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SECTION 1. Theoretical research in mathematics

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APPLICATION OF ADOMIAN DECOMPOSITION METHOD, TAYLOR SERIES METHOD AND A VARIATIONAL ITERATIONS METHOD TO SOLVING A SECOND ORDER ORDINARY DIFFERENTIAL EQUATIONS

***Abstract**: In this paper, the Cauchy problem with the second order ordinary differential equations is solved analytically using the Adomian decomposition method, Taylor series method and the variational iteration method. It is shown that these methods are the most effective and convenient for solving some evolution equations. The obtained approximate solutions were compared, the results of these methods are the same; while the method of decomposition of Adomian can be much simpler, more convenient and more efficient to approach such problems as compared to the method of variational iteration and other traditional methods.*

***Key words**: second order ordinary differential equations, Cauchy problem, variational iteration method, Adomian decomposition method, Taylor series method, exact solutions.*

***Language**: English*

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I. Introduction.

Nonlinear phenomena are of fundamental importance in various fields of science and technology. Nonlinear models of real-world problems are still difficult to solve either numerically or theoretically. Recently, much attention has been paid to the search for better and more efficient approximate or exact, analytical or numerical methods for solving for nonlinear models [6, 7, 10, 11]. There are many standard semi-analytical methods for solving linear and nonlinear partial or ordinary differential equations, for example, the Adomian decomposition method, Taylor series method and the variational iterations method [6-12].

The Adomian decomposition method and the variational iterations method is one of the wellknown methods for solving various linear and nonlinear evolution equations. Many studies have proven that these methods are reliable and effective for a wide range of scientific applications, linear and nonlinear equations with bounded and unbounded domains [1-7]. These methods have no special requirements, such as linearization, small parameters, and so on for nonlinear operators. Below, the Cauchy problem with the second order ordinary differential equations are solved analytically using the Adomian decomposition method, Taylor series method and variational iterations method.

II. Analysis of the methods.

1) *Adomian decomposition method*.

Considering the differential equation below in an operator form as

$$Lu + Ru = f \tag{1}$$

In this case L is mostly the lower order derivative assumed to be invertible, R is other differential operator while f is the source term. Applying L^{-1} to both sides of equation (1) and imposing the given conditions, we have

$$u = h - L^{-1}(Ru) \tag{2}$$

where the function h represents given conditions and the source term. The standart Adomian decomposition method gives the solution of u by and infinite series of components written as

$$u = \sum_{n=0}^{\infty} u_n \tag{3}$$

where the components $u_0, u_1, u_2, ...$ are determined recursively. Substituting (3) into (2) yields

$$\sum_{n=0}^{\infty} u_n = h - L^{-1}\left[R\left(\sum_{n=0}^{\infty} u_n\right)\right] \tag{4}$$

We then determine the solution by identifying the zeroth components as

$$u_0 = h \tag{5}$$

and the remaining components are written as the recursive relation

$$u_{n+1} = -L^{-1}[R(u_n)], \quad n \ge 0 \tag{6}$$

2) *Variational iterations method.*

Considering the differential equation below in an operator form as

$$Lu + Nu = f(t) \tag{7}$$

where L is linear operator, N is a nonlinear operator and $f(t)$ is known analytical function.

According to the variational iterations method, we can construct a correction functional as follows:

$$u_{n+1}(t) = u_n(t) + \int_0^t \lambda(\xi)[Lu_n(\xi) + N\tilde{u}_n(\xi) - f(\xi)]d\xi \tag{8}$$

where λ is a general Lagrange multiplier, which can be identified optimally via the variational theory (He, 2007), the subscript n denotes the n th approximation and $\tilde{u}_n$ is considered as a restricted variation, i.e., $\delta\tilde{u}_n = 0$.

It is obvious now that the main steps of the variational iterations method require first the determination of the Lagrangian multiplier λ that will be identified optimally. Having determined the Lagrangian multiplier, the successive approximations u_{n+1}, $n \ge 0$, of the solution u will be readily obtained upon using any selective function u_0. Consequently, the solution

$$u = \lim_{n\to\infty} u_n. \tag{9}$$

Lagrange multiplier can be easily identified as:

$$\lambda(\xi) = (-1)^m \frac{1}{(m-1)!}(\xi - t)^{m-1}. \tag{10}$$

III. Application methods and results.

Example. Consider the second order ordinary differential equations

$y'' - 5y' = 5$, $y(0) = 1$, $y'(0) = 1$, $0 < x < 3$.

We find the exact solution to the problem $y(x) = 0{,}4e^{5x} + 0{,}6 - x$.

