



Employing the Outputs of Overlapped, Delayed Narrowband Receivers for Reconstruction of Wideband Signals

W.A. Azeez¹, Gilbert A. Ibitola²

¹Department of Physics, The Polytechnic, Ibadan, Nigeria

²Department of Physical Sciences, Ondo State University of Science and Technology, Nigeria

Abstract A signal which possesses bandwidth that exceeds that of available receivers can be reconstructed from the outputs of spectrally overlapped receivers. Adaptive filters were used to compensate for varying amounts of overlap, relative delay between receiver outputs, and a small amount of frequency shift. The preferred architecture contains adaptive multiple transversal filters, each of which incorporates an adaptive lattice filter to pre-whiten the data, thereby enabling faster convergence. Testing was conducted in the cases that the modulations were QPSK and 16-QAM and when two overlapping receivers were employed. Results indicate that, for a wide range of overlaps and even when the relative delay between receiver outputs is several symbols, the equalizer structure will converge in roughly the same time as a conventional equalizer applied to an unfiltered signal.

Keywords Quaternary-Phase Shift Keying (QPSK), Quadrature Amplitude Modulation (QAM), Overlapping Receivers, Wideband Signals, Signal Processing and Modelling

1. Introduction

Signals with bandwidths in excess of available receiver bandwidths can still be demodulated and information extracted, if multiple receivers are available which can capture overlapping frequency bands which cover the entire signal spectrum. Indeed signal reconstruction methods which make use of multiple channel outputs are well known [1]. The outputs of quadrature mirror filters, for example, may be up-sampled, heterodyned, and summed to reconstruct the signal. In this paper, a robust signal reconstruction method is described which consists of a variant of the constant modulus algorithm (CMA) [2] applied to the different receiver outputs, which can be delayed with respect to one another and which can be overlapped in frequency by variable amounts (unlike quadrature mirror filters, which are carefully overlapped).

Simulation results are provided in the case of quaternary-phase shift keyed (QPSK) and quadrature amplitude modulated (QAM) signals with a 4x4 signal constellation (16-QAM), two receivers, and two adaptive filters.

The fundamentals of signal combining and modelling are provided in Section 2. Section 3 sheds light on the implementation and evaluation of the two adaptive filtering (signal processing) architectures. In Section 4, the results obtained and the discussions that follow are clearly presented. The tap update equations are derived in section 5. The concluding remarks are given in Section 6.

2. Signal Combining and Modelling

If there were no delay between receiver outputs, and if the overlap between receivers is such that the “combined” (*i.e.* summed) transfer function is relatively flat across the signal’s frequency band, then the obvious solution to signal reconstruction is simply to add the appropriately heterodyned receiver outputs and apply an adaptive equalizer. This technique was implemented and tested when the number of receivers was two.



It works well if there is no delay or frequency offset between signals, since the CMA can compensate well for a non-flat amplitude response of the effective channel filter resulting from non-ideal overlap of the receivers' frequencies. However, if the relative delay between receiver outputs exceeds the duration of a symbol then the technique fails completely. The technique also fails if there is a frequency offset, as would occur if the signal were shifted in frequency by different amounts at the two receivers.

To enable compensation for relative delays and, in general, to provide additional robustness to the receiving system, a multiple equalizer system was developed. If delays are indeed a potential problem, this robustness can ease system requirements which are designed to minimize such delays.

Here and henceforth, the signal model derives from the reasonable assumption that the signal was captured by two receivers, although the results for two receivers are generalized to an arbitrary number of overlapping receivers. Neither receiver had sufficient bandwidth to collect the entire signal, although the bandwidth of each was slightly greater than one-half of the signal bandwidth. The two receivers were tuned so that each receiver collected somewhat in excess of one-half the frequency band occupied by the signal.

The amount of overlap between the receivers has been observed to vary. The delay τ between receiver outputs varied between 0 and 3 symbols. The receiver outputs were heterodyned by (complex) sinusoids differing in frequency by the difference in the center frequencies of the receivers. Frequency offsets varied between 0.0 % and about 0.4 percent of the symbol rate. The signal-circuitry model is shown in Figure 1.

