\rangle_{i-ps}^{i},$$

where ps is the averaging time interval.

4. The production rate y_i is determined by the following formulas:

$$y_{i+1} = \left(y_i + \frac{ym - y_i}{ty}\right)\cdot A(S_i), \tag{7}$$

$$A(S_i) = \begin{cases} 1, if\ S_i < S_m - 2, \\ 0{,}5\ other\ wise, \end{cases} \tag{8}$$

where y_i is the production capacity in the i^{th} period; y_m is the planned value of the production capacity; S_m is the maximum inventory level at the wholesale warehouse. These formulas allow avoiding the overflow of the wholesale warehouse.

5. The inventory level at the wholesale warehouse S_i is calculated using the following formula:

$$S_{i+1} = S_i + Td\cdot(y_i - so_i), \tag{9}$$

Where y_i is the rate of goods flow which goes in to the wholesale warehouse from the enterprise.

6. To determine the net income of the enterprise, the following formulas are applied:

$$M_i = (1-kp)\cdot[(1-kad)\cdot p\cdot r_i - p\cdot c\cdot yp_i - \\ -k2\cdot S_i - z\cdot Rm - qz\cdot Z] - B(y_i), \tag{10}$$

$$B(y_i) = \begin{cases} 0, if, i<1, \\ qy\left|y_i - y_{i-1}\right|, other\ wise, \end{cases} \tag{11}$$

where *c* is the share of the prime cost in the price for products; *p* is the price for a production unit; *zR*, *zS* are the costs for the storage of a production unit during one period at the retail and wholesale warehouse, respectively; *kp* is the income tax rate; kad is the value-added tax rate; *qy* is the cost of 'including', 'excluding' a unit of production capacity.

Analysis of the results of the calculations. Before carrying out the optimization, we need to check out the adequacy of the model (4) – (11). For this purpose, we will perform calculations for the model (4) – (11) with the following values of its parameters:

$$\begin{aligned} &Rm0 = 100, qy = 100, Q = 1200, \\ &T = 1, n = 0.0001, k1 = 0.33, \\ &k2 = 0.01, So = 100, Sm = 200, \\ &Ro = 50, n1 = 0.1, kp = 0.25, kad = 0.06, \\ &c = 0.6, p = 10, z = 0.01, Se = 180, qy = 50. \end{aligned} \tag{12}$$

The enterprise decides to set the production capacity at the level of *ym* = 4.5. Let us calculate the main indicators for this case. Figure 3 shows the behavior of the basic rates of the logistics system.

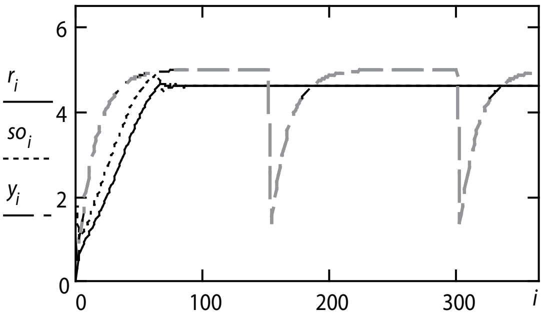

Fig. 3. Behavior of the basic rates of the logistics system

It can be seen that the production rates in some periods significantly decrease. The reason for this behavior of the production rates is clear from Figure 4. During these periods, the inventory level at the wholesale warehouse S_i approaches the maximum capacity of the wholesale warehouse *Sm* = 200. Thus, to avoid overflowing the wholesale warehouse, the enterprise's production capacity should be significantly reduced during these periods in accordance with formulas (7), (8).

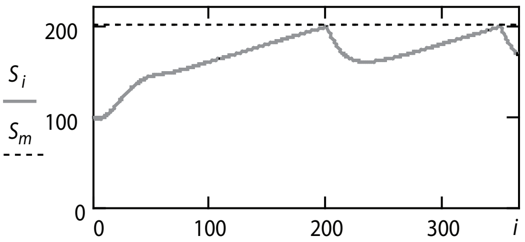

Fig. 4. Behavior of the inventory level at the warehouse S_i

In this case, the enterprise's comprehensive income for the year will amount to

$$\sum_{i=1}^{365} M_i = 3386.9.$$

Figure 5 shows the behavior of the operating income for the case when the behavior of the production capacity is determined by formulas (7), (8).

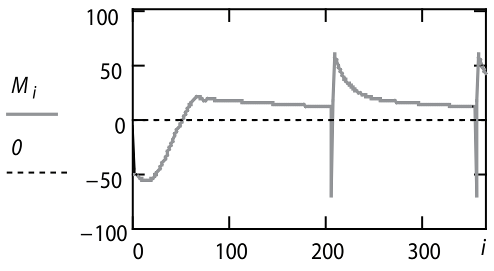

Fig. 5. Behavior of the enterprise's operating income

The time dependence of other characteristics is shown in Figures 6 and 7.

