SECTION 7. Mechanics and machine construction.

QR – Issue   QR – Article

Denis Chemezov
M.Sc.Eng., Corresponding Member of International
Academy of Theoretical and Applied Sciences, Lecturer of
Vladimir Industrial College, Russian Federation
https://orcid.org/0000-0002-2747-552X
chemezov-da@yandex.ru

Evgeny Knyazkov
Student of Vladimir Industrial College,
Russian Federation

Irina Pavluhina
Lecturer of Vladimir Industrial College,
Russian Federation

Alexandra Strunina
Lecturer of Vladimir Industrial College,
Russian Federation

Irina Medvedeva
Master of Industrial Training, Vladimir Industrial
College, Russian Federation

Nina Melenteva
Master of Industrial Training, Vladimir Industrial
College, Russian Federation

Polina Nikitina
Lecturer of Vladimir Industrial College,
Russian Federation

BENDINGS OF CANTILEVER AND DOUBLY SUPPORTED STEEL I-BEAMS

Abstract: Plastic deformations and internal stresses in cantilever and doubly supported steel I-beams are calculated under conditions of application of concentrated or distributed forces and concentrated moment by means of finite element modeling in the Comsol Multiphysics software environment. Analytical equations characterizing a change of physical and mechanical properties of material at different degrees of plastic deformation of the I-beam are presented.

Key words: an I-beam, force, moment, a support, a scheme, temperature, physical and mechanical properties.
Language: English
Citation: Chemezov, D., et al. (2019). Bendings of cantilever and doubly supported steel I-beams. ISJ Theoretical & Applied Science, 01 (69), 261-267.
Soi: http://s-o-i.org/1.1/TAS-01-69-36   Doi: cross https://dx.doi.org/10.15863/TAS.2019.01.69.36

Introduction

Metal or wooden I-beams are the most optimal structural elements in construction. Strength characteristics of the metal I-beam are in several times higher than strength of the square or rectangular beams. High rigidity of the I-beam is provided by a stiffening rib (a web). Radii, performed between flanges and the web, reduce local

stress concentration of material at the elements junction of the I-beam. Some strength calculations of the metal I-beams are presented in the works [1 – 10].

Bending deformation of the metal I-beams is studied in the corresponding section of the discipline "Strength of materials". Solution of such problems is reduced to preparation of a design scheme of loading of the beam, preparation and solution of equilibrium equations, diagrams construction of shearing forces and bending moments and definition of the dangerous (the most loaded) sections of the beam. Representation impossibility of material volume deformation at the sections of the I-beam at time of application and after removing of various loads is main solution disadvantage of these problems. The mathematical calculation by means of a computer will allow to visually obtain a deviation from a normal, surface and internal stresses of material, changes of physical and mechanical properties of the I-beam after removing of external active forces.

Materials and methods

The computer calculations of stress-strain condition of the I-beams models, loaded by external active forces and moments, taking into account changing of material temperature were the purpose of researches. The three-dimensional solid model of the I-beam is shown in the Fig. 1 [11].
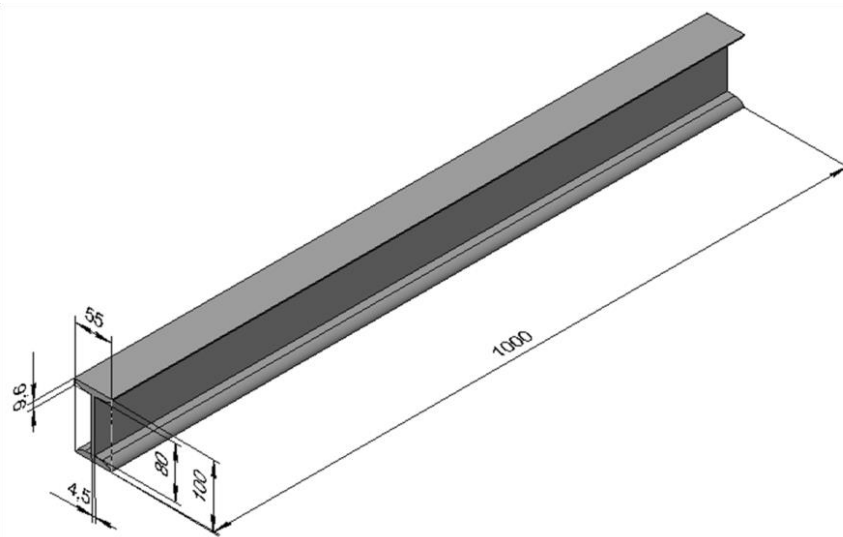

Figure 1 – The dimensions of the I-beam model.