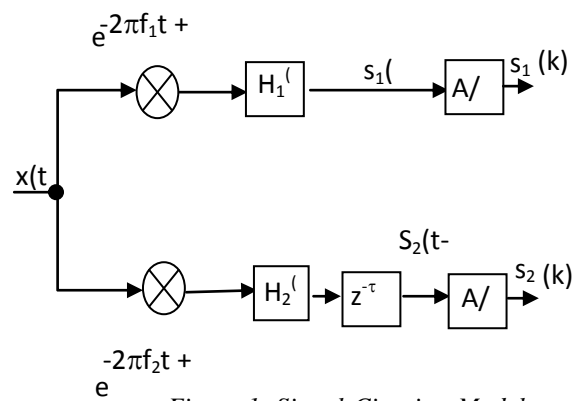


Figure 1: Signal-Circuitry Model

The heterodynes and filters in Figure 1 represent the receivers, while f_1 and f_2 are the frequencies to which they are tuned. They are separated ($= f_2 - f_1$) so that the two receivers cover the entire RF band containing the signal. The delay τ is included to indicate that the receiver outputs could be delayed with respect to one another. In the simulations, the delay could be as great as the duration of several symbols.

3. Signal Processing Architectures

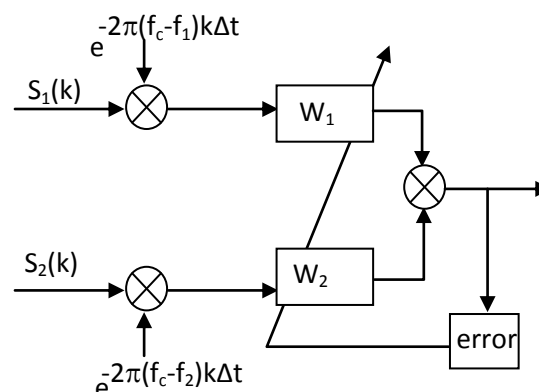


Figure 2: Model of Processing System I (Here, two weight coefficients are simultaneously being varied in order to eliminate the CMA error in the output signal)

Two signal processing architectures for processing the two outputs of the system in Figure 1 were implemented and evaluated. The first architecture is shown in Figure 2. Each input signal was heterodyned by a complex sinusoid at frequencies which differed by the difference in center frequencies ($= f_{c1} - f_{c2}$) of the receivers. The outputs of adaptive transversal equalizers were added, with the feedback taps of each equalizer adjusted to minimize the CMA-based error in the output signal.

The heterodynes shown in Figure 2 are intended to overlap the spectra of the receiver outputs by the appropriate amounts, and to center the spectrum of the combined signal at the frequency 0 Hz. The frequency f_c is an “intermediate frequency” whose purpose is to place the signal at the appropriate frequency for an existing receiver/demodulator. Generalization of the two-equalizer system of Figure 2 to an N-equalizer system capable of processing inputs from N overlapped channels is straightforward.

The system depicted in Figure 2 worked as expected, with the convergence time comparable to that for the conventional CMA algorithm. Most of the experiments were conducted using thirteen taps in each equalizer. To speed up the convergence time, an attempt was made to replace the two transversal equalizers with a combination of a lattice filter and transversal equalizer, like that shown in Ref. [3]. The equalizers would not converge in this case. The reason was that the taps in the two equalizers were updated jointly, as they would be when a single signal were being processed, but unlike the usual situation when lattice filters are employed, not all of the signals propagating through the two lattices were orthogonal.