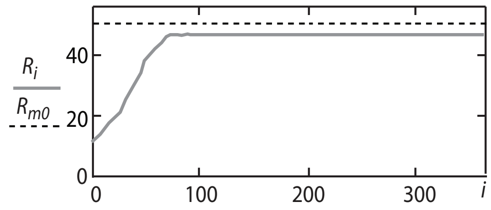

Fig. 6. Behavior of the inventory level at the retail

Figure 6 shows that the inventory level at the retail is stabilizing at the level that is close to the maximum one. This means that the retail network operates effectively.

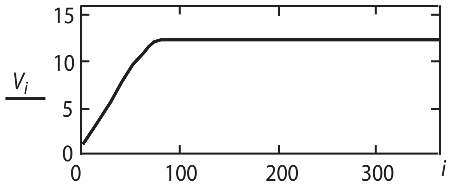

Fig. 7. Behavior of the inventory level in the hands of the consumer

Figure 7 makes it clear that the inventory level in the hands of the consumer stabilizes at 12.2.

However, expression (8) is not the only way to avoid overflowing the warehouse. For this purpose, the enterprise may choose another way of reducing its production capacity:

$$A(S_i) = \begin{cases} 1, if\ S_i < S_e, \\ \dfrac{S_e}{S_i}, other\ wise. \end{cases} \tag{13}$$

Then the results shown in Figures 8–10 will be obtained.

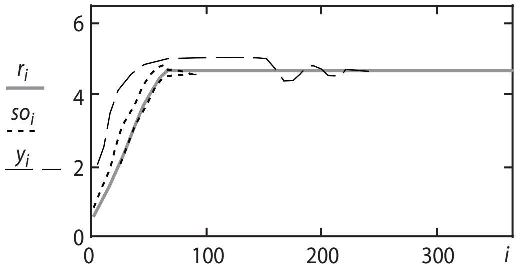

Fig. 8. Behavior of the basic rates of the logistics system for case (13)

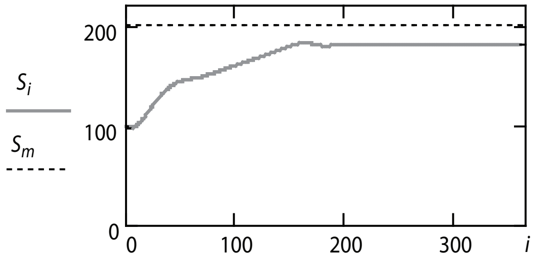

Fig. 9. Behavior of the inventory level at the warehouse S_i for case (13)

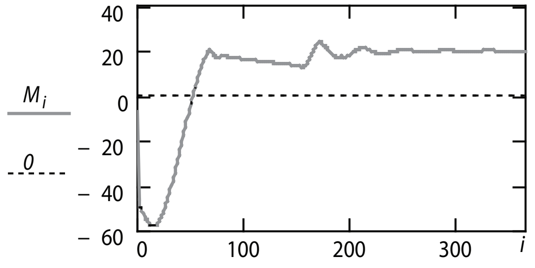

Fig. 10. Behavior of the operating income of the enterprise

The comparison of Figures 3 – 5 with the corresponding Figures 6 – 10 demonstrates that the way to reduce the production capacity (13) is more efficient and more realistic. In this case, the comprehensive income for the year will amount to

$$\sum_{i=1}^{365} M_i = 3653.4,$$

which is noticeably larger, than for the previous case.

Let us now formulate a series of optimization tasks with sequentially increasing complexity in order to plan the life time of the project and the relevant advertising campaign.

As the target function of the optimization task we will take the income received over the chosen period of time *T* (planning horizon):

$$F_T(Z)=\sum_{i=1}^{T} M_i \to max.$$

However, if the project must be fully completed by the end of the planning period, it is necessary to decide what to do with the unsold warehouse inventory whose level at the end of the project accounts for S_{im} ($T = im$). Since the inventory level at the warehouse was S_0 at the beginning of the project, all excess inventory ($S_{im} - S_0$) should be sold at a slightly reduced price, with this amount being included in the target function. As a result, we get the optimization task:

$$F1(Z)=\sum_{i=1}^{T} M_i + p\cdot pq\cdot(S_{im}-S_0)\to max \tag{14}$$

where *pq* is the coefficient of an effective discount for the products ($pq < 1$, with $pq = 0.25$ being chosen for the calculations).