The computer researches were performed according to nine loading schemes of the I-beams (the Fig. 2).

The first scheme is the cantilever I-beam having a fixed support constraint (left). L is the beam length. Concentrated moment M was applied clockwise at a loose end of the beam.

The second scheme is the cantilever I-beam having the fixed support constraint (left). Concentrated force F was applied at the angle of 90 degrees at the loose end of the beam.

The third scheme is the cantilever I-beam having the fixed support constraint (left). Concentrated force F was applied at the angle of 90 degrees at $1/2$ of the span length.

The fourth scheme is the cantilever I-beam having the fixed support constraint (left). Distributed force q was applied along the entire length of the beam.

The fifth scheme is the I-beam placed on the hinged immovable support (A) and the hinged movable support (B). Concentrated moment M (clockwise) was applied to the beam at the support A.

The sixth scheme is the I-beam placed on the hinged immovable support (A) and the hinged movable support (B). Concentrated force F was applied at the angle of 90 degrees at $1/2$ of the beam length.

The seventh scheme is the I-beam placed on the hinged immovable support (A) and the hinged movable support (B). Distributed force q applied along the entire length of the beam (from the support A to the support B).

The eighth scheme is the I-beam placed on the hinged immovable support (A) and the hinged movable support (B). Concentrated force F was applied at the angle of 90 degrees at $1/3$ of the beam length (from the support A).

The ninth scheme is the I-beam placed on the hinged immovable support (A) and the hinged movable support (B). Distributed force q applied at $1/2$ of the beam length (from the support A).
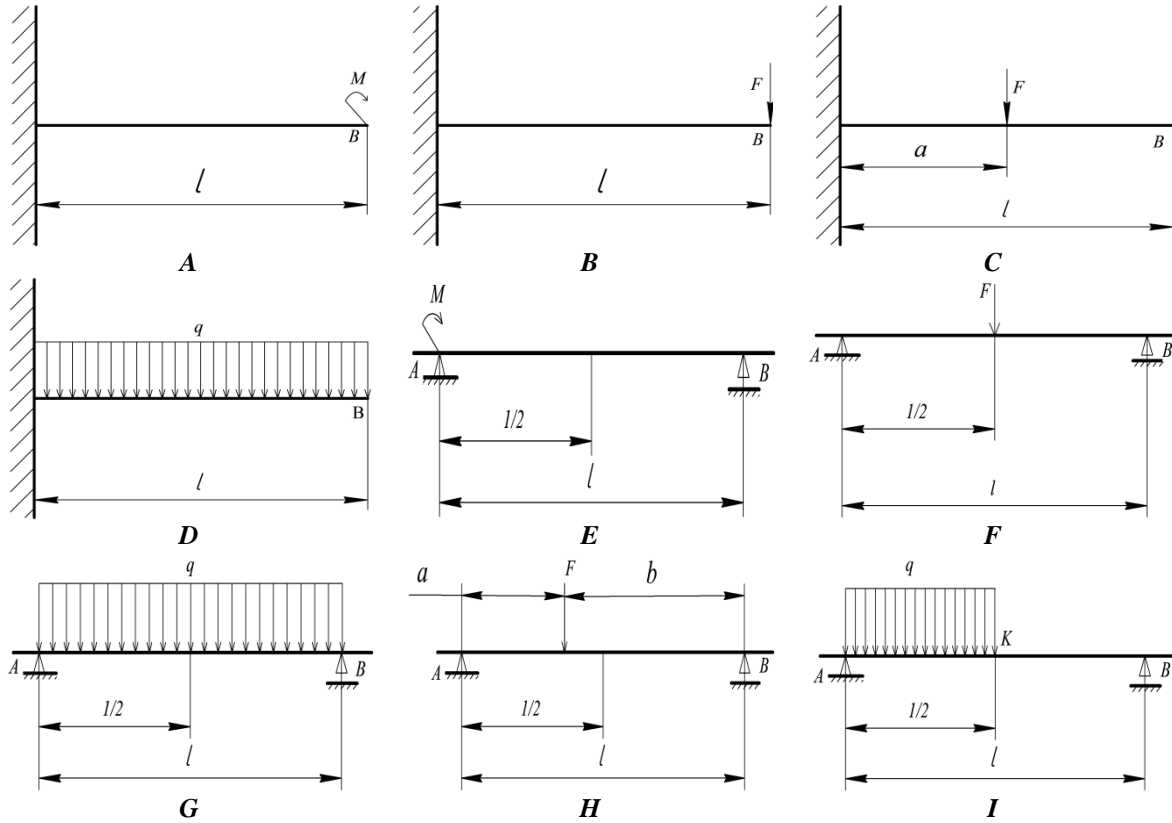
Figure 2 – The loading schemes of the I-beams: *A* – the first scheme; *B* – the second scheme; *C* – the third scheme; *D* – the fourth scheme; *E* – the fifth scheme; *F* – the sixth scheme; *G* – the seventh scheme; *H* – the eighth scheme; *I* – the ninth scheme.