Specifically, the signals in one lattice filter were not orthogonal to those of the other lattice filter. More precisely, System I can be represented as a linear system,

$$y = \underline{W}^T x,$$

where $\underline{W}^T = (\underline{W}_1^T, \underline{W}_2^T)$ and $x^T = (S_1^T, S_2^T)$. The vector x represents the vector output of the lattice filters. The autocorrelation function is therefore given by:

$$R_{xx} = \begin{bmatrix} R_{S_1S_1} & R_{S_2S_1} \\ R_{S_1S_2} & R_{S_2S_2} \end{bmatrix}$$

While the two lattice filters applied at the inputs to the filters, \underline{W}_1 and \underline{W}_2 , render $R_{S_1S_1}$ and $R_{S_2S_2}$ diagonal, the cross correlation matrices, $R_{S_1S_2}$ and $R_{S_2S_1}$, are not necessarily zero.

To remedy the difficulty encountered when attempting to apply lattice filters to the system in Figure 2, a second architecture for processing the outputs of multiple receivers was developed. It is illustrated in Figure 3. By adding the first signal to the filtered version of the second signal prior to applying filter \underline{W}_1 , lattice filters can be applied at the inputs to both filters, \underline{W}_1 and \underline{W}_2 . By so doing ensures that all of the outputs that multiply the equalizer coefficients are orthogonal, and the equalizer converges normally. The disadvantage of this approach is that it is more complex, both in hardware implementation and in the tap update equations, which are given in Section 5.

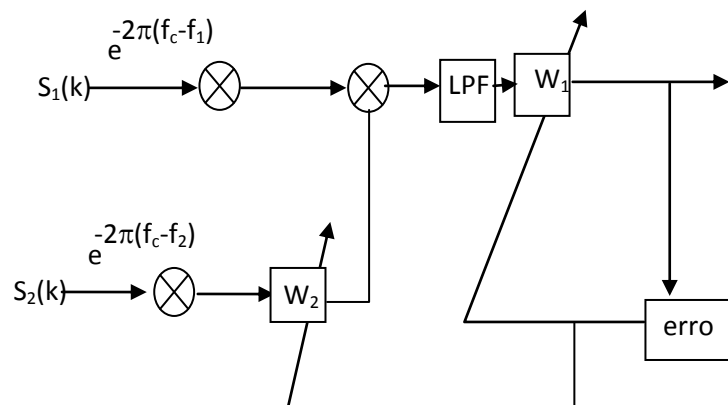


Figure 3: Processing System II block diagram (Here, two weight coefficients are separately, successively controlled in order to eliminate any error in the output signal)



The system shown in Figure 3 can be generalized to the one in Figure 4, which can be applied to outputs from N overlapping channels, for any finite arbitrary N . While no experiments were conducted for $N > 2$, and while the tap update equations become more complicated as N increases, the previous argument about orthogonalization of filter outputs applies also to the system of Figure 4.

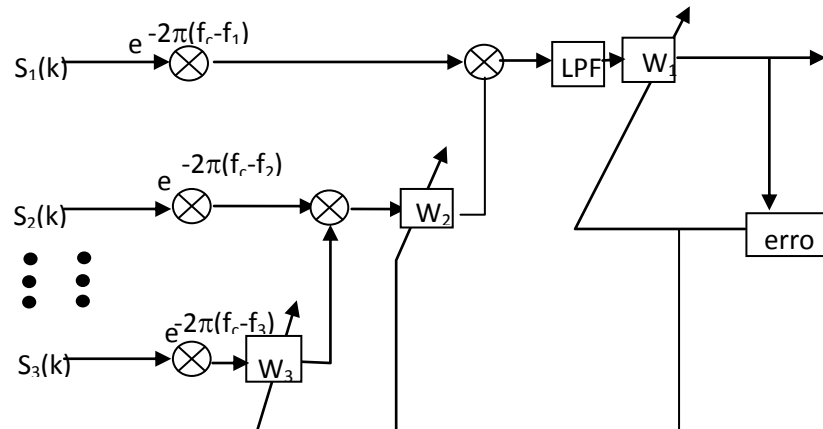


Figure 4: Improved, General Block Diagram of Processing System II (In which, three weight coefficients are separately, successively varied to eliminate any CMA error in the output signal).

