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SECTION 1. Theoretical research in mathematics.

ABOUT ONE APPROACH OF SOLVING THE PROBLEM OF OPTICAL SENSING

Abstract: *The paper considers one inverse problem of hydro-optics. The importance of these tasks taking into account multiple scattering increases with the necessity of interpretation of optical observations in probing. The aqueous medium in which the radiation propagates is supposed layered-homogeneous. The linkage between the characteristics and parameters of the test medium contributes to the formulation and successful solution of inverse problems. After that by direct simulation of photon trajectories on a computer the problem of determining the scattering coefficient is solved using the Monte Carlo method.*

Key words: *Monte-Carlo methods, integral equations, optics of the water environment.*
Language: *English*
Citation: Seytkerim AS (2017) ABOUT ONE APPROACH OF SOLVING THE PROBLEM OF OPTICAL SENSING. ISJ Theoretical & Applied Science, 05 (49): 75-79.
Soi: http://s-o-i.org/1.1/TAS-05-49-14   *Doi*: crossref https://dx.doi.org/10.15863/TAS.2017.05.49.14

UDC 519.245+519.642

Introduction

Description of the task. Let in the aquatic environment $0 < H_{sur.} \leq z \leq H_{bot}$, $(x,y) \in R^2$ spreads narrow light beam of constant wavelength, emitted from a point source $\vec{r}_{sou.} = (0,0,0)$, placed in «vacuum» above the surface of the medium, isotropically in a circular conical solid angle $\Omega_{sou.} = [\cos\theta_{sou.}, 1] \times [0, \pi]$, where $\Omega_{sou.}$ – the angle of the source relative to the unit vector $\vec{n}_{sou.} = (0,0,1)$ the axis of the source. The density distribution emitted by the source of light quanta is given by function [1]

$$\psi(\vec{r}, \vec{\omega}, t) = p(\vec{\omega})\delta(\vec{r} - \vec{r}_{sou.})\delta(t),$$

where $p(\vec{\omega}) = \begin{cases} \{2\pi(1 - \cos\theta_{sou.})\}^{-1}, & if \quad \vec{\omega} \in \Omega_{sou.} \\ 0, & \text{otherwise.} \end{cases}$

The environment is expected to horizontally homogeneous and broken z axis into n sufficiently thin layers with different constant values of scattering coefficient $\sigma_{s_1}^*, \sigma_{s_2}^*, ..., \sigma_{s_n}^*$ inside them. The value of the absorption coefficient is considered in all layers of constant and equal σ_a^*. The numbering of the layers are from top to bottom.

The bottom $z = H_{bot}$, with a value of albedo *Al*-probability of survival, reflects radiation according to Lambert law: the density distribution of the cosine of the angle θ_0 between the reflected beam from the bottom surface and the normal to it has the form

$$p_0(\mu) = 2\mu, \quad \mu = \cos\theta_0.$$

Receiver with optical axis along the vector $\vec{n}_{rec.} = (0,0,1)$, measuring the intensity of scattered radiation environment, combined with the source and perceives only the light, supplied in directions $\vec{\omega}$ such that $(\vec{\omega}, \vec{n}_{rec}) \geq \cos\theta_{rec.}$, that is $\vec{\omega} \in \Omega_{rec.} = [\cos\theta_{rec.}, 1] \times [0, \pi]$ – the angle of the receiver.

Let us consider the temporal distribution of the intensity of the radiation entering the receiver. The total value of the intensity is

$$I(t) = \int_{\Omega_{rec.}} \Phi(\vec{r}_{rec.}, \vec{\omega}, t) d\vec{\omega}.$$

The temporal distribution of $I_k = \dfrac{1}{t_k - t_{k-1}} \int\limits_{t_{k-1}}^{t_k} I(t)dt$ this intensity in intervals $[t_{k-1}, t_k]$, where t_k – the nodes of the breaking length of time $[0,T]$, $k = 1,...,m$, it is possible to measure experimentally.

Statement of the problem. Knowing the measured intensity values $I_1, I_2,..., I_m$ in m different time intervals, find the value of the scattering coefficient of the medium in layers $1,..., n,$ at a known and fixed value of absorption coefficient in all layers [2].