The mathematical calculations were carried out in the **Solid Mechanics** module of the **Comsol Multiphysics** software environment. The following initial conditions were used for the calculations of bending of the I-beam:

1. The equations (1 – 7) for linear elastic material of the beam (1010 steel).

$$0 = \nabla \cdot S + F_V \ (1)$$

$$S = S_{ad} + C : \varepsilon_{el} \ (2)$$

$$\varepsilon_{el} = \varepsilon - \varepsilon_{inel} \ (3)$$

$$S_{ad} = S_0 + S_{ext} + S_q \ (4)$$

$$\varepsilon_{inel} = \varepsilon_0 + \varepsilon_{th} + \varepsilon_{hs} + \varepsilon_{pl} + \varepsilon_{cr} \ (5)$$

$$\varepsilon = \frac{1}{2}\left[(\nabla u)^T + \nabla u\right] \ (6)$$

$$C = C(E, \nu) \ (7)$$

where ∇ is gradient; S is the second Piola-Kirchhoff stress tensor; Fv is load defined as force per an unit volume; S_{ad} is additive stress; C is the fourth-order elasticity tensor; $:$ is contraction over two indices; ε_{el} is elastic strain; ε is total strain tensor; ε_{inel} is inelastic strain; S_0 is initial stress; S_{ext} is external stress; S_q is stress (viscous damping); ε_0 is initial strain; ε_{th} is thermal strain; ε_{hs} is hygroscopic strain; ε_{pl} is plastic strain; ε_{cr} is creep strain; u is displacement field; T is temperature; E is Young's modulus; ν is Poisson's ratio.

2. The equation (8) for the fixed support constraint of the beam (rigid restraint in wall).

$$u = 0 \ (8)$$

3. The equation (9) for the rollers.

$$n \cdot u = 0 \ (9)$$

where n is outward unit normal vector.

4. The equations (10 – 16) for specifying of concentrated moment.

$$a_f = a_{cen} + a_{cor} + a_{eul} \ (10)$$

$$a_{cen} = \Omega \cdot (\Omega \cdot r_p) \ (11)$$

$$a_{cor} = 0 \ (12)$$

$$a_{eul} = 0 \ (13)$$

$$\Omega = -\Omega_{e_{ax}} \ (14)$$

$$e_{ax} = \{1,0,0\} \ (15)$$

$$r_p = \{X, Y, Z\} + u \ (16)$$

where a_f is a frame acceleration; a_{cen} is centrifugal force; a_{cor} is Coriolis force; a_{eul} is Euler force; Ω is angular velocity; r_p is rotation position vector that contains coordinates with respect to any point on an axis of rotation; e_{ax} is axial direction vector; X, Y, Z are the coordinate axes.

5. The equations (17 – 18) for specifying of concentrated and distributed forces.

$$S \cdot n = F_A \ (17)$$

$$F_A = \frac{F_{tot}}{A} \quad (18)$$

where F_A is load defined as force per an unit area; F_{tot} is total force; A is the cross section area.