4. Results and Discussions

Very little frequency offset between $S_1(t)$ and $S_2(t)$ can be compensated by the adaptive filters in either system. Such offsets manifest themselves as oscillations in the mean square error of the CMA, with the frequency of the oscillations varying in proportion to the magnitude of the offset. These oscillations are depicted in Figure 5 for a frequency offset of 0.1 percent of the symbol rate. To eliminate these frequency offsets, the value of the heterodyne frequency f_c can be adjusted for one of the receiver outputs to eliminate the oscillations, although this was not done in the experiments. Such an adjustment offset is not too great. In the experiments, the equalizer could compensate for a frequency offset of about 0.01 percent of the symbol rate.

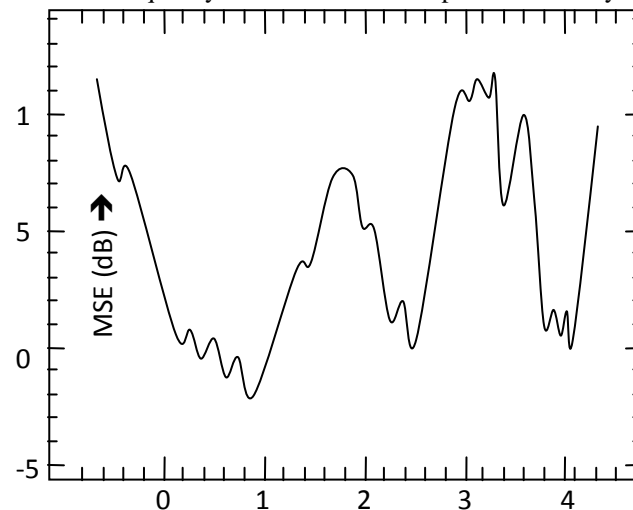


Figure 5: Oscillations in mean squared error due to frequency offset

Figure 6 shows an overlay of two overlapping spectra that represent two receiver outputs. The signal that was filtered to obtain these spectra is a 16-QAM. The SNR is 30dB. Figure 7 shows the constellation after the CMA had converged and when the delay between inputs to the system in Figure 4 was one symbol. Both systems perform well when the delay τ is greater than the duration of a symbol. In system II, the equalizers had converged by approximately 2400 symbols, which is roughly the convergence for the conventional (lattice-



based) orthogonal CMA algorithm [3-4]. In the tests, the convergence time was approximately the same for SNR = 30dB and SNR = 20dB.

dB

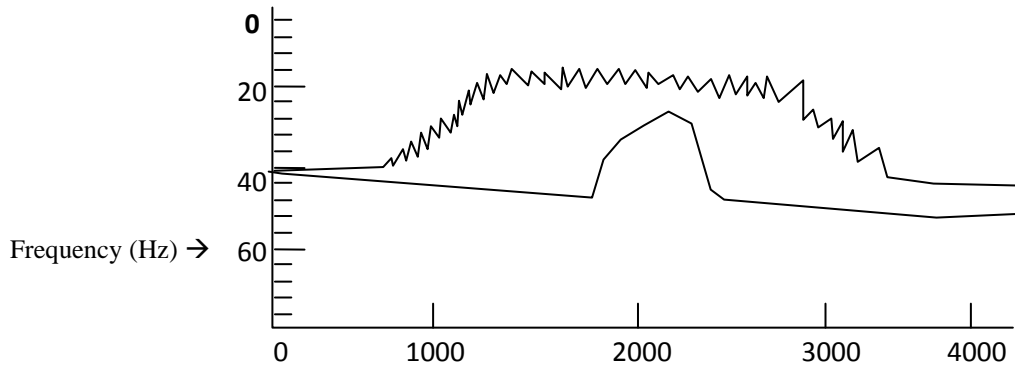


Figure 6: Overlapped spectra covering signal band

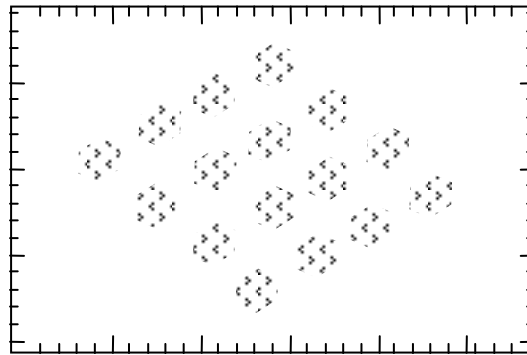


Figure 7: Constellation after CMA convergence delay between signal inputs s_1 and s_2 is one symbol