1) *Adomian decomposition method.*

$y'' = 5 + 5y' \quad \Rightarrow \quad L^{-1}y'' = L^{-1}(5 + 5y')$,

where $L(\cdot) = \dfrac{d^2(\cdot)}{dx^2}$ and $L^{-1}(\cdot) = \int_0^x\int_0^x (\cdot)d\xi$, then

$$y(x) = 1 + x + \frac{5x^2}{2} + 5\int_0^x\int_0^x y'(\xi)d\xi.$$

According to ADM we search for the solution as follows: $y(x) = \sum_{k=0}^{\infty} y_k(x)$.

This,

$$y_0 + y_1 + y_2 + ... = 1 + x + \frac{5x^2}{2} + $$

$$+5\int_0^x\int_0^x [y_0' + y_1' + y_2' + ...]d\xi$$

now

$$y_0 = 1 + x + \frac{5x^2}{2};$$

$$y_1 = 5\int_0^x\int_0^x y_0' d\xi = \frac{5x^2}{2} + \frac{25x^3}{3!};$$

$$y_2 = 5\int_0^x\int_0^x y_1' d\xi = \frac{25x^3}{3!} + \frac{125x^4}{4!};$$

$$y_n = 5\int_0^x\int_0^x y_{n-1}' d\xi = \frac{5^n x^{n+1}}{(n+1)!} + \frac{5^{n+1}x^{n+2}}{(n+2)!}$$ and so on.

$$y(x) = \sum_{k=0}^{\infty} y_k(x) = 1 + x + \sum_{n=1}^{\infty}\frac{2\cdot 5^n x^{n+1}}{(n+1)!} =$$

$$= 1 - x + \sum_{n=0}^{\infty}\frac{2\cdot 5^n x^{n+1}}{(n+1)!} = 1 - x + +0{,}4\cdot\sum_{n=0}^{\infty}\frac{(5x)^{n+1}}{(n+1)!}$$

$$= 0{,}6 - x + 0{,}4\cdot\sum_{n=0}^{\infty}\frac{(5x)^n}{n!} = 0{,}6 - x + 0{,}4\cdot e^{5x}.$$

The found solution to be compatible with the exact solution.

2) *Variational iterations method.* To solve the VIM problem, we first use the replacement

$$y(x) = 1 + \int_0^x z(\xi)d\xi, \qquad z(0) = 1.$$

The formula of VIM is

$$z_{k+1}(x) = z_k(x) + \int_0^x \lambda(\xi)(z_k' - 5z_k - 5)d\xi.$$

Where $\lambda(\xi)$ - Lagrange multiplier, and for the stationary case $\lambda'(\xi)\big|_{\xi=x} = 0$; $1 + \lambda(\xi)\big|_{\xi=x} = 0$ and from here we have $\lambda(\xi) = -1$. Then we have

$$z_{k+1}(x) = z_k(x) - \int_0^x (z_k' - 5z_k - 5)d\xi.$$

Now applying VIM, we get the following results:

$z_0 = 1$; $z_1 = 1 + 10x$; $z_2 = 1 + 10x + 25x^2$;

$$z_3 = 1 + 10x + 25x^2 + \frac{125x^3}{3};$$

$$z_4 = 1 + 10x + 25x^2 + \frac{125x^3}{3} + \frac{625x^4}{12};\ldots;$$

$$z_n = 2\cdot\sum_{k=0}^{n}\frac{(5x)^n}{n!} - 1$$ and so on.

According to them

$y_0 = 1 + x$;

$y_1 = x + 5x^2$;

$$y_2 = x + 5x^2 + \frac{25x^3}{3};$$

$$y_3 = x + 5x^2 + \frac{25x^2}{3} + \frac{125x^4}{12};$$

$$y_4 = x + 5x^2 + \frac{25x^2}{3} + \frac{125x^4}{12} + \frac{625x^5}{120};\ldots;$$

$$y_n = 1 - x + 0{,}4\cdot\sum_{k=0}^{n}\frac{(5x)^{n+1}}{(n+1)!}$$ and so on.

3) *Taylor series method.*

$y'' = 5 + 5y'$, $y(0) = 1$, $y'(0) = 1$,

$y^{(n)} = 5y^{(n-1)} \Rightarrow y^{(n)}\big|_{x=0} = 5y^{(n-1)}\big|_{x=0}$, $n > 2$.

$y''(0) = 10$; $y'''(0) = 50$; $y^{IV}(0) = 250$; $y^{V}(0) = 1250$; …; $y^{(n)}(0) = 2\cdot 5^{n-1}$ and so on.