The dimensions of the I-beam model: X – the width, Y – the height and Z – the length were oriented along the coordinate axes of the Cartesian coordinate system. A deformed configuration of the beam model was oriented on the local coordinate system: the first ($t1$), the second ($t2$) and the third (n). Structural transient behavior of material included inertial parameters. F and q were accepted by the value of 5 kN. Initial temperature of the beam material before the deformation process was accepted by the value of 293.15 K. The number of the elements of the beam model after dividing was 4881. The minimum element quality – 0.009426, the mesh volume – 1171000 mm³. The stationary deformation process of the steel I-beam was researched. The calculation was performed by means of the MUMPS solver under the following conditions: the nonlinear method – automatic (Newton); the initial damping factor – 1; the minimum damping factor – $1 \cdot 10^{-4}$; restriction for step-size update – 10; the recovery damping factor – 0.75.

Results and discussion

Stress-strain condition of the I-beams models loaded by active forces and moments is presented in the Fig. 3.
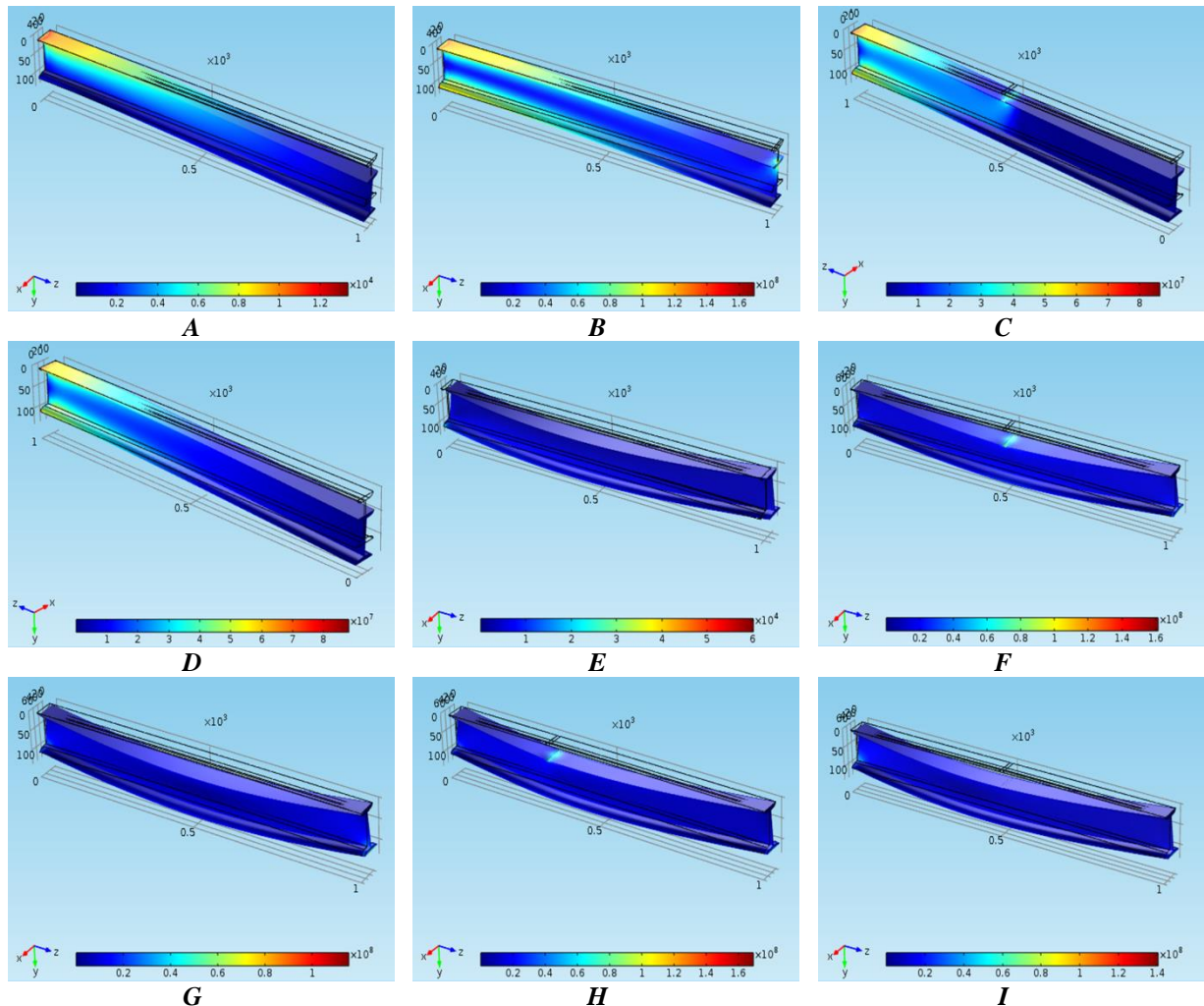

Figure 3 – Stress-strain condition of the I-beams models after loads removing: *A* – the first scheme; *B* – the second scheme; *C* – the third scheme; *D* – the fourth scheme; *E* – the fifth scheme; *F* – the sixth scheme; *G* – the seventh scheme; *H* – the eighth scheme; *I* – the ninth scheme. The color contours on the models are von Mises stress, N/m².