As a rule, if the planned duration of the project is *T*, the suspension of production occurs somewhat earlier – at $T_1 < T$. The enterprise's management is also supposed to plan the suspension of the related advertising campaign at T_1. Then the optimization task will be to maximize the enterprise's profit with two variable parameters: *Z* (advertising costs) and T_1, which is expressed by the formula:

$$F1(Z,T_1)=\sum_{i=1}^{T} M_i + p\cdot pq\cdot(S_{im}-S_0)\to max. \tag{15}$$

Expression (10) for calculating the operating income should be modified as follows:

$$\begin{aligned} M_i &= (1-kp)\cdot[(1-kad)\cdot p\cdot r_i - p\cdot c\cdot yp_i - \\ &-k2\cdot S_i - z\cdot Rm - qz\cdot Z\cdot \begin{cases} 1, \; if \; i<T_1 \\ 0, \; other \; wise \end{cases}. \end{aligned} \tag{16}$$

The formula for calculating the production capacity should be changed in the same way:

$$y_{i+1}=\left(y_i+\frac{ym-y_i}{ty}\right)\cdot A(S_i)\cdot\begin{cases} 1, if \;\; i<T_1 \\ 0, other \; wise \end{cases}. \tag{17}$$

Having solved optimization task (15) with modifications (16), (17) we will obtain:

$$\begin{pmatrix} T_1\_opt \\ Z\_opt \end{pmatrix}=\begin{pmatrix} 312 \\ 60 \end{pmatrix}.$$

With these values, target function (15) is equal to

$$F1 = 8141.4. \tag{18}$$

The dependences shown in Figures 8 –10 will now be presented as follows:

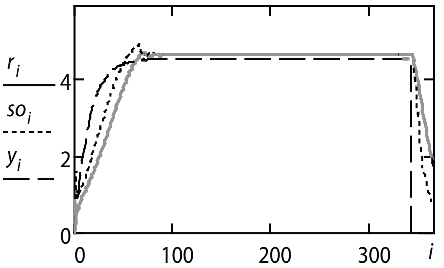

Fig. 11. Behavior of the basic rates of the logistics system

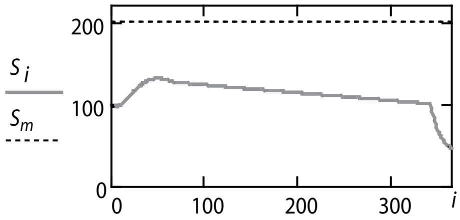

Fig. 12. Behavior of the inventory level at the warehouse S_i

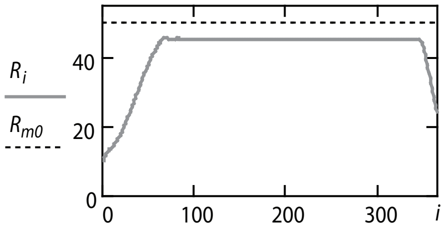

Fig. 13. Behavior of the inventory level at the retail

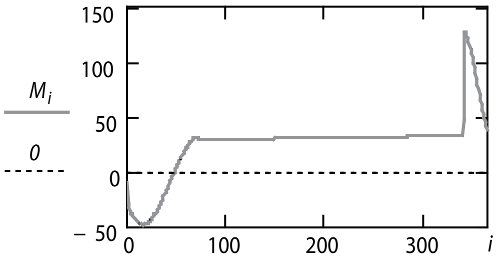

Fig. 14. Behavior of the enterprise's operating income

The optimization task can be solved by the enterprise's management in a more general setting: to find the maximum of target function (15) with the following variable parameters: *ym* is the planned value of the production capacity; *Z* is advertising costs (constant for each period); *Ty* is the time of suspension of the production; *Tz* is the time of suspension of the advertising campaign.

Solving this optimization task leads to the following result:

$$\begin{pmatrix} ym_{opt} \\ Z_{opt} \\ Ty_{opt} \\ Tz_{opt} \end{pmatrix}=\begin{pmatrix} 4.14 \\ 40 \\ 319.3 \\ 304.6 \end{pmatrix}.$$

In this case, the value of the target function (the profits received for the entire planned period T) will be as follows:

$$F1 = 9085.4.$$

This significantly surpasses the previous results. It should be mentioned that this result is obtained with a significant reduction in the production capacity: $ym_{opt} < 4.5$.

Conclusions. It t-nomic and mathematical model of the activity of an enterprise's logistics system is developed. The model takes into account the relationship between the production parameters and the current market conditions. A scheme of the model for co-optimization of an enterprise's production capacity and advertising is developed. The detailed information about the market conditions, which is contained in the model, contributes to the co-optimization of the production capacity and advertising in order to achieve the desired economic result.

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