Recall that the Taylor expansion of $y(x)$ is given by

$$y(x) = y(0) + \frac{y'(0)}{1!}x + \frac{y''(0)}{2!}x^2 + + \frac{y'''(0)}{3!}x^3 + \ldots + \frac{y^{(n)}(0)}{n!}x^n + \ldots$$

then

$$y(x) = 1 + x + 5x^2 + \frac{50}{3!}x^3 + \ldots + \frac{2\cdot 5^{n-1}}{n!}x^n + \ldots = = 0{,}6 - x + 0{,}4e^{5x}.$$

The result of the difference between the exact and approximate solution alongside the Absolute error E_A, shown in the Table I below:

Table I.

X	Exact solution y(x)	Approximate solutions $y_n(x)$			Absolute error E_A		
		n=20	n=25	n=30	n=20	n=25	n=30
0	1	1	1	1	0	0	0
0,2	1,487312731	1,48731273 1	1,48731273 1	1,48731273 1	2,22045E-16	2,22045E-16	2,22045E-16
0,4	3,15562244	3,15562244	3,15562244	3,15562244	1,86517E-14	8,88178E-16	8,88178E-16
0,6	8,034214769	8,03421476 9	8,03421476 9	8,03421476 9	9,47278E-11	3,55271E-15	0

0,8	21,63926001	21,63925997	21,63926001	21,63926001	4,19982E-08	5,22959E-12	1,06581E-14
1	58,96526364	58,96525883	58,96526364	58,96526364	4,81408E-06	1,81063E-09	2,77112E-13
1,2	160,7715174	160,7712826	160,7715172	160,7715174	0,000234813	2,16923E-07	7,92966E-11
1,4	437,8532634	437,8469049	437,8532509	437,8532634	0,006358429	1,25177E-05	9,80179E-09
1,6	1191,383195	1191,271149	1191,382771	1191,383194	0,112045749	0,000423469	6,40278E-07
1,8	3240,033571	3238,609853	3240,024037	3240,033545	1,423717867	0,009533795	2,5708E-05
2	8809,186318	8795,19281	8809,030544	8809,185615	13,99350766	0,155773566	0,000703419
2,2	23948,05669	23836,186	23946,09161	23948,04257	111,8706827	1,96507678	0,014118601
2,4	65100,11657	64345,08165	65080,08112	65099,89707	755,0349205	20,03544883	0,219500729
2,6	176963,3568	172537,1328	176792,4021	176960,6019	4426,223989	170,9546813	2,754952192
2,8	481039,5137	457993,5703	479785,1559	481010,6859	23045,94337	1254,35775	28,82776695
3	1307604,549	1199111,21	1299517,126	1307346,541	108493,3385	8087,423202	258,0080457

Since the exact solution function is grown, also is absolute error high, so we also investigate relative errors E_R, shown in the Table II below:

Table II.

x	Relative error E_R		
	n=20	n=25	n=30
0	0	0	0
0,2	1,49292E-16	1,49292E-16	1,49292E-16
0,4	5,91064E-15	2,81459E-16	2,81459E-16
0,6	1,17905E-11	4,42198E-16	0
0,8	1,94083E-09	2,41672E-13	4,92537E-16
1	8,16426E-08	3,07067E-11	4,69957E-15
1,2	1,46054E-06	1,34927E-09	4,93225E-13
1,4	1,45218E-05	2,85889E-08	2,2386E-11
1,6	9,40468E-05	3,55443E-07	5,37424E-10
1,8	0,000439415	2,9425E-06	7,93449E-09
2	0,001588513	1,76831E-05	7,98506E-08
2,2	0,004671389	8,20558E-05	5,89551E-07
2,4	0,011598058	0,000307764	3,37174E-06
2,6	0,025012093	0,000966046	1,55679E-05
2,8	0,047908629	0,002607598	5,99281E-05
3	0,082971062	0,006184915	0,000197314

IV. Conclusions.

In this work, ADM, TSM and VIM were successfully applied to solve the Cauchy problem with the second order ordinary differential equations. It is obvious that VIM an ADM are very powerful and effective methods for finding analytical solutions for wide classes of problems. It is worth noting that these two methods are a quick convergence of solutions. Application of ADM to the problems discussed has more advantages than VIM and most other methods; it overcomes the difficulties in calculating other methods and auxiliary parameters; it helps us to obtain a solution for smaller approximations. Also, the ADM does not require changing some parameters in the equation, therefore, the calculations are simple and straightforward.

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