Predicted displacement of the I-beam from bending was determined by the distance between the color contours of the deformed model and the contours before deformation. The beam deflection at the first, second and fourth loading schemes was observed at $^2/_3$ of the length from the loose end.

Application of concentrated force at $^1/_2$ of the length of the I-beam led to maximum deflection on the right side. Deflection of the I-beams, placed on two supports, had almost the same value. Maximum deflection was calculated at $^1/_2$ of the length of the I-beam.

Surface stress of material was considered in the article. The top and bottom flanges from the side of rigid restraint were subjected to stress at loading of the cantilever beam by concentrated and distributed forces. Action of concentrated moment (the first scheme) led to stress only the top flange from the side of restraint. Maximum von Mises stress was determined in surface layers of two flanges under action of concentrated force at the loose end of the I-beam. Stresses of the I-beams, placed on two supports, were concentrated in the inner layers of material. Material stress of the I-beam, loaded according to the fifth scheme, is in five times more

than material stress of the I-beam loaded according to the first scheme. The ratio of stresses in the conditions of application of concentrated and distributed forces (the sixth and seventh schemes) on the top flange of the I-beams was 0.75. One-sided concentrated and distributed loading of the I-beam (according to the eighth and ninth schemes, respectively) was accompanied by increasing of von Mises stress by 15% in comparison with von Mises stress of the I-beams loaded according to the sixth and seventh schemes.

Material temperature of the I-beams increased in the process of plastic deformation. Temperature changing affects the physical and mechanical properties of the beam material. The equations for determining of the physical and mechanical properties in the conditions of temperature changing of the deformed steel I-beam are presented in the summary table 1.

Table 1. The changing dependencies of the physical and mechanical properties of deformed material of the I-beams from temperature.

Intervals	Equations
dL	
0 – 30	-0.00197
30 – 110	$-0.001910556 - 3.518812 \cdot 10^{-6} \cdot T + 5.140443 \cdot 10^{-8} \cdot T^2 - 5.849422 \cdot 10^{-11} \cdot T^3$
110 – 215	$-0.002967233 + 1.81617 \cdot 10^{-5} \cdot T - 9.276156 \cdot 10^{-8} \cdot T^2 + 2.54081 \cdot 10^{-10} \cdot T^3$
215 – 960	$-0.002341855 + 4.083948 \cdot 10^{-6} \cdot T + 1.512293 \cdot 10^{-8} \cdot T^2 - 6.083782 \cdot 10^{-12} \cdot T^3$
CTE	
91 – 400	$-3.482149 \cdot 10^{-6} + 1.265098 \cdot 10^{-7} \cdot T - 3.745646 \cdot 10^{-10} \cdot T^2 + \\ + 4.692131 \cdot 10^{-13} \cdot T^3 - 1.879297 \cdot 10^{-16} \cdot T^4$
400 – 550	$3.718357 \cdot 10^{-5} - 1.761777 \cdot 10^{-7} \cdot T + 3.878031 \cdot 10^{-10} \cdot T^2 - 2.554622 \cdot 10^{-13} \cdot T^3$
550 – 960	$-7.251939 \cdot 10^{-6} + 6.376279 \cdot 10^{-8} \cdot T - 4.206483 \cdot 10^{-11} \cdot T^2$
Thermal conductivity (k)	
122 – 1200	$76.84794 - 0.03185824 \cdot T - 4.196927 \cdot 10^{-5} \cdot T^2 + 2.819746 \cdot 10^{-8} \cdot T^3$
Resistivity (res)	
293 – 1000	$1.038288 \cdot 10^{-8} + 4.780562 \cdot 10^{-10} \cdot T - 8.034806 \cdot 10^{-14} \cdot T^2 + 5.447716 \cdot 10^{-16} \cdot T^3$
1000 – 1255	$1.323461 \cdot 10^{-6} - 5.259924 \cdot 10^{-9} \cdot T + 7.937115 \cdot 10^{-12} \cdot T^2 - 3.047727 \cdot 10^{-15} \cdot T^3$
Coefficient of thermal expansion (alpha)	
0 – 960	$6.78762 \cdot 10^{-6} + 2.878178 \cdot 10^{-8} \cdot T - 7.594277 \cdot 10^{-11} \cdot T^2 + 1.43191 \cdot 10^{-13} \cdot T^3 - \\ - 1.357067 \cdot 10^{-16} \cdot T^4 + 4.830576 \cdot 10^{-20} \cdot T^5$
Heat capacity at constant pressure (C)	
73 – 973	$105.6645 + 1.794069 \cdot T - 0.002752955 \cdot T^2 + 1.768457 \cdot 10^{-6} \cdot T^3$
973 – 1000	$-6453.402 + 7.5312 \cdot T$
Electrical conductivity (sigma)	
293 – 1000	$\dfrac{1}{5.447716 \cdot 10^{-16} \cdot T^3 - 8.034806 \cdot 10^{-14} \cdot T^2 + 4.780562 \cdot 10^{-10} \cdot T + 1.038288 \cdot 10^{-8}}$
1000 – 1255	$\dfrac{1}{-3.047727 \cdot 10^{-15} \cdot T^3 + 7.937115 \cdot 10^{-12} \cdot T^2 - 5.259924 \cdot 10^{-9} \cdot T + 1.323461 \cdot 10^{-6}}$
Density (rho)	
0 – 30	7906.743
30 – 53	$7905.632 + 0.06730164 \cdot T - 0.001025121 \cdot T^2 + 4.962534 \cdot 10^{-7} \cdot T^3 + 1.383301 \cdot 10^{-9} \cdot T^4$
53 – 190	$7924.0 - 0.430436 \cdot T + 0.002196832 \cdot T^2 - 6.010914 \cdot 10^{-6} \cdot T^3$
190 – 960	$7910.967 - 0.100271 \cdot T - 3.511597 \cdot 10^{-4} \cdot T^2 + 1.470685 \cdot 10^{-7} \cdot T^3$

