SECTION 7. Mechanics and machine construction

QR – Issue   **QR – Article**

Shamil Usmanovich Galiev
Doctor of Technical Sciences, Professor to Department
of Mechanical Engineering, The University of Auckland,
Private Bag 92019, Auckland 1142, New Zealand,
s.galiyev@auckland.ac.nz

Khayrulla Khudoynazarov
Doctor of Technical Sciences, Professor to Department
of Theoretical and Applied Mechanics of Mechanical and
Mathematical Faculty, Samarkand State University,
Uzbekistan, khayrullakhudoynazarov@gmail.com

Ablakul Abdirashidov
Candidate of Physical and Mathematical Sciences, Docent
to Department of Theoretical and Applied Mechanics, of
Mechanical and Mathematical Faculty, Samarkand
State University, Uzbekistan, abdira@mail.ru

Akmaljon Ablakulovich Abdurashidov
Assistant to Department of Theoretical and Applied
Mechanics, of Mechanical and Mathematical Faculty,
Samarkand State University, Uzbekistan

NONLINEAR ELASTOPLASTIC DEFORMATION OF A HOLLOW FINITE LENGTH CYLINDRICAL SHELL UNDER HYDRODYNAMIC LOADING

Abstract: The process of nonlinear elastoplastic deformation of a cylindrical shell under influence of pulse and hydrodynamic loadings is numerically investigated. Calculations of shell were carried out for elastic and elastoplastic models on the basis of the Kirchhoff-Love and Timoshenko theories. The influences of the loading duration, its intensity and amplitude, as well as geometrical and mechanical characteristics of the structure on the nonstationary behavior of the hydroelastoplastic system are analyzed.

Key words: cylindrical shell, Kirchhoff-Love theory, Timoshenko theory, elastoplastic solution, numerical solution.
Language: English

Citation: Galiev, S.U., Khudoynazarov, K., Abdirashidov, A., & Abdurashidov, A.A. (2018). Nonlinear elastoplastic deformation of a hollow finite length cylindrical shell under hydrodynamic loading. ISJ Theoretical & Applied Science, 10 (66), 256-264.

Soi: http://s-o-i.org/1.1/TAS-10-66-29   Doi: crossref https://dx.doi.org/10.15863/TAS.2018.10.66.29

Introduction.

A large number of publications have been devoted to solving problems of studying the stressed-strain state of elastic and elastoplastic deformable systems under impact, pulse and hydrodynamic loadings. Various aspects of the appearance of elastic and elastoplastic deformations of structures under influences of pulse and hydrodynamic loadings [3, 4, 10] and compression waves [5, 6, 8] are investigated, leading to damage of

the barrier and the subsequent flow of the containing fluid.

Because of the impossibility of obtaining closed solutions for most of such problems, using of numerical methods seems to be the most efficient [9, 11]. In this case, classical Kirchhoff-Love theory [2] and the refined S.P. Tymoshenko type theories are used to describe behavior of the shell [6]. For the interacting fluid, depending on the problem, various fluid models

are used as ideal [4, 8], viscous [2, 7], metastable [3, 10] and others [1, 2, 5].

In additional, in recent years, the development of methods for numerical modeling of high-speed deformation [11, 12] and nonlinear analysis [13, 14] of the dynamic behavior of cylindrical shells of different structures under pulsed [15] and shock [16] loading have been noted.

Therefore, in the future, the study of the dynamic reaction and strength under hydrodynamic loading of structural elements are actual. Below, we investigate the process of nonlinear elastoplastic deformation of a sloping cylindrical shell of finite length under the action of an internal axisymmetric impulse and hydrodynamic loads.

Statement of problem.

The problem of numerical investigation of the nonstationary interaction of elastoplastic shell of finite length containing deformable liquid medium is considered. It is assumed that the shell ends are free and deformed under the influence of external forces acting its inner surface. As a source of disturbances, the non-stationary hydrodynamic pressure that occurs when an underwater explosion charge of cylindrical form is taken, and in the case of the absence of liquid, it is assumed that a short-time pulse acts on the inner surface of the structure.

To describe the behavior of the shell basic relations of the geometrically nonlinear theory of S.P. Timoshenko are assumed. The motion of a cylindrical shell is axisymmetric, so the terms containing the circumferential displacement and derivatives with respect to the angular coordinate in the equations of motion are zero. Then

$$\frac{\partial N_x}{\partial x} = \rho h \frac{\partial^2 u}{\partial t^2};$$

$$\frac{\partial Q_x}{\partial x} + \frac{1}{R} N_y + \frac{\partial}{\partial x}\left(N_x \frac{\partial w}{\partial x}\right) = P - \rho h \frac{\partial^2 w}{\partial t^2}; \quad (1)$$

$$\frac{\partial M_x}{\partial x} - Q_x = -\rho \frac{h^3}{12} \frac{\partial^2 \psi_x}{\partial t^2}.$$

where y, x are district and axial coordinate; u, w are longitudinal and radial displacements; N_x, N_y are longitudinal and circumferential forces; M_x is axial moment; Q_x is shear force; P is pressure in fluid; ψ_x is angle of rotation; t are time; ρ, R, h are density, radius and thickness of a shell respectively.