The solution of the task. I_k it is a function of the value $\sigma = \sigma_s + \sigma_a$: $I_k = I_k(\sigma)$. It's known [3,4], that

$$I_k(\sigma) = \sum_{j=0}^{\infty} \int_X ... \int_X \psi(\vec{x}_0) \prod_{i=0}^{j-1} k(\vec{x}_i, \vec{x}_{i+1}, \sigma) \varphi_k(\vec{x}_j, \sigma) d\vec{x}_0 ... d\vec{x}_{j-1} d\vec{x}_j ,$$

where

$$k(\vec{x}_i, \vec{x}_{i+1}, \sigma) = \frac{q(\vec{r}_i) g(\mu) \sigma \exp(-\sigma | \vec{r}_{i+1} - \vec{r}_i |)}{2\pi | \vec{r}_{i+1} - \vec{r}_i |^2} \delta\left(\vec{\omega}_{i+1} - \frac{\vec{r}_{i+1} - \vec{r}_i}{| \vec{r}_{i+1} - \vec{r}_i |} \right) \times$$

$$\times \delta\left(t_{i+1} - t_i - \frac{| \vec{r}_{i+1} - \vec{r}_i |}{v} \right), \qquad (1)$$

$\vec{r}_i$ – the radius vector of point i-th collision, $\vec{\omega}_i$ – a unit vector in the direction of the photon before i-th collision, t_i – time i-th collision, measured from the moment of penetration of a photon in the environment,

$$\vec{x}_i = (\vec{r}_i, \vec{\omega}_i, t_i),$$

$$\mu = (\vec{\omega}_i, \vec{\omega}_{i+1}),$$

$$\varphi_k(\vec{x}_j, \sigma) = \sigma \exp(-\sigma | \vec{r}_{sur.} - \vec{r}_j |) F_1,$$

$$F_1 = \frac{q(\vec{r}_j) g(\mu_1)}{2\pi | \vec{r}_{sur.} - \vec{r}_j |^2} \Delta_{\Omega_{rec.}} (\vec{\omega}_{rec.}) \Delta_k(t^*) \{1 - Y(\vec{\omega}_{sur.})\},$$

$\vec{r}_{sur}$ – the radius of the intersection vector of the photon with the medium surface when returning to the receiver from point $\vec{r}_j$, $\vec{\omega}_{sur.} = (\vec{r}_{sur.} - \vec{r}_j)/| \vec{r}_{sur.} - \vec{r}_j |$,

$\mu_1 = (\vec{\varpi}_j, \vec{\varpi}_{sur})$ – cosine of the angle between the $\vec{\varpi}_j$ and $\vec{\omega}_{sur.}$,

$q(\vec{r}_j)$, $q(\vec{r}_i)$ probability of survival, $\vec{\omega}_{rec.} = (\vec{r}_{rec..} - \vec{r}_{sur.})/| \vec{r}_{rec.} - \vec{r}_{sur.} |$,

$$\Delta_{\Omega_{rec}}(\vec{\omega}_{rec.}) = \begin{cases} 1, & if \quad \vec{\omega}_{rec.} \in \Omega_{rec.}, \\ 0, & otherwise \end{cases} - \text{ the }$$

angle indicator of solubility of the receiver,

$t^* = t_0 + t_j + t_{sur.} + t_{rec.}$ – the total time of a photon in transit from source to receiver,

$t_0 = H_{sur.}/\mu_0 c$, where $\vec{\omega}_0 = (\mu_0, \beta_0)$ – the random vector of the direction of departure of a photon from a source with a given distribution density $p(\vec{\omega})$, c – the speed of light in vacuum. t_j – the time of the collision at the point $\vec{r}_j$, starting from the contact with the medium, $t_{sur} = | \vec{r}_{sur.} - \vec{r}_j |/v$ – time to reach the surface of the medium photon at time t_j, v – the speed of light in water, $t_{rec.} = | \vec{r}_{rec} - \vec{r}_{sur.} |/c$ – the time of reaching the photon receiver after it's left of the environment,

$$\Delta_k(t^*) = \begin{cases} 1, & if \quad t^* \in [t_{k-1}, t_k], \\ 0, & otherwise \end{cases} - \text{ indicator } k\text{-th}$$

time interval, $Y(\vec{\omega}_{sur.})$ – frenel reflection coefficient at point $\vec{r}_{sur.}$.

To find $\sigma_{s_1}^*, \sigma_{s_2}^*,..., \sigma_{s_n}^*$ in the work [6] the proposed recurrence system of linear equations

$$\sum_{j=1}^{m} q_{ij}^{(p)} \Delta \sigma_{s_j}^{(p+1)} = d_i^{(p)} \qquad (2)$$

where

$$i = 1,...,n, \quad q_{ij}^{(p)} = \sum_{k=1}^{m} a_{ki}^{(p)} a_{kj}^{(p)}, \quad d_i^{(p)} = \sum_{k=1}^{m} a_{ki}^{(p)} b_k^{(p)}, \quad a_{ki}^{(p)} = \left. \frac{\partial I_k(\vec{\sigma})}{\partial \sigma_{s_i}} \right|_{\vec{\sigma} = \vec{\sigma}^{(p)}}, \quad a_{kj}^{(p)} = \left. \frac{\partial I_k(\vec{\sigma})}{\partial \sigma_{s_j}} \right|_{\vec{\sigma} = \vec{\sigma}^{(p)}},$$

$$b_k^{(p)} = I_k^* - I_k(\vec{\sigma}^{(p)}), \qquad \Delta\sigma_{s_j}^{(p+1)} = \sigma_{s_j}^{(p+1)} - \sigma_{s_j}^{(p)},$$

I_k^* – the values of the intensity of scattered radiation observed by the receiver in time intervals

$$k = 1,...,m, \quad \vec{\sigma} = \vec{\sigma}_s + \vec{\sigma}_a^*, \quad \vec{\sigma}_s = (\sigma_{s_1},...,\sigma_{s_n}). \quad \vec{\sigma}_a^* = (\sigma_a^*,...,\sigma_a^*),$$

$\sigma_{s_1}^{(p)},...,\sigma_{s_n}^{(p)}$ – p-th approximation to the expected values.

In research we will use the following well-known formulas the approximate formulas[7]:

$$I_k(\vec{\sigma}^{(p)}) = \frac{1}{N} \sum_{l=1}^{N} \xi_l^{(p)}(k), \quad a_{kh}^{(p)} = \frac{1}{N} \sum_{l=1}^{N} \eta_l^{(p)}(k,h), \text{ where}$$

$$\xi_l^{(p)}(k) = \sum_{j=0}^{\gamma(l)} \varphi_k(x_j^{(l)}, \vec{\sigma}^{(p)}), \quad \eta_l^{(p)}(k,h) = \sum_{j=0}^{\gamma(l)} \varphi_k(x_j^{(l)}, \vec{\sigma}^{(p)}) w_j^{(l)}(\sigma_h^{(p)}),$$

$$w_j^{(l)}(\sigma_h^{(p)}) = \frac{M_h^{(j+1)}}{\sigma_h^{(p)}} - \sum_{i=0}^{j-1} l_h^{(i,i+1)} - l_h^{(j)},$$

$\vec{r}_0^{(l)}, \vec{r}_1^{(l)},...,\vec{r}_j^{(l)}$ - the collision point l-th the simulated trajectory.

$l_h^{(i,i+1)}$, $l_h^{(j)}$ – the length of those parts of the segments $|\vec{r}_{i+1}^{(l)} - \vec{r}_i^{(l)}|$ и $|\vec{r}_{sur}^{(l)} - \vec{r}_j^{(l)}|$, which accounted for h-th layer,

$M_h^{(j+1)}$ the number of points from the set $\vec{r}_0^{(l)}, \vec{r}_1^{(l)},..., \vec{r}_j^{(l)}$, caught in h-th layer, $\gamma(l)$– random number collision, which ends up with l-th trajectory [8].

The objective of the evaluation values $I_k^{(p)} = I_k(\vec{\sigma}^{(p)})$ and their derivatives $a_{ki}^{(p)}$, $p = 0,1,2,...$ shall be determined by direct simulation of the photon trajectories on a computer, as a homogeneous Markov chain of random points on collision $x_0, x_1,..., x_n,...$ of a photon with the elements of the water environment. Distribution of the initial state of the Markov chain is specified by the function $\psi(x)$, physical distribution density of the emitted light quanta. The transition density of the Markov chain from one state to another state is set by the physical density of transitions, the given function (1). The solution in this article the inverse problem, in fact, it is the solution of a series of direct problems. Namely, at the beginning of the true values $\{\sigma_{s_i}^*\}_{i=1,...n}$ estimated intensity values $\{I_k^*\}_{k=1,...m}$ and its derivatives $\{a_{ki}^*\}_{k=1,...m,i=1,...n}$ and

they are accepted as "measured" by the device, in the General case getting a faux, normally distributed random error $\{\varsigma_k\}_{k=1,...m}$, owing to the inaccuracy of physical experiments. Next selected prognostic values of the unknown factor $\{\sigma_{s_i}^{(0)}\}_{i=1,...n}$ in layers, and for them again solved the direct problem in the calculation of $\{I_k^{(0)}\}_{k=1,...m}$ and $\{a_{ki}^{(0)}\}_{k=1,...m,i=1,...n}$. According to the latest system of equations is solved (2) relatively $\{\sigma_{s_i}^{(1)}\}_{i=1,...n}$ and etc.

Practical solution of the problem by Monte Carlo. To study the effectiveness of various modifications of the process simulation of radiation transfer with strongly anisotropic scattering can use the following process model. The particle moves from point $x = 0$ along the axis x random runs, distributed with density $e^{-x}(x > 0)$. At the end of the path there is a collision, in which a particle can be absorbed with a probability $1 - q$; otherwise, she's moving on. At the point $x = H$ occurs sortie, i.e. torn off trajectory. Required to calculate the probability that the particle will leave, having experienced at least one collision. The transition density for the chain of collisions is determined by the formula [9]:

$$k(x', x) = qe^{-(x-x')}, x' \le x \le H \qquad (3)$$

The density of the primary collision $f(x) = e^{-x} (0 \le x \le H)$. Therefore, the total density of the collision $\varphi(x)$ satisfies the equation

$$\varphi(x) = q \int_0^\pi e^{-(x-x^/)} \varphi(x^/) dx^/ + e^{-x}$$

It is easy to check that here $\|K\| \prec q$. It is known that in this case the required probability $P = \varphi(H)$. Thus, it is required to compute the solution of the integral equation at a single point $x = H$. To do this, you can use a local assessment, putting $h(x) = k(x, H) = qe^{-(H-x)}$, i.e. $P = M\xi$,

where $\xi = \sum_{n=0}^N h(x_n) = \sum_{n=0}^N qe^{-(H-x_n)}$, if implemented direct modeling a chain collision [10]. Direct substitution into the equation can see that here

$$\varphi(x) = e^{-(1-q)x}, 0 \le x \le H$$

It is easy to obtain the value of the required probability

$$P = \varphi(H) = (\varphi, h) = e^{-(1-q)H}(1 - e^{-qH}).$$

The adjoint equation in this case has the form

$$\varphi^*(x) = q \int_x^H e^{-(x^/-x)} \varphi^*(x^/) dx^/ + qe^{-(H-x)}.$$

Him meets function $\varphi^*(x) = qe^{-(1-q)(H-x)}$. The variance of the random evaluation ξ is determined by the equality

$$M\xi^2 = (\varphi, h(2\varphi^* - h)) = q^2 \frac{1+2q}{1+q} e^{-(1-q)H} - 2q^2 e^{-H} + \frac{q^2 e^{-2H}}{1+q}$$

Consider now the evaluation on acquisitions

$$\eta = \frac{h(x_N)}{g(x_N)} = \frac{qe^{-(H-x_N)}}{1 - q + qe^{-(H-x_N)}}.$$

For her

$$M\eta^2 = (\varphi, \frac{h^2}{g}) = q^2 e^{-(1-q)H} \int_0^H \frac{e^{-(1+q)x}}{1 - q + qe^{-x}} dx.$$

In this case, we can consider a binary evaluation ζ, which is equal to 1, if the particle flew, and equal to 0 otherwise. It is obvious that

$M\zeta^2 = M\zeta = P = e^{-(1-q)H}(1 - e^{-qH})$. To compare variance estimates for a large H it is enough to consider the corresponding coefficients of the functions $e^{-(1-q)H}$, i.e. value

$$C_\xi = q^2 \frac{1+2q}{1+q}, C_\eta = q^2 \int_0^\infty \frac{e^{-(1+q)x}}{1 - q + qe^{-x}} dx, C_\zeta = 1$$

Values of C_ξ and C_η given in the table 1.

Table 1

The values of the coefficients C_ξ and C_η.

q	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1
C_ξ	0,0101	0,047	0,111	0,21	0,33	0,45	0,62	0,93	1,19	1,5
C_η	0,0096	0,037	0,080	0,14	0,21	0,31	0,42	0,55	0,72	1

Conclusion.

Thus, can be seen, evaluation on absorption multiple is more accurate than the other evaluation. This is due to the fact, in this case, the probability is calculated from the absorbance of one species – sortie. In the transition to the real problems of the relationship between $D\xi$ and $D\eta$ changes in the valuation advantage ξ because of the possibility of more changes $g(x)$. Also in the works, knowing the measured values of intensity in different time intervals, found the value of the scattering coefficient of the medium in layers at a known and fixed value of absorption coefficient.

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