TD	
116 – 977	$4.099893 \cdot 10^{-5} - 1.280394 \cdot 10^{-7} \cdot T + 2.42349 \cdot 10^{-10} \cdot T^2 - 2.385192 \cdot 10^{-13} \cdot T^3 + 8.798585 \cdot 10^{-17} \cdot T^4$
977 – 1033	$-9.756661 \cdot 10^{-5} + 2.606505 \cdot 10^{-7} \cdot T - 1.594388 \cdot 10^{-10} \cdot T^2$
1033 – 1122	$-0.6504472 + 0.002392029 \cdot T - 3.298482 \cdot 10^{-6} \cdot T^2 + 2.021377 \cdot 10^{-9} \cdot T^3 - 4.644974 \cdot 10^{-13} \cdot T^4$
Young's modulus (E)	
4 – 273	$2.217366 \cdot 10^{11} + 5020008.0 \cdot T - 305140.4 \cdot T^2 + 926.6601 \cdot T^3 - 1.145454 \cdot T^4$
273 – 1050	$2.109875 \cdot 10^{11} + 3.572844 \cdot 10^7 \cdot T - 106319.6 \cdot T^2$
1050 – 1500	$2.024261 \cdot 10^{11} - 6.77381 \cdot 10^7 \cdot T$
Poisson's ratio (nu)	
4 – 120	$0.2850355 - 1.662951 \cdot 10^{-6} \cdot T$
120 – 273	$0.2848011 - 7.147353 \cdot 10^{-6} \cdot T + 6.558945 \cdot 10^{-8} \cdot T^2$
273 – 1053	$0.2712267 + 7.030261 \cdot 10^{-5} \cdot T - 3.856929 \cdot 10^{-8} \cdot T^2 + 1.246582 \cdot 10^{-11} \cdot T^3$
1053 – 1500	$0.3165268 - 1.242823 \cdot 10^{-6} \cdot T + 1.661461 \cdot 10^{-9} \cdot T^2$
Shear modulus (mu)	
4 – 273	$8.626526 \cdot 10^{10} + 1636497.0 \cdot T - 108981.6 \cdot T^2 + 291.1261 \cdot T^3 - 0.3377859 \cdot T^4$
273 – 1050	$8.301552 \cdot 10^{10} + 9184755.0 \cdot T - 38834.5 \cdot T^2$
1050 – 1500	$7.694034 \cdot 10^{10} - 2.580357 \cdot 10^7 \cdot T$
Bulk modulus (kappa)	
4 – 100	$1.800573 \cdot 10^{11} - 3478177.0 \cdot T + 151512.1 \cdot T^2 - 5485.327 \cdot T^3 + 28.28329 \cdot T^4$
100 – 273	$1.818111 \cdot 10^{11} - 4.023792 \cdot 10^7 \cdot T + 84204.68 \cdot T^2 - 93.09453 \cdot T^3$
273 – 1500	$1.842649 \cdot 10^{11} - 2.509462 \cdot 10^7 \cdot T - 28588.37 \cdot T^2$