Nonlinear relations between components of the deformation tensor and the displacement vector, which allow expressing forces and moments in (1) through displacements, are taken in the form [3]

$$\varepsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 + z \frac{\partial \psi_x}{\partial x};$$

$$\varepsilon_y = -\frac{w}{R}; \quad \varepsilon_{xz} = \left(\frac{\partial w}{\partial x} + \psi_x\right) f(z).$$

Relations between the components of stress and deformation tensors is established in accordance with the theory of flow, described in detail in [5], in respect to problems of nonstationary deformation of thin-walled structures

$$\sigma_x = \frac{E}{1-\nu^2}\left[\varepsilon_x + \nu\varepsilon_y - \sum_{n=1}^{n}\left(\Delta_n\varepsilon_x^p + \nu\,\Delta_n\varepsilon_y^p\right)\right];$$

$$\sigma_y = \frac{E}{1-\nu^2}\left[\varepsilon_y + \nu\varepsilon_x - \sum_{n=1}^{N}\left(\Delta_n\varepsilon_y^p + \nu\,\Delta_n\varepsilon_x^p\right)\right]; \quad (2)$$

$$\sigma_{xz} = \frac{E}{2(1+\nu)}\left[\varepsilon_{xz} - \sum_{n=1}^{N}\Delta_n\varepsilon_{xz}^p\right].$$

Substituting expressions (2) in formulas of forces and moment, we obtain the refined expressions for forces and the moment:

$$N_x = \frac{Eh}{1-\nu^2}\left[\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - \frac{\nu}{R}w - \right.$$

$$\left. - \sum_{n=1}^{N}\left(\Delta_n^\alpha\varepsilon_x + \nu\,\Delta_n^\alpha\varepsilon_y\right)\right];$$

$$N_y = \frac{Eh}{1-\nu^2}\left\{\frac{w}{R} + \nu\left[\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right] - \right.$$

$$\left. - \sum_{n=1}^{N}\left(\Delta_n^\alpha\varepsilon_y + \nu\,\Delta_n^\alpha\varepsilon_x\right)\right\};$$

$$M_x = D\frac{\partial^2 w}{\partial x^2} - $$
$$- \frac{Eh}{1-\nu^2}\sum_{n=1}^{N}\left(\Delta_n^\beta\varepsilon_x + \nu\,\Delta_n^\beta\varepsilon_y\right). \quad (3)$$

The shear force is defined as:

$$Q_x = \frac{hEk^2}{2(1+\nu)}\left(\frac{\partial w}{\partial x} + \psi_x\right) - $$
$$- \frac{Ek^2}{2(1+\nu)}\int_{-h/2}^{h/2} f(z)\sum_{n=1}^{N}\Delta_n\varepsilon_{xz}^p dz. \quad (4)$$

Taking into account of (3) and (4) equations of motion (1) we will write in the form:

$$\frac{\partial^2 u}{\partial x^2} = \rho \frac{1-v^2}{E}\frac{\partial^2 u}{\partial t^2} - F_1(w) + F_1^p;$$

$$-\frac{hEk^2}{2(1+v)}\frac{\partial^2 w}{\partial x^2} + \frac{hE}{R^2(1-v^2)}w =$$

$$= P - \rho h \frac{\partial^2 w}{\partial t^2} - F_2(u,w,\psi_x) + F_2^p;$$

$$\frac{E}{\rho(1-v^2)}\frac{\partial^2 \psi_x}{\partial x^2} + \frac{6Ek^2}{\rho h^2(1+v)}\psi_x =$$

$$= \frac{\partial^2 \psi_x}{\partial t^2} + F_3(w) - F_3^p,$$

(5)

where

$$F_1(w) = \frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x^2} + \frac{v}{R}\frac{\partial w}{\partial x};$$

$$F_1^p = \sum_{n=1}^N \frac{\partial}{\partial x}\left(\Delta_n^\alpha \varepsilon_x + v\,\Delta_n^\alpha \varepsilon_y\right);$$

$$F_2(u,w,\psi_x) = -\frac{hEk^2}{2(1+v)}\frac{\partial \psi_x}{\partial x} +$$

$$+\frac{hEk^2}{2(1-v^2)}\left[\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2\right] -$$

$$-\frac{hE}{1-v^2}\left[\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x^2} + \frac{v}{R}\frac{\partial w}{\partial x}\right)\frac{\partial w}{\partial x} +\right.$$

$$\left. + \left(\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 + \frac{v}{R}w\right)\frac{\partial^2 w}{\partial x^2}\right];$$

$$F_2^p = \frac{Ek^2}{2(1+v)}\int_{-h/2}^{h/2} f(z)\sum_{n=1}^N \frac{\partial}{\partial x}\Delta_n \varepsilon_{xz}^p dz +$$

$$+\frac{hE}{R(1-v^2)}\sum_{n=1}^N\left(\Delta_n^\alpha \varepsilon_y + v\Delta_n^\alpha \varepsilon_x\right) -$$

$$-\frac{hE}{1-v^2}\left[\frac{\partial w}{\partial x}\sum_{n=1}^N \frac{\partial}{\partial x}\left(\Delta_n^\alpha \varepsilon_x + v\Delta_n^\alpha \varepsilon_y\right) +\right.$$