5. Tap Update Equations

The steepest (gradient) descent algorithm was used to adapt the filter weights for the first two-channel system (Figure 2). The tap update equations are very similar to those in the case of one signal and one equalizer. The combined output $\{y(k)\}$ of the adaptive filters is given by:

$$y(k) = \sum_{i=-n}^n w_{1i} s_1(k-i) + \sum_{i=-m}^m w_{2i} s_2(k-i), \quad (\text{I})$$

Where: $\underline{W}_1 = \{W_{1i}\}$, $\underline{W}_2 = \{W_{2i}\}$ represent the complex tap weights of the two filters. The constant Modulus error function at time k is:

$$E^2(k) = (|y(k)|^2 - 1)^2, \quad (\text{II})$$

Assuming reasonably that the signals $S_1(t)$ and $S_2(t)$ have been appropriately normalized.

In the gradient descent method, the tap weights $\underline{W}_1 = \{W_{1i}\}$, $\underline{W}_2 = \{W_{2i}\}$ are updated at time $k+1$ according to the formulas:

$$\underline{W}_1(K+1) = \underline{W}_1(K) - \mu_1 \nabla_{\underline{W}_1}(k), \quad (\text{III})$$

and

$$\underline{W}_2(K+1) = \underline{W}_2(k) - \mu_2 \nabla_{\underline{W}_2}(k). \quad (\text{IV})$$

Where: $\nabla_{\underline{W}_1}(k)$ and $\nabla_{\underline{W}_2}(k)$ are the gradients of E^2 with respect to \underline{W}_1 and \underline{W}_2 , respectively, at time k , and μ_1 , μ_2 are the adaptation gains.

The gradients [2] are given by:

$$\nabla_{\underline{W}_1}(k) = -E y^T S_1^*; \quad (\text{V})$$

and

$$\nabla_{\underline{W}_2}(k) = -E y^T S_2^*; \quad (\text{VI})$$

where: “T” denotes “transpose” and “*” denotes “complex conjugate”.



>> Thus, tap update equations for System I become:

$$\underline{W}_1(k+1) = \underline{W}_1(k) + \mu_1 E(k) y(k)^T S_1^*(k), \tag{VII}$$

$$\underline{W}_2(k+1) = \underline{W}_2(k) + \mu_2 E(k) y(k)^T S_2^*(k). \tag{VII}$$

The tap update equations for system II (Figure 3) are considerably more complicated, since the system is non-linear.

- The tap update equation for register 1 tap(s) in Figure 3 is the standard one, given as follows:

$$\underline{W}_1(k+1) = \underline{W}_1(k) + \mu_1 E y(k) x^*(k), \tag{VIII}$$

Where x denotes the input to the filter

- The update equation for the taps associated with register 2 is:

$$\underline{W}_2(k+1) = \underline{W}_2(k) - \mu_2 \nabla_{\underline{W}_2}(k), \tag{IX}$$

>>The gradient vector $\nabla_{\underline{W}_2}(k)$ consists of the partial derivatives $(\partial E^2 / \partial \text{Re}(\underline{W}_2))$, and $\partial E^2 / \partial \text{Im}(\underline{W}_2)$. These quantities are given by:

$$\frac{\partial E^2}{\partial \text{Re}(\underline{W}_2)} = 4 \{ \text{Re}(y(k)) \cdot \text{Re}(\underline{W}_1^T(k) S_2(k)) + \text{Im}(y(k)) \cdot \text{Im}(\underline{W}_1^T(k) S_2(k)) \} E; \tag{X}$$

$$\frac{\partial E^2}{\partial \text{Im}(\underline{W}_2)} = 4 \{ \text{Re}(y(k)) \cdot \text{Im}(\underline{W}_1^T(k) S_2(k)) - \text{Im}(y(k)) \cdot \text{Re}(\underline{W}_1^T(k) S_2(k)) \} E. \tag{XI}$$