The intervals, characterizing an intensity degree of plastic deformation of the I-beams material, are presented in the left part of the table. The changing equations of dL, CTE, thermal conductivity, resistivity, coefficient of thermal expansion, heat capacity at constant pressure, electrical conductivity, density, *TD*, Young's modulus, Poisson's ratio, shear modulus and bulk modulus from material temperature at different phases of plastic deformation are written in the right part of the table. Let us consider the some equations on the example of changing of the mechanical material property of the beam. The value of material shear modulus of the I-beams is described by three equations. Shear modulus of the beam material is reduced by 0.95 times at plastic deformation of low intensity (the interval of 4 – 100). Shear modulus of the beam material is reduced by 0.99 times at plastic deformation of medium intensity (the interval of 273 – 1050). Shear modulus of the beam material is reduced by 0.9 times at plastic deformation of high

intensity (the interval of 1050 – 1500). Thus, changing of shear modulus of the I-beam material is minimal at plastic deformation of medium intensity.

Conclusion

Action of concentrated moment leads to the less degree of deformation of the beam material than action of external concentrated and distributed forces. Maximum stress in material of the deformed I-beam, placed on two supports, is concentrated in the inner layers. Residual stresses in the middle part of the web of the cantilever I-beam (loading according to the scheme No. 2) are minimal. The basic physical and mechanical properties can be determined by substituting in the resulting equations the values of material temperature at the different degrees of plastic deformation of the I-beam. For example, the values of Young's modulus and shear modulus at the medium degree of plastic deformation practically do not change.

References:

1. Chemezov, D., Osipov, T., & Pesenko, A. (2016). A static calculation of an I-beam. *ISJ Theoretical & Applied Science, 11 (43)*, 49-52.

2. Pilkey, W. D. (2002). *Analysis and Design of Elastic Beams. Computational Methods.* (p.462). NY: John Wiley & Sons, Inc.

Clarivate Analytics indexed

3. Aldabergenov, A. K. (2015). On deformation of bending of a beam subjected to uniformly distributed load. *Structural mechanics of engineering constructions and buildings, №4*, 32-34.

4. Aldabergenov, A. K. (1994). *Strength of Materials with Bases of Theory of Elasticity.* (p.468). Almaty: Rauan.

5. Goel, S. C., Stojadinovic, B., & Lee, K.-H. (1997). Truss Analogy for Steel Moment Connections. *AISC Engineering Journal, 2nd Qtr.*, 43-53.

6. Renton, J. D. (1995). Generalized Beam Theory and Modular Structures. *Int. J. Solid Structures, Vol. 33, No. 10*, 1425-1438.

7. Manakova, N. A., & Vasiuchkova, K. V. (2017). Numerical investigation for the start control and final observation problem in model of an I-beam deformation. *J. Comp. Eng. Math., 4:2*, 26–40.

8. Tusnin, A. R., & Prokic, M. (2015). Experimental research of I-beam under bending & torsion actions. *Magazine of Civil Engineering, No. 1*, 24-31.

9. Pi, Y., Bradford, M., & Trahair, N. (2000). Inelastic analysis & behavior of steel I-beams curved in plan. *Journal of structural engineering, Vol. 126(7)*, 772-779.

10. Mishichev, A. I., & Sapozhnikov, A. I. (2013). The analysis of stability and elastoplastic strain of I-beam with holes in its side. *Building materials, the equipment, technologies of XXI century, 3(170)*, 32-34.

11. (n.d.). *GOST 26020-83. Hot-rolled steel I-beam with parallel flange edges. Dimensions.*