$$\left. + \frac{\partial^2 w}{\partial x^2}\sum_{n=1}^N\left(\Delta_n^\alpha \varepsilon_x + v\Delta_n^\alpha \varepsilon_y\right)\right];$$

$$F_3(w) = \frac{6Ek^2}{\rho h^2(1+v)}\frac{\partial w}{\partial x};$$

$$F_3^p = -\frac{12E}{\rho h^2(1-v^2)} \times$$

$$\times \sum_{n=1}^N \frac{\partial}{\partial x}\left(\Delta_n^\beta \varepsilon_x + v\Delta_n^\beta \varepsilon_y\right) -$$

$$-\frac{6Ek^2}{\rho h^3(1+v)}\int_{-h/2}^{h/2} f(z)\sum_{n=1}^N \Delta_n \varepsilon_{xz}^p dz.$$

(6)

Thus pressure P in the right hand side of the second equation of (1) is from the following equations for ideal fluid in cylindrical system of coordinates r, x [16]:

- fluid conservation equations

$$\frac{\partial \vartheta_r}{\partial t} = -\frac{1}{\rho}\frac{\partial(P+q)}{\partial r},$$

$$\frac{\partial \vartheta_x}{\partial t} = -\frac{1}{\rho}\frac{\partial(P+q)}{\partial x},$$

$$\frac{\partial \rho}{\partial t} + \rho\left(\frac{\partial \upsilon_r}{\partial r} + \frac{\partial \vartheta_x}{\partial x}\right) = 0$$

where r, x are radial and axial coordinates; ϑ_r, ϑ_x are radial and axial speed of fluid respectively; ρ - current density of liquid; q – artificial viscosity.

- State equation of liquid:
  a) Bubble:

$$P = \frac{1}{2\chi_0}\left(f + \sqrt{f^2 + 4nV_0 P_0 \chi_0}\right);$$

where $f = 1 + \chi_0 \rho_0 - \dfrac{\rho_0}{\rho}$; $\chi_0 = 2100$ MPa; nV_0 - the volume of the gas-containing bubbles in fluid;
  b) Ideally elastic (Tet's equation):

$$P = -\chi(P_0 + B)\frac{\rho - \rho_0}{\rho} \times$$

$$\times\left[1 + \frac{\chi-1}{2}\frac{\rho-\rho_0}{\rho} + \frac{(\chi-1)(\chi-2)}{2}\left(\frac{\rho-\rho_0}{\rho}\right)^2\right],$$

where χ, B - some constants (for water: $B = 304,5$ MPa; $\chi = 7,15$);

c) cavitating (destroyed): $P \le P_k$; at the same time it is considered that cavitation in a point of fluid disappears, if $P > P_k$, where P_k - critical value of pressure, P – pressure on Tet;

d) metastable (widely-band equation of Kuznetsov):

$$P = \frac{305(\overline{\rho}^{-7,3}-1)}{1+0,7(\overline{\rho}-1)^4} \times$$

$$\times(1-0,012\overline{\rho}^2 F) + 0,47\overline{\rho}F(T-273)$$

if $1 \le \overline{\rho} < 2,3$;

$$P = 0,1\xi^4 - 47\overline{\rho}F\xi + 0,47F\overline{\rho}(T-273),$$

if $0 < \overline{\rho} < 1$, where

$$\xi = \begin{cases} 10(1-\overline{\rho}) + 66(1-\overline{\rho})^2 - 270(1-\overline{\rho})^3, \\ 0,8 \le \overline{\rho} \le 1, \\ 6,6(1-\overline{\rho})^{0,57}\overline{\rho}^{-0,25}, 0 < \overline{\rho} < 0,8; \end{cases}$$

$$\overline{\rho} = \frac{\rho}{\rho_0}; \qquad \rho_0 = 1000 кг/м^3;$$

$$F = F(\overline{\rho}) = \frac{1+3,5\overline{\rho}-2\overline{\rho}^2+7,27\overline{\rho}^6}{1+1,09\overline{\rho}^6}.$$

THe boundary conditions of the problem with allowance for the free ends of the shell have the form $N_x = 0$; $M_x = 0$; $Q_x = 0$. On the boundary of the contact between the fluid and the shell, the linearized kinematic condition takes place $\frac{\partial w}{\partial t} = \vartheta_r\big|_{r=R}$. The initial conditions of the problem for the shell are zero.

Solution algorithm of the problem.

For the solution of an objective the finite difference algorithm in the form of the implicit scheme is used [5].

Let's construct difference equations for the motion equation of the shell (5). We receive the implicit scheme by means of the following representations u, w and ψ_x on m th step of time:

$$\overline{u}^m = \gamma u^{m+1} + \xi u^{m-1};$$

$$\overline{w}^m = \gamma w^{m+1} + \xi w^{m-1};$$

$$\overline{\psi}_x{}^m = \gamma \psi_x{}^{m+1} + \xi \psi{}^{m-1}. \qquad (7)$$

Here γ, ξ - weight coefficients of the scheme, and $\gamma + \xi = 1$.

We will substitute (7) in the left hand side of the equations (5). On a step on time of m receive

$$\frac{\partial^2 \overline{u}^m}{\partial x^2} = \rho\frac{1-v^2}{E}\frac{\partial^2 u^m}{\partial t^2} - F_1(w) + F_1^P;$$

$$-\frac{hEk^2}{2(1+v)}\frac{\partial^2 \overline{w}^m}{\partial x^2} + \frac{Eh}{R^2(1-v^2)}\overline{w}^m =$$

$$= P - \rho h\frac{\partial^2 w^m}{\partial t^2} - F_2(u,w,\psi_x) + F_2^P;$$

$$\frac{E}{\rho(1-v^2)}\frac{\partial^2 \overline{\psi}_x{}^m}{\partial x^2} - \frac{6Ek^2}{\rho h^2(1+v)}\overline{\psi}_x{}^m = \qquad (8)$$

$$= \frac{\partial^2 \psi^m}{\partial t^2} + F_3(w) - F_3^P;$$

where functions $F_1(w)$, F_1^P, $F_2(u,w)$, F_2^p are in the right parts of the equations (8) on formulas (6).

Using the central differences and taking into account of boundary conditions, receive the difference equations, corresponding (8):

$$\overline{B}_1 u_{i-1}^{m+1} - \overline{B}_2 u_i^{m+1} + \overline{B}_3 u_{i+1}^{m+1} = -\overline{Q}_1^m;$$

$$\overline{A}_1 w_{i-1}^{m+1} - \overline{A}_2 w_i^{m+1} + \overline{A}_3 w_{i+1}^{m+1} = -\overline{Q}_2^m; \qquad (9)$$

$$\overline{C}_1 \psi_{i-1}^{m+1} - \overline{C}_2 \psi_i^{m+1} + \overline{C}_3 \psi_{i+1}^{m+1} = -\overline{Q}_3^m.$$

Here

$$\overline{B}_1 = \frac{\gamma E\tau^2}{\rho h_x(1-v^2)}; \quad \overline{B}_2 = 1 + 2\overline{B}_1; \quad \overline{B}_3 = \overline{B}_1;$$

$$\overline{A}_1 = \frac{\gamma E\tau^2 k^2}{2\rho h_x^2(1+v)}; \overline{A}_2 = 1 + 2\overline{A}_1 + \frac{\gamma E\tau^2}{\rho R^2(1-v^2)};$$

$$\overline{A}_3 = \overline{A}_1; \quad C_1 = -\frac{\gamma E\tau^2}{\rho h_x^2(1-v^2)};$$

$$\overline{C}_2 = 1 + 2\overline{C}_1 + \frac{\gamma E\tau^2}{\rho R_x^2(1-v^2)}; \overline{C}_3 = \overline{C}_1;$$

$$\overline{Q}_1^m = 2u_i^m - u_i^{m-1} + \frac{E\tau^2}{\rho(1-v^2)} \times$$

$$\times \left[\xi \frac{\partial^2 u^{m-1}}{\partial x^2} + F_1(w^m) - F_1^p \right];$$

$$\overline{Q}_2^m = 2w_i^m - w_i^{m-1} + \frac{E\tau^2 k^2}{2\rho h(1+v)} \xi \frac{\partial^2 w^{m-1}}{\partial x^2} -$$

$$- \frac{E\tau^2}{\rho R^2(1-v^2)} \xi w_i^{m-1} -$$

$$- \frac{\tau^2}{\rho h} \left[F_2(u^m, w^m, \psi_x^m) - P - F_2^P \right];$$

$$\overline{Q}_3^m = 2\psi_i^m - \psi_i^{m-1} + \frac{E\tau^2}{\rho(1-v^2)} \xi \frac{\partial^2 \psi^{m-1}}{\partial x^2} -$$

$$- \frac{6E\tau^2 k^2}{\rho h^2(1+v)} \xi \psi_i^{m-1} - \xi \psi_i^{m-1} \tau^2 F_3(w^m) + \tau^2 F_3^P,$$

where h_x - step on length of the shell.

Difference expressions for Q_i^m, F_i^m can be easily obtained by replacing the derivatives of u^{m-1}, w^{m-1}, ψ_x^{m-1}, u^m, w^m, ψ_x^m with the corresponding difference relations.

The right hand side of the equations (8) nonlinearly depend to u^{m-1}, u^m, w^{m-1}, w^m, ψ_x^{m-1}, ψ_x^m. There are for brevity not all derivatives entering in $\overline{Q}_1^m$, $\overline{Q}_2^m$, $\overline{Q}_3^m$, are written difference form, in particular composed $F_1(w^m)$, $F_2(u^m, w^m, \psi_x^m)$, $F_3(w^m)$ turn out from $F_1(w)$, $F_2(u, w, \psi_x)$, $F_3(w)$ (6) when replacing derivatives from u, w, ψ_x corresponding central differences.

In finite differences, the boundary conditions for the shell $N_x = 0$; $M_x = 0$; $Q_x = 0$ at $x = 0$ were written in the form: at $i = 2$

$$u_2^{m+1} = \frac{1}{6h_x}\left(w_3^m - w_1^m\right)^2 + \frac{4vh_x}{3R} w_2^m +$$

$$+ \frac{\left(8u_3^m - 2u_4^m - 3u_2^{m-1}\right)}{3} - \frac{4h_x}{3}\sum_{n=1}^{N}\left(\Delta_n^\alpha \varepsilon_x + v\Delta_n^\alpha \varepsilon_y\right);$$

$$\psi_2^{m+1} = \frac{\left(8\psi_3^m - 2\psi_4^m - 3\psi_2^{m-1}\right)}{3} +$$

$$+ \frac{16h_x}{h^2}\sum_{n=1}^{N}\left(\Delta_n^\beta \varepsilon_x + v\Delta_n^\beta \varepsilon_y\right);$$

$$w_2^{m+1} = \frac{\left(8w_3^m - 2w_4^m - 3w_2^{m-1}\right)}{3} + \frac{4h_x}{3}\psi_2^m +$$

$$+ \frac{4h_x}{3h}\int_{-h/2}^{h/2} f(z)\sum_{n=1}^{N}\Delta_n \varepsilon_{xz}^p dz,$$

at $i = N$

$$u_N^{m+1} = \frac{1}{6h_x}\left[\left(w_{N-1}^m - w_{N+1}^m\right)^2 + \frac{4vh_x}{3R} w_N^m\right] +$$

$$+ \frac{\left(8u_{N-1}^m - 2u_{N-2}^m - 3u_N^{m-1}\right)}{3} +$$

$$+ \frac{4h_x}{3}\sum_{n=1}^{N}\left(\Delta_n^\alpha \varepsilon_x + \Delta_n^\alpha \varepsilon_y\right);$$

$$\psi_N^{m+1} = \frac{\left(8\psi_{N-1}^m - 2\psi_{N-2}^m - 3\psi_N^{m-1}\right)}{3} -$$

$$- \frac{16h_x}{h^2}\sum_{n=1}^{N}\left(\Delta_n^\beta \varepsilon_x + v\Delta_n^\beta \varepsilon_y\right);$$

$$w_N^{m+1} = \frac{\left(8w_{N-1}^m - 2w_{N-2}^m - 3w_N^{m-1}\right)}{3} +$$

$$+ \frac{4h_x}{3}\psi_2^m - \frac{4h_x}{3h}\int_{-h/2}^{h/2} f(z)\sum_{n=1}^{N}\Delta_n \varepsilon_{xz}^p dz.$$

Here the central differences were applied. In the derivation of the first and last relationships, one-sided differences were also used, for example

$$\frac{\partial u}{\partial x} = \frac{1}{4h_x}\left(-3u_2^{m+1} + 4u_3^{m+1} - u_4^{m+1} - 3u_2^{m-1} + 4u_3^{m-1}\right.$$

$$\left. - u_4^{m-1}\right) = \frac{1}{4h_x}\left(-3u_2^{m+1} + 8u_3^m - 2u_4^m - 3u_2^{m-1}\right) \quad (10)$$

Boundary conditions in the cross section $x = L/2$ (or $x = L$) were written analogously. Equations (9), (10) are represented in the form convenient for solutions by a tridiagonal matrix algorithm. Let us briefly discuss the method for solving equations (9). The system (9) has a matrix of tridiagonal structure and can be solved by the tridiagonal matrix algorithm. As is known, in this case the solution of the system is represented as follows

$$u_{i,k}^{m+1} = \alpha_i u_{i+1,k}^{m+1} + \beta_{i,k} \; ;$$

$$w_{i,k}^{m+1} = \alpha_i^{I} w_{i+1,k}^{m+1} + \beta_{i,k}^{I} \; ; \qquad (11)$$

$$\psi_{i,k}^{m+1} = \alpha_i^{II} \psi_{i+1,k}^{m+1} + \beta_{i,k}^{II} \; .$$

The first stage of the solution, so-called direct algorithm, consists in the sequential determination of the coefficients by the recurrence formulas α_i, $\beta_{i,k}$, α_i^{I}, $\beta_{i,k}^{I}$, α_i^{II}, $\beta_{i,k}^{II}$, . At the beginning, the $\alpha_3, \beta_{3,k}, \alpha_3^{I}, \beta_{3,k}^{I}, \alpha_3^{II}, \beta_{3,k}^{II}$ are calculated, and α_2, $\beta_{2,k}$ α_2^{I}, $\beta_{2,k}^{I}$, α_2^{II}, $\beta_{2,k}^{II}$ are used, which are found from the boundary conditions at $x = 0$. Then, starting from $\alpha_3, \beta_{3,k}$ $\alpha_3^{I}, \beta_{3,k}^{I}, \alpha_3^{II}, \beta_{3,k}^{II}$, they are found in $\alpha_4, \beta_{4,k}, \alpha_4^{I}, \beta_{4,k}^{I}, \alpha_4^{II}, \beta_{4,k}^{II}$, etc., up to $\alpha_{N+2}, \beta_{N+2,k}$ $\alpha_{N+2}^{I}, \beta_{N+2,k}^{I}, \alpha_{N+2}^{II}, \beta_{N+2,k}^{II}$.

The second stage, or the reverse run, consists in determining, by (11), the required values $u_{i,k}^{m+1}$, $w_{i,k}^{m+1}$, $\psi_{i,k}^{m+1}$. In the beginning are calculated $u_{N+1,k}^{m+1}, w_{N+1,k}^{m+1}, \psi_{N+!,k}^{m+1}$ as $u_{N+1,k}^{m+1}, w_{N+1,k}^{m+1}, \psi_{N+!,k}^{m+1}$ it is known from the condition at $x = L$.

The equations of hydrodynamics are solved numerically using the difference algorithm of M. Wilkins [14]. The finite-difference formulation of the original differential equations includes discretization with respect to time and space coordinates. Discretization with respect to spatial variables was carried out by finite-difference operators using quadrangular mesh nodes with a second order of accuracy. In calculations, the relative volume in the mass conservation equation was identified with the volume of the cell of the finite-difference node, and the derivatives in the equation of motion of continuous media were calculated from the position of the four mesh nodes approaching the node under consideration. In this case, the velocities and displacements were calculated for the mesh nodes, and the density, stresses, damage parameters and deformation were calculated for the center of the cell. An explicit difference scheme is used for time integration.

Results of calculations.

As an example, we study the reaction of steel cylindrical shell of finite length to the action of an internal hydrodynamic loading. Geometrical dimensions of the shell: $R = 0.014$ m; $h = 0.001$ m; $L = 0.2$ m. The material characteristics are as follows: $E = 200,000$ MPa; $\nu = 0.25$; $\rho = 7850$ kg / m³; $\sigma_t = 400$ MPa; $E_1 = 500$ MPa. Hydrodynamic loading occurs when a cylindrical charge of explosive, located along the axis of the cylinder filled with fluid, is undermined.

The pressure is determined by the dependence defined for spherical charges [6]. Ends of the shell are free.

The problem is axisymmetric with respect to the central point along the length, and therefore consider half of the computational domain. We divide the computational domain into mesh with step $h_x = L/20$; $h_r = R/10$; $\tau = h^*/(ka)$, $h^* = \min(h_x, h_r)$, and the coefficient k was determined from the stability condition of the scheme [5]. The values of the accumulated plastic deformation were refined through a certain step τ. The charge is located in the volume of two calculated cells at the point x = L / 2.

The shell was divided in thickness into 4 parts. The step along this coordinate was 0.00025 m, a $\tau = 0.2$ mks. Accumulated plastic deformation was refined through step $\tau = \tau_1$. Calculations showed that a change $5\tau > \tau_1$, and an increase in the thickness division up to 8 parts, had practically no effect on the residual displacements.

Further increases led to approximation of the results for the elastic shell, for example, at $\tau_1 = 20\tau$, the maximum deflections differed by a factor of two than $\tau_1 = \tau$.

Fig. 1 plotted deflection w (Fig. 1, a) and the pressure variation curves P (Fig. 1, b) in a bubble liquid of small gas content ($nV = 0.000009$ [5]) of the shell at the point $x = L / 2$ at $q_0 = 1000$ MPa (P_0 is the initial pressure in the gas). The solid curves determine w and P calculated with allowance for, dashed curves - without taking into account the plasticity of the material of the shell. The solid curve selected by the letter Э was obtained experimentally [6].

Theoretical and experimental solid curves agree well with each other. It follows from the comparison that taking into account the plastic properties of the material leads to a significant increase in the maximum values of deflections. Also Fig. 1 shows that ductility leads to decrease of the pressure wave amplitude in the fluid.

The influence of the fluid model on the deflections of the shell was investigated. Figure 2 shows the time variation of deflections of the central point of the shell, depending on the fluid models, respectively. The curve numbers correspond to model numbers. The calculations were carried out taking into account the plasticity (a) and without it (b). It can be seen that the maximum deflections of the shell, determined with allowance for ductility, exceed the corresponding values for the elastic shell. In addition, models of bubble and cavitation of fluids give similar results, and the model of Theta - with the model of Kuznetsov. This can be explained by the fact that the Theta model of the fluid admits rarefaction waves of arbitrarily large amplitude, and the Kuznetsov model (boiling liquid at room temperature) is close to the

Theta equation. Models of bubble and cavitation of liquid do not allow significant negative pressures.
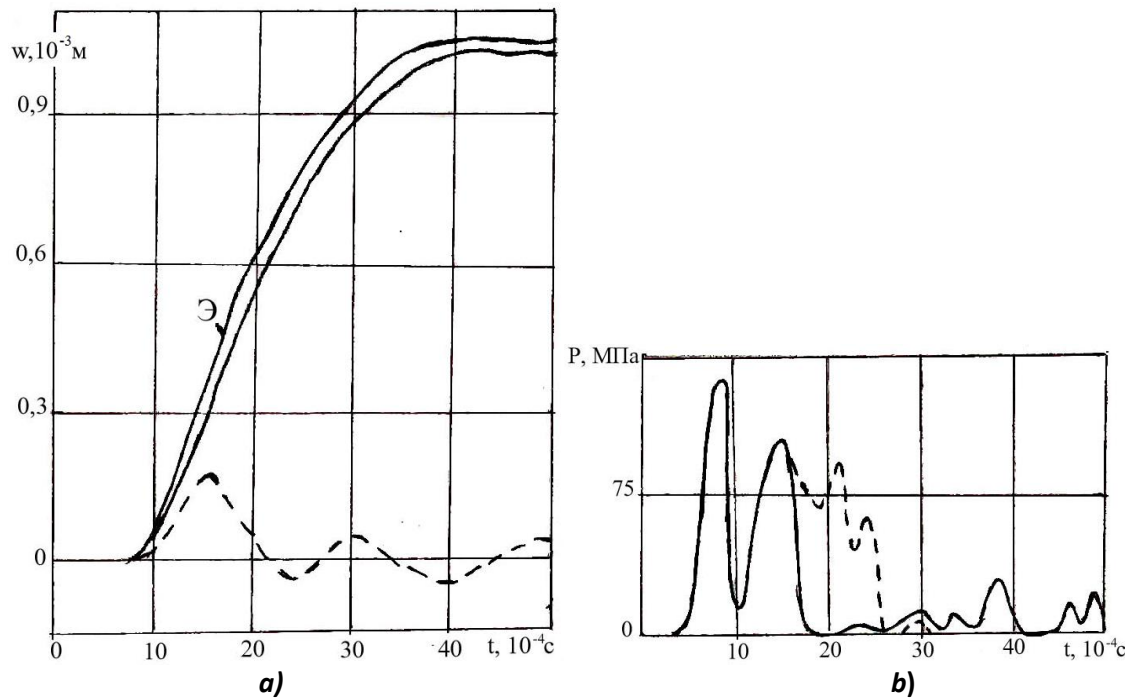

Fig.1. Dependences of the deflection _w_ (_a_) of the shell and the pressure P (_b_) in a bubble liquid of a small gas content with time. Continuous curves are calculated taking into account, a dasheds - without taking into account the plasticity of the shell material. The solid curve, isolated by the letter Э, was obtained experimentally [5].
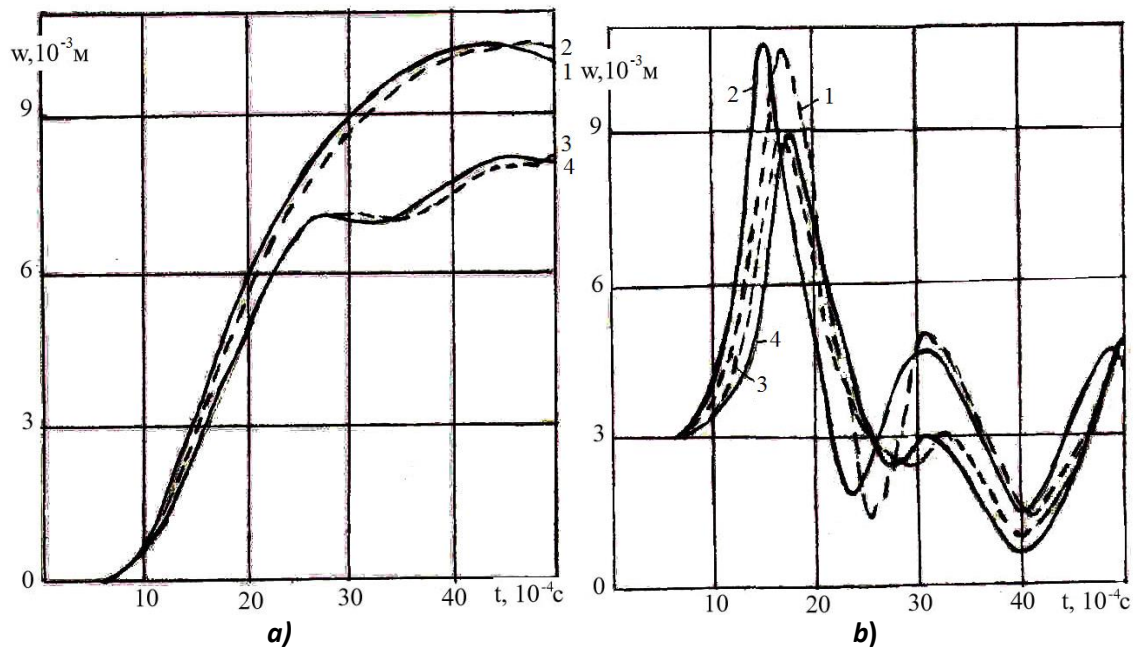

Fig.2. Time variation in deflections of the central point of the shell as a function of time for different fluid models (_a_ - with allowance for, _b_ - without plasticity).
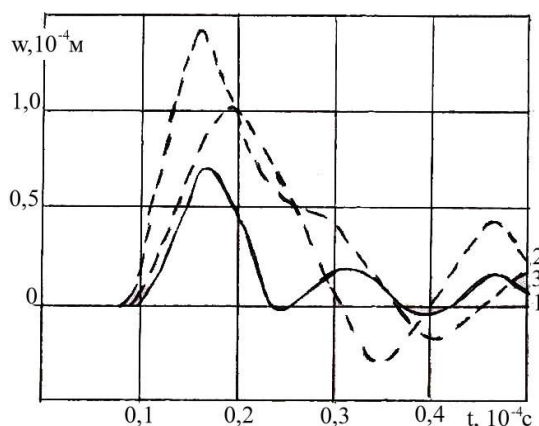
Fig.3. Influence of changes in the parameters of the shell material (curve 1 - steel, 2 - CuSi3Mn1, 3 - D16AT) on the deflections of the central point of the elastic shell.
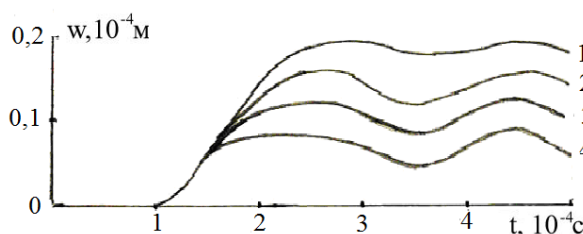

Fig.4. Step-by-step account of the influence of the accumulated plastic deformation of the shell on its deflections (curve 1 - through 1 step, curve 2 - through 5 steps, curve 3 - through 10 steps, curve 4 - through 20 steps)

The effect of changes in material parameters and shell geometry is not significant for the stability of computations. The calculations were carried out until such a time that the edge effects do not affect the residual deflection acquired by the shell (Fig. 3).

A step-by-step account of the accumulated plastic deformation of the shell significantly affects the deflections of its central point (Fig. 4).

Conclusions.

In the process of calculations, it is established that the influence of the hydrodynamic load generates a complex stressed-strain state in the hydro-elastic-plastic system. The influence of the loading parameters and the medium state is essential for evaluating the strength and load-carrying capacity of the shell. Therefore, the use of the theory of elastoplastic flow in the analysis of this class of problems seems necessary. Thus, the developed numerical method and the results of the work make it possible to more reasonably approach the dynamic calculation of some technical objects and designs, more accurately mathematically simulate and solve a number of problems of non-stationary hydroelasticity, taking into account the nonlinear nature of structural elements in the nonstationary interaction with the medium. The proposed numerical method for solving this problem based on Timoshenko's nonlinear model for the shell and for various models (bubble, cavitation, metastable) of an ideal fluid can be applied to a number of related problems in mathematical physics.

References:

1. Bochkarev, S.A., Lekomtsev, S.V., & Matveenko, V.P. (2017). Aeroelastic stability of heated functionally graded cylindrical shells containing fluid. *Mechanics of Advanced Materials and Structures, 24, 16,* 1391–1400.

2. Volmir, A.S. (1972). *Nonlinear dynamics of plates and shells.* Moscow, Nauka Publ, pp.1-432.

3. Galiev, S.U. (1988). *Nonlinear waves in the restricted continuous medium.* - Kiev: Naukova Dumka publ, pp.1-263.

THOMSON REUTERS
Indexed in Thomson Reuters

4. Gorshkov, A.G., Morozov, V.I., Ponomarov, A.T., & Shklyarchuk, F.N. (2000). *Aerohydroelasticity of structures.* Moscow: FIZMAT LIT, pp. 1-592.

5. Guz, A.N. (2002). Dynamics of system of the shells immersed in a liquid. *Appl. mech. 38, № 3,* 260–301.

6. Ilgamov, M.A. (1991). *Introduction to nonlinear hydroelasticity.* Moscow: Nauka Publ., pp. 200.

7. Blinkov, Y.A., Kovaleva, I.A., & Mogilevich, L.I. (2013). Nonlinear waves dynamics modeling in coaxial geometrically and physically nonlinear shell containing viscous incompressible fluid in between. *Saratov: Sarat. gos. tekhn. Univer, №3,* 42-51.

8. Smetanin, B.I., Tarasov, A.E. (2012). Harmonic vibrations circular cylindrical shell finite length in the ideal fluid. *Modern problems of science and education, № 1,* 1-8.

9. Wilkins, M.L. (1967). *Computation of elastoplastic flows.* Computational methods in the hydrodynamics. Moscow: Mir publ., pp. 212-263.

10. Khudoynazarov, K.K., Abdirashidov, A. (2005). *Nonstationary interaction elastoplastic deformable elements of structures with fluid.* Tashkent: Fan publ., pp. 1-220.

11. Vorobev, Y.S., Kolodyajniy, V.I., Sevryukov, V.I., & Yanyutin, E.I. (1989). *High speed deformation of structural elements.* Kiev: Nauk dumka publ, pp. 1-192.

12. Chernobrivko, M.V., Vorobev, Y.S., & Yaarijko, A.V. (2012). Adequacy of simplified models high speed deformation of cylindrical shells. Kharkov: *Vestnik KhNADU, №57,* 146-149.

13. Meyers, M.A. (1994). *Dynamics behavior of materials.* New York: Wiley, pp. 1-283.

14. Abrosimov, N.A., & Novosel'tseva, N.A. (2015). The identification of material parameters in nonlinear deformation models of metallic-plastic cylindrical shells under pulsed loading. *Materials Physics and Mechanics, 23,* 66-70.

15. Abrosimov, N.A., Elesin, A.V., Lazarev, L.N., & Novoselceva N.A. (2013). Numerical analysis of strength of glass-fibre plastic cylindrical shells various structure at pulsing loading. *Problems of strength and plasticity, Issue. 75 (4),* 288-295.

16. Zefirov, S.V., Kochetkov, A.V., & Savikhin, A.O. (2012). Numerical modelling of deformation of the underwater pipeline with filling liquid at a shock loading. *Problems of strength and plasticity, Issue. 74,* 160-166.