Therefore,

$$\nabla_{\underline{W}_2}(k) = 4 \cdot E \cdot y(k) \cdot (\underline{W}_1^T(k) S_2(k))^* \tag{XII}$$

And the tap update equation for

$$\underline{W}_2 \text{ is: } \underline{W}_2(k+1) = \underline{W}_2(k) - \mu_2 E \cdot y(k) \cdot (\underline{W}_1 S_2(k))^*. \tag{XIII}$$

Note that the product $\underline{W}_1^T S_2$ is not available directly in Figure 3. Figure 8 shows the computations required for the adaptive filtering.

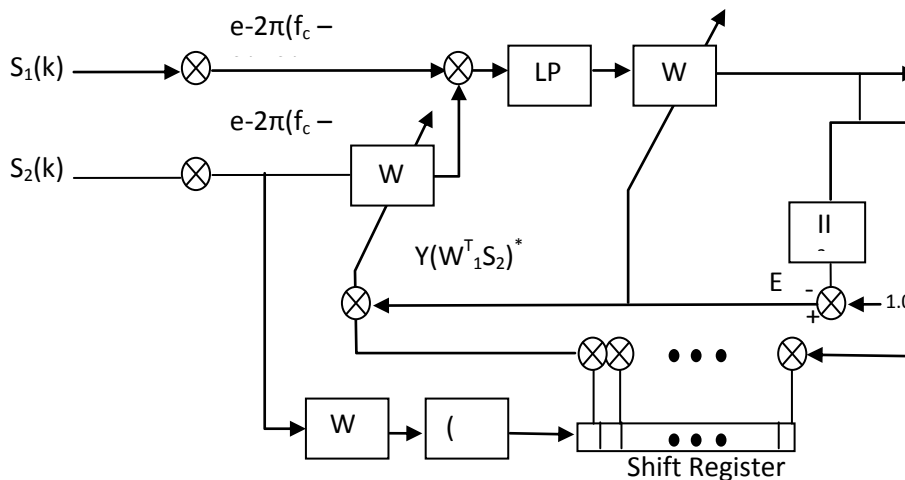


Figure 8: Detailed processing flow block diagram – System II

The additional filtering (by \underline{W}_1) associated with system II is its major disadvantage. For the three-receiver case, three new filtering operations are required, for a total of six. In general, when going from N-1 to N receivers, N new filtering operations are introduced, so that the number of filtering operations is $N \cdot (N + 1) / 2$. Clearly, the system quickly becomes prohibitively complex as N increases.

6. Concluding Remarks

- ⇒ Thus, it has been demonstrated that a wideband signal can be reconstructed from the outputs of spectrally overlapped, narrowband N ($= 2, 3, \dots$) receivers.
- ⇒ Using adaptive transversal equalizers, orthogonal filters and a modified version of the constant modulus algorithm, we obtain a useful wideband signal constructed from the outputs of

overlapped, delayed, narrowband N receivers, when the received signal is of type QPSK with 16-QAM [5-6].

- ⇒ Tap update equations are derived for the following cases:
 - One narrowband signal, one equalizer system;
 - Two narrowband signals, one-loop error corrector processing system I;
 - Two narrowband signals, one-loop error corrector processing system II;
 - Three narrowband signals, two-loop error corrector processing system II; and
 - The combined output $y(k)$ signal.
- ⇒ The processing system design block diagram becomes highly complex as the number of tapped input signal lines (N) increases.
- ⇒ These reconstructed wideband signals improve performance of equalizers, improve signal-to-noise ratio (SNR) and are utilized for research and trouble-shooting programs or